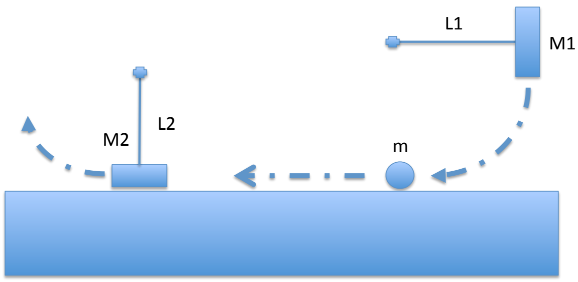
Høsten 2017

# FYS100 Fysikk Eksamen/Exam Solutions

## Problem 1: Plok-plump! (ca. 25 %)




*A hammer of mass $M_1 = 2.00$ kg on a massless stick of length $L_1 = 30.0$ cm is nailed to a wall, so that it is free to turn around the axis (se Figure). Starting at rest from the horizontal position, it swings down and hits a ball of mass $m = 0.100$ kg in an elastic collision. The ball is initially at rest, and when hit proceeds to slide frictionlessly along the table until it reaches a second hammer of mass $M_2 = 1.50$ kg on a massless stick of length $L_2 = 20.0$ cm. In this collision, the ball attaches itself to the hammer, and the hammer and ball swing up to a height h. There is gravity $g = 9.80$ $m/s^2$.*

a) *What is the speed of the ball as it slides along the table?*

**Solution:** There is energy conservation as the hammer swings down, so that its speed at the bottom is

$$\frac{1}{2}M_1v_1^2 = M_1gL_1 \rightarrow v_1 = \sqrt{2gL_1}. \tag{1}$$

Then it performs an elastic collision with the ball, with the ball initially at rest, so

$$mv_b + M_1v_h = M_1v_1, \tag{2}$$

$$\frac{1}{2}mv_b^2 + \frac{1}{2}M_1v_h^2 = \frac{1}{2}M_1v_1^2. \tag{3}$$

Solving this gives

$$v_b = \frac{2v_1}{1+\frac{m}{M_1}} = \frac{\sqrt{8gL_1}}{1+\frac{m}{M_1}} = 4.62 \text{ m/s}. \tag{4}$$

b) *What is the height h?*

**Solution:** The second collision is completely inelastic, so that

$$(M_2+m)v_2 = mv_b \rightarrow v_v = \frac{m}{M_2+m}v_b. \tag{5}$$

(Mechanical) Energy is not conserved in an inelastic collision.

Then energy conservation applies as the hammer swings up

$$(M_2+m)gh = \frac{1}{2}(M_2+m)v_2^2 \rightarrow h = \frac{1}{2g}v_2^2 = 4L_1\frac{m^2}{(M_2+m)^2}\frac{M_1^2}{(M_1+m)^2}, \tag{6}$$

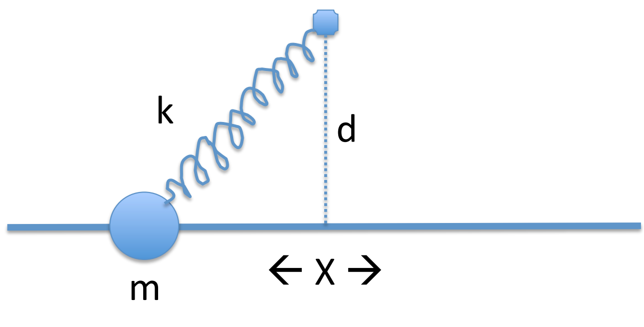
which for the numbers given is 4.25 mm.

c) *How much of the initial energy in the hammer $M_1$ ends up in the hammer/ball ($M_2+m$) system at the end (as mechanical energy)?*

**Solution :** Simply compute the ratio of potential energies at the beginning and at the end:

$$\frac{(M_2+m)gh}{M_1gL_1} = \frac{4m^2M_1}{(M_1+m)^2(M_2+m)} = 0.0113 \tag{7}$$

## Problem 2: Bead-on-a-stick (ca. 25 %)




*A bead on a horizontal stick is connected to a nail by a spring. The bead has mass m, the spring has spring constant k and the vertical distance from the* *stick to the nail is d. We take the unstretched equilibrium length of the spring to be very small, relative to d. We define an x-axis along the stick, with $x = 0$ directly below the nail. There is gravity, g.*

a) *Draw a force diagram of the system. Find, as a function of x, the force on the bead from the spring 1) in the direction along the spring and 2) in the direction along the stick.*

**Solution:** The string force is given by how much it is stretched according to Stoke's Law. From Pythagoras, and using the information that the equilibrium length is "very small"$\simeq 0$, we have 1)

$$F_s = -kL = -k\sqrt{d^2 + x^2}. \tag{8}$$

Along the stick, we just have to project the spring force on the direction of the stick, to find

$$F = F_s \frac{x}{L} = -kL\frac{x}{L} = -kx. \tag{9}$$

b) *Is the motion of the bead along the stick like that of a harmonic oscillator, and if so, what is the angular frequency of the motion?*

**Solution:** Yes, it is a harmonic oscillator, because the force along the stick is $F = -kx$ which gives a Newton's 2. law

$$m\frac{d^2x}{dt^2} = -kx. \tag{10}$$

The angular frequency is $\omega = \sqrt{k/m}$ as usual.

*The system is now tilted by 90 degrees, so that the stick is vertical.*

c) *Find the potential function $U(x)$ corresponding to the motion up and down the stick. Find the equilibrium point(s).*

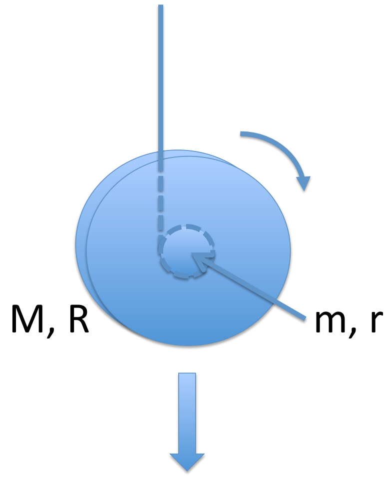
**Solution:** The potential function is composed of two parts, potential energy of gravity and potential energy of the spring. We have

$$U(x) = mgx + \frac{1}{2}kL^2 = mgx + \frac{1}{2}kx^2 + \frac{1}{2}kd^2. \tag{11}$$

The equilibrium point is when the derivative with respect to $x$ is zero. We have

$$-\frac{dU}{dx} = -mg - kx = 0 \rightarrow x = -\frac{mg}{k}. \tag{12}$$

## Problem 3: Yo! Yoda! Yo-Yo! (ca. 30 %)




*A yo-yo roughly speaking consists of two round, uniform discs, sandwiched around a third smaller disc. A string is wound around the middle disc, and so the yo-yo may roll up and down as the string winds and unwinds. Consider such a yo-yo, with the two bigger discs having radius $R = 4.00$ cm and mass $M = 30.0$ g each; and the smaller disc in the middle having radius $r = 0.700$ cm and mass $m = 5.00$ g. The string is taken to be massless, and infinitely thin. The moment of inertia of a disc of mass $m$ and radius $r$ is $I = mr^2/2$. There is gravity $g = 9.80$ $m/s^2$.*

a) *What is the total moment of inertia of the yo-yo, around an axis going through the centre of the discs?*

**Solution:** Each big disc has $MR^2/2$ and the small one $mr^2/2$. Add them up to get

$$I = 2 \times MR^2/2 + mr^2/2 = 4.81 \times 10^{-5} \text{ kg m}^2. \quad (13)$$

*The end of the string is now fastened to something at a fixed position (like a finger), and the yo-yo is let drop towards the floor.*

b) *Identify the forces acting on the yo-yo, and for each, indicate whether they provide torque, work, impulse and/or acceleration to the yo-yo.*

**Solution:** The finger acts as a normal force on the string. The string tension acts on the finger. The (end of the) string doesn't move so no work, impulse, acceleration is given to it.
The string tension acts on the yo-yo ultimately via the static friction at the string-inner-disc interface, which we may model as a force acting tangentially at the edge of the inner disc. This force provides a contribution to the

acceleration; a torque relative to the CM; and impulse; but no work, since the point of application does not move relative to the string (it is static, not kinetic friction).
Gravity works at the centre of mass of the yo-yo. It contributes to the acceleration, the impulse and does work, but does not give any torque relative to the CM, because the lever arm is zero.

c) *What is the acceleration of the yo-yo downwards? What is its angular acceleration? How large is the string force?*

**Solution:** We can now write down the relevant equations, focusing on linear acceleration of the CM $a$ and angular acceleration $\alpha$.

$$(2M+m)a = F_g - T, \tag{14}$$
$$I\alpha = Tr, \tag{15}$$
$$a = r\alpha, \tag{16}$$

where the last equation is the rolling without sliding criterion. Using $F_g = (2M+m)g$, we can solve for $a$, $\alpha$ and $T$

$$a = \frac{2M+m}{M\left(2+\frac{R^2}{r^2}\right)+\frac{3}{2}m}g = 0.608 \text{ m/s}^2, \tag{17}$$
$$\alpha = \frac{a}{r} = 86.9 \text{ rad/s}^2, \tag{18}$$
$$T = (2M+m)(g-a) = 0.597 \text{ N}. \tag{19}$$

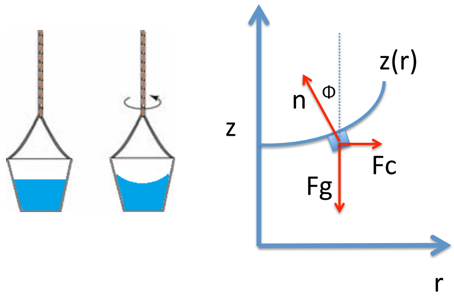
d) *How big a fraction of the total kinetic energy goes into the rotating motion?*

**Solution:** When at some point the speed of the CM is $v$, we have

$$E_{kin,tot.} = E_{kin,CM} + E_{kin,rot.} = \frac{1}{2}(2M+m)v^2 + \frac{1}{2}\frac{I}{r^2}v^2, \tag{20}$$
$$\frac{E_{kin,rot.}}{E_{kin,tot.}} = \frac{1}{1+\frac{(2M+m)r^2}{I}} = 0.938. \tag{21}$$

We have used the rolling without sliding condition $v = r\omega$ (the string winds around the inner disc).

## Problem 4: Spinning water! (ca. 20 %)




*Consider a bucket of water spinning with constant angular speed $\omega$. We are interested in finding the shape of the water's surface, $z(r)$. We do this by first noting that in the rotating frame of the bucket, a small water element of mass $m$ on the surface of the water is in static equilibrium. The figure shows the three forces acting on the water element: Centrifugal force $\mathbf{F_c}$, gravity $\mathbf{F_g}$ and the normal force $\mathbf{n}$ of the rest of the water on the water element.*

a) *Find the angle $\phi$ as a function of $r$, the distance from the axis of rotation.*

**Solution:** We write down stability equations in x and y

$$F\cos\phi = mg, \tag{22}$$
$$F\sin\phi = m\omega^2 r, \tag{23}$$

which gives

$$\tan\phi = \frac{\omega^2 r}{g}. \tag{24}$$

*We are interested in the curve describing the water surface height, $z(r)$.*

b) *Given that the normal force is perpendicular to the surface, express the slope $dz/dr$ in terms of $\phi$.*

**Solution:** The angle $\phi$ appears so that the slope is $\tan\phi$. Hence, we have

$$\frac{dz}{dr} = \tan\phi. \tag{25}$$

c) *Combine the results of a) and b) to find $z(r)$. Does it make sense, and why?*

**Solution:** We solve the differential equation by simple integration:

$$\frac{dz}{dr} = \frac{\omega^2 r}{g} \to z(r) = \frac{\omega^2}{2g} r^2. \tag{26}$$

This makes sense because: Units match. Larger $\omega$ gives larger slope. Larger $g$ gives smaller slope.