Høsten 2016

# FYS100 Fysikk
# Re-Exam
# Solutions

Exam number **must** be written on every sheet.

The problem set is composed of 5 problems, that all need to be solved for a full score.
The standard formula sheet for FYS100 Fysikk is part of this problem set.
The problems are also attached in Norwegian.

Hints:
- Don't panic! - Read all the problems through first. - Do the easy parts first. - Include units on your answers. - 3 significant figures in the results. - Don't plug in numbers until the end. - Don't panic!!

You **must** draw sketches illustrating the problems and their solution!

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Below, you will find solutions, as well as some of the most common mistakes.

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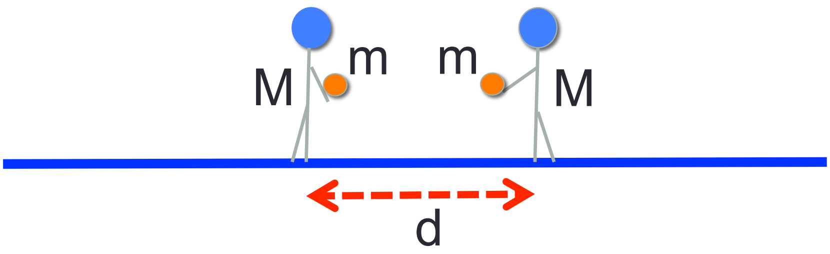
## Problem 1: Get off the lake! (Hand-in problem) (20 points)




*Two men, each of mass $M = 75.0$ kg, find themselves standing on an icy lake, a distance $d = 5.00$ m apart. They each happen to carry a basketball, weighing $m = 0.600$ kg. In order to move, they get the clever idea of throwing the balls to each other. They throw with the same speed $v = 10.0$ m/s, at the same angle with horizontal, $\theta$, and at the same time. We also assume that they somehow manage for the balls to not hit each other in mid-air, and that they throw and catch the balls at the same height above the ground. There is gravity, $g = 9.80$ m/s$^2$.*

a) *With what angle $\theta$ should they throw, in order to hit the other guy? Is there more than one solution?*

**Solution :** As soon as they throw, they start moving. If it takes the ball $\Delta t$ to make the journey, it will have to cover a distance $d + v_x^g \Delta t$, where $v_x^g$ is the speed of the receiving guy, after he has thrown his own ball. So we have

$$v_x^b \Delta t = d + v_x^g \Delta t, \qquad \Delta t = \frac{d}{v\cos\theta - v_x^g}. \tag{1}$$

where $v_x^b = v\cos\theta$ is the x-component of the ball velocity. Momentum conservation in the x-direction of guy + ball tells us that

$$v\cos\theta m = M v_x^g \rightarrow v_x^g = \frac{m}{M} v\cos\theta. \tag{2}$$

We also need the range equation, so that

$$\frac{v^2}{g}\sin 2\theta = v_x^b \Delta t. \tag{3}$$

Solving the whole system of equations, we find

$$\theta = \frac{1}{2}\sin^{-1}\frac{gd}{v^2(1-m/M)} = 14.8^\circ \text{ or } 75.2^\circ. \tag{4}$$

Which makes sense, since in the limit $m = 0$, the guys don't move and the distance is just $d$. And when $m = M$, the guy is moving as fast as the ball, and it will never catch up.

b) *How far are the two guys apart, when they receive the balls?*

**Solution :** Either way, they are at a distance

$$d + 2v_x^g \Delta t = 2\frac{v^2}{g}\sin 2\theta - d = 5.08 \text{ m}. \tag{5}$$

c) *What is their speeds after they receive the balls?*

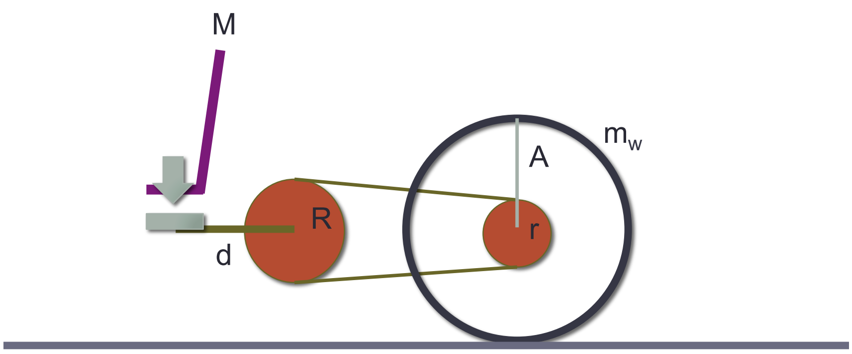
**Solution :** Momentum conservation tells us that $mv_x^b + Mv_x^g = (m+M)v_f$, so that

$$v_f = \frac{mv_x^b + Mv_x^g}{m+M} = \frac{2m}{m+M}v_x^b = \frac{2m}{m+M}v\cos\theta = 0.153 \text{ or } 0.0405 \text{ m/s}. \tag{6}$$

*For each question, provide an algebraic expression, a sketch and a numerical result.*

**Typical mistakes:** This was a known hand-in problem, and the majority of you did it right, or at least remembered what you were supposed to do. Mistakes were typically putting in the numbers wrong, forgetting one of the solutions to the angle in b) and/or the final speed in c). Some of you did very little, and for those, it was mostly not realising that momentum conservation had to be taken into account. In total, 64% of available points were given.

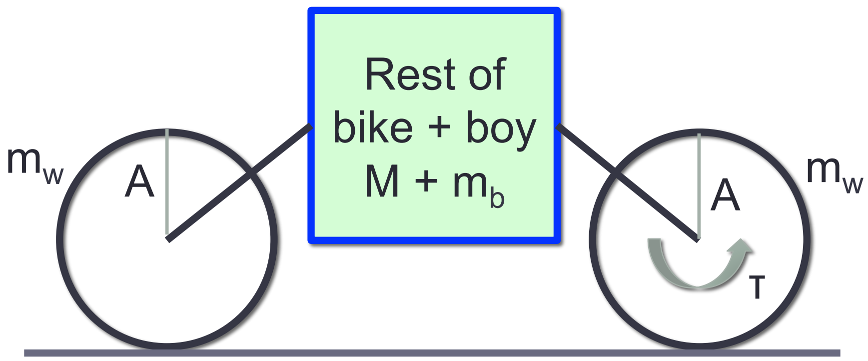
## Problem 2: Bike-boy (20 points)



*A boy of mass $M$ is on his bike going along a horizontal road, and puts all his weight on the pedal to force the wheel round. The pedal is attached by a rod of length $d$ to the center of a cog wheel of radius $R$. Which is in turn connected by a chain to a smaller cog wheel of radius $r$, which is attached to the back wheel (as on a standard bike). All rods, chains and cog wheels can be taken to be massless. There is no kinetic friction, and everything is turning without sliding. There is gravity $g$.*

a) *If the boy stands on the pedal when it is at "9 o'clock" (see figure), what is the torque provided to the back wheel? The bike starts from rest.*

**Solution:** The torque of the pedal on the first cog wheel is $\tau = Mgd$. That becomes a force along the chain of $F = \tau/R = Mgd/R$. This force is transported by the chain to give the torque $Mgrd/R$ around the back wheel.




*The bike itself is taken to have mass $m_b$ plus the mass of its two wheels, each providing $m_w$, plus $M$ for the boy. The wheels have radius $A$ and the moment of inertia of a ring $I = m_w A^2$.*

b) *Assuming that the back wheel and front wheel roll without sliding on the ground, what is the acceleration of the bike + boy? Hint: Consider the back wheel, front wheel and middle part (bike+boy) as sub-systems.*

**Solution:** Now we have to write down the torque and force equations for each system. The force between back-wheel and middle part is $f_3$; between front wheel and middle part $f_4$. The friction forces on the front and back wheels are $f_2$ and $f_1$, respectively (one acting forwards, the other backwards!). With a sensible choice of positive directions of rotation, and using Newton 3., we get the set of 5 equations, with in addition the no-slipping condition $a = A\alpha$

(both wheels have same radius):

$$(M + m_b)a = f_3 - f_4, \tag{7}$$
$$m_w a = f_1 - f_3, \tag{8}$$
$$m_w a = f_4 - f_2, \tag{9}$$
$$I\alpha = \tau - f_1 A, \tag{10}$$
$$I\alpha = f_2 A. \tag{11}$$

Adding all these up on the left- and right-hand sides, all the forces $f_{1-4}$ cancel out, and we get

$$a = \frac{\tau}{A}\left(\frac{1}{M + m_b + 4m_w}\right) = Mg\frac{dr}{RA}\left(\frac{1}{M + m_b + 4m_w}\right). \tag{12}$$

c) *Assuming he started at rest, how fast is he going by the time he gets the pedal to position "6 o'clock"? Ignore all sources of kinetic friction.*

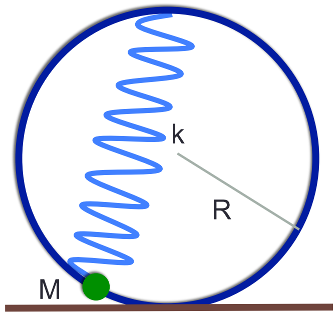
**Solution:** We use energy conservation. The boy releases gravitational potential energy, which is transformed into kinetic energy of the bicycle. We write (remembering both translational and rotational kinetic energy contributions)

$$Mgd = \frac{1}{2}(M + m_b + 2m_w)v^2 + 2\frac{1}{2}I\omega^2 = \frac{1}{2}\left(M + m_b + 4m_w\right)v^2 \tag{13}$$
$$\rightarrow v = \sqrt{\frac{2Mgd}{M + m_b + 4m_w}}. \tag{14}$$

*For each question, provide an algebraic expression and a sketch.*

**Typical mistakes:** This apparently was challenging, although we have done rolling of wheels with and without torques from engines at lecture. Only a handful understood how to transport the torque that the boy gives to the cog-wheel, via the chain to the back wheel. For b) many of you tried invoking forces and torques, but most forgot the static friction with the road, responsible for the overall acceleration and torques on the two wheels. In c), some realised that energy conservation was in play, but almost no-one included the rolling energy and translational energy of all the components. In total, 17 % of available points were given for this problem.

## Problem 3: Bead it! (20 points)




*Note: Similar, but not the same as hand-in problem.*

*A bead of mass $m$ is sliding on a ring of radius $R$, the ring standing on its side. A spring is attached to the top of the ring at one end, and to the bead at the other end. The spring has spring constant $k$ and we will assume that the equilibrium point of the spring is when it is unstretched. There is gravity, $g$.*

a) *If the bead starts from rest (almost) exactly at the bottom (at 6 o'clock), what is its speed when it reaches the top (at 12 o'clock)?*

**Solution:** Spring potential energy is converted into gravitational potential energy and kinetic energy. We write

$$\frac{1}{2}Mv^2 = \frac{1}{2}k(2R)^2 - Mg(2R) \to v = 2\sqrt{\frac{k}{M}R^2 - gR} \tag{15}$$

b) *Find a requirement on $k$, $M$, $g$ and $R$ for the bead getting to the top at all.*

**Solution:** Obviously, this only works if the inside of the sqrt is positive, so

$$\frac{k}{M}R^2 - gR > 0 \to \frac{kR}{Mg} > 1. \tag{16}$$

c) *Find the potential energy function $E(\theta)$, as a function of the angle along the ring, including both gravitational and spring potential energy. We take $\theta = 0$ when the bead is at the bottom of the ring. Find the stable equilibrium point, depending on whether the criterion in b) is fulfilled or not.*

**Solution:** Now we need to find the height as a function of $\theta$ (for the gravitational potential) and the distance from the top as a function of $\theta$ (for the spring length). We have

$$E(\theta) = Mgh(\theta) + \frac{1}{2}kd(\theta)^2 = MgR(1-\cos\theta) + \frac{1}{2}kR^2(2+2\cos\theta) \quad (17)$$

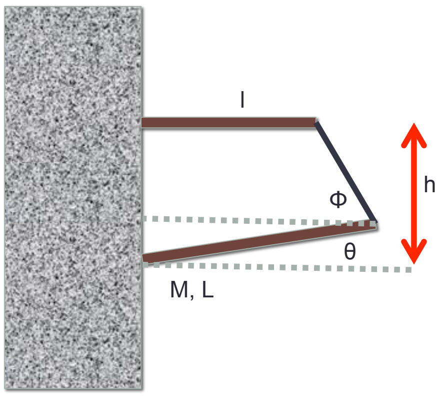
$$= MgR\left[\left(\frac{kR}{Mg}+1\right) + \left(\frac{kR}{Mg}-1\right)\cos\theta\right]. \quad (18)$$

This has extrema at $\theta = 0$ and $\theta = \pi$. If $kR/Mg > 1$, the potential has a minimum at the top of the ring; if $kR/Mg < 1$, potential has a minimum at the bottom of the ring.

*For each question, provide an algebraic expression and a sketch.*

**Typical mistakes:** Getting the sign of one of the energy terms wrong, so that there is a sum inside the sqrt, rather than a difference. That of course made b) quite hard to get right. A bunch of factors of two in the two terms also got lost here and there. Some got the equality sign the wrong way around in b). In c) nobody got the exact energy function right, but quite a few got some of the way. In total, 40% of the available points were given.

## Problem 4: Draw a Bridge! (30 points)




*A drawbridge is composed of a wooden plank attached to the wall on a pivot at one end, and with a rope at the other end (see figure). The rope goes up at an angle of $\phi$ with horizontal, and is attached to a horizontal wooden beam, a height $h = 3.00$ m above the pivot. The bridge in turn makes an angle of $\theta$ with horizontal, as it is pulled up. The plank is $L = 5.00$ m long, is uniform and weighs $M = 200$ kg. The rope is taken to be massless. The wooden beam is $l = 4.00$ m long. There is gravity $g = 9.80$ m/s$^2$.*

a) *First consider the case, where the bridge is horizontal ($\theta = 0$). Make a sketch of all the relevant forces acting on the bridge. Find an expression for $\phi$ in terms of the other quantities.*

**Solution:** There is gravity $Mg$ in the middle of the bridge, there is a pivot force at the left-hand end $F = (F_x, F_y)$ and a string tension $T$ along the rope. The relation for $\phi$ follows from trigonometry

$$\tan\phi = \frac{h}{L-l} = 3. \tag{19}$$

b) *Given that the bridge is in static equilibrium, find the magnitude of the rope tension $T$ and of the pivot force $F$.*

**Solution:** Write down force equations along $x$ and $y$,

$$F_x = T\cos\phi, \tag{20}$$

$$F_y + T\sin\phi = Mg, \tag{21}$$

and a torque equation, most conveniently around the pivot

$$Mg\frac{L}{2} = TL\sin\phi. \tag{22}$$

Solving for $F_x$, $F_y$ and $T$, we find

$$|T| = Mg\frac{1}{2\sin\phi} = 1033\text{ N}, \tag{23}$$

$$|F| = \sqrt{F_x^2 + F_y^2} = \frac{Mg}{2}\sqrt{1 + \frac{1}{\tan^2\phi}} = Mg\frac{1}{2\sin\phi} = 1033\text{ N}. \tag{24}$$

*The bridge can now be pulled up, to a maximum angle $\theta_{max}$ where the end touches the wooden beam above it.*

c) *Solve for the magnitude of the tension and the pivot force, for a general $\theta$ and $\phi$.*

**Solution:** Keeping in mind how the angles are defined, we have the same force equations

$$F_x = T\cos\phi, \tag{25}$$

$$F_y + T\sin\phi = Mg, \tag{26}$$

but the torque equation is a little different

$$Mg\frac{L}{2}\cos\theta = TL\sin(\theta+\phi). \tag{27}$$

Solving for $F_x$, $F_y$ and $T$, we find

$$|T| = Mg\frac{\cos\theta}{2\sin(\theta+\phi)}, \tag{28}$$

$$|F| = Mg\sqrt{\left(\frac{\cos\theta\cos\phi}{2\sin(\theta+\phi)}\right)^2 + \left(1-\frac{\cos\theta\sin\phi}{2\sin(\theta+\phi)}\right)^2}. \tag{29}$$

d) *Find a relation between $\theta$ and $\phi$ as the bridge is raised.*

**Solution:** From geometry, one finds

$$\tan\phi = \frac{h - l\sin\theta}{L\cos\theta - l}. \tag{30}$$

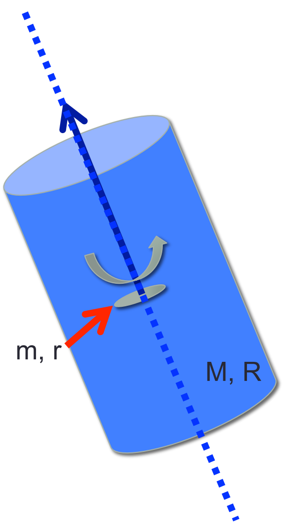
e) *By inserting some numbers, make a sketch of the rope tension $T(\theta)$ and find its maximum value on the interval $\theta \in [0, \theta_{max}]$.*

**Solution:** One quickly realises that $T$ decreases with increasing $\theta$, and so the maximum magnitude is the one we found in b), 1033 N.

*For each question, provide an algebraic expression, a sketch, and a number.*

**Typical mistakes:** Forgetting the pivot force. Thinking that it only goes in the x or only in the y-direction. Thinking that the beam above is somehow involved in the dynamics. Not writing down the torque equations. Few got to the right number, but some got to a number, using additional (incorrect) assumptions. 18% of the available points were given.

## Problem 5: Discs. In. Space! (10 points)



*A spaceship is travelling with engines off far away from everything. Inside, a CD-player suddenly goes on, spinning the CD with 4 revolutions per second. The spaceship (not including the CD) can be thought of as a hollow cylinder ($I = MR^2$) of radius $R = 1.00$ m and mass 1000 kg. The CD is a uniform disc ($I = mr^2/2$) of mass $m = 15.0$ g and radius $r = 6.00$ cm. The axis of the spinning CD coincides with the symmetry-axis of the cylinder, and it is spinning according to the right-hand-rule around this axis.*

a) *In what direction and with what angular speed does the spaceship start rotating? The spaceship is assumed to not spin initially. Hint: Yes, it is very small.*

**Solution :** Angular momentum is conserved and so $L_{CD} + L_{Ship} = 0$. We write:

$$I_{CD}\omega_{CD} = -I_{Ship}\omega_{Ship} \rightarrow \omega_{Ship} = -\frac{mr^2/2}{MR^2}\omega_{CD} = \frac{0.015 \times 0.06^2}{2 \times 1000 \times 1^2} 8\pi = 6.79 \times 10^{-7}\, s^{-1}. \tag{31}$$

*Provide an algebraic expression, a sketch, and a number.*

**Typical mistakes:** Putting in numbers wrong. Using energy conservation instead of angular momentum conservation. Thinking that the space-ship starts turning in the same direction as the CD. In total, 43% of the points available were given.