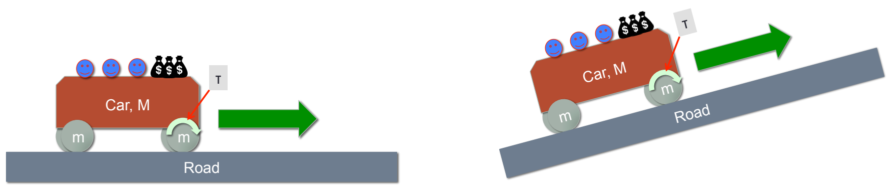
Høsten 2017

FYS100 Fysikk
Konte-Eksamen/Re-Exam
Solutions and comments

Problem 1: Bank robbers! (25)



Figur 1: a) + b) On flat road. c) Up an incline.

Three bank robbers have robbed a bank, and are now anxious to get away before the police arrive. They jump into their getaway car, and floor the accelerator. As a result, the engine provides a torque $\tau = 200$ Nm to each of the two front wheels. Each of the four wheels of the car has mass $m = 15.0$ kg, while the rest of the car (including robbers and money, but not the wheels) has mass $M = 1000$ kg. The wheels can be taken to have the moment of inertia of a uniform disc, and have a radius $R = 35.0$ cm. The wheels roll without slipping on the ground.

a) *What is the acceleration of the car?*

Solution: Split up into 3 systems (front wheels, back wheels, rest of the car). Contact forces are f_1 (between front wheels and car) and f_2 (between car and back wheels). Friction forces are $f_{1,s}$ on front wheels (positive forwards) and $f_{2,s}$ on back wheels (positive backwards). Contact forces act at the center of the wheels and friction forces at edge R. Now write down all the relevant equations:

$$\begin{aligned} a &= R\alpha, && (1)\\ I &= mR^2/2, && (2)\\ Ma &= 2f_1 - 2f_2, && (3)\\ ma &= f_{1,s} - f_1, && (4)\\ ma &= f_2 - f_{2,s}, && (5)\\ I\alpha &= \tau - f_{1,s}R, && (6)\\ I\alpha &= f_{2,s}R. && (7) \end{aligned}$$

Adding them all up, all the forces cancel, and one gets

$$a = \frac{2\tau/R}{M + 6m} = 1.05 \text{ m/s}^2. \tag{8}$$

b) *How long does it take to get to a speed of* 100 *km/h, and how far has the car gone by then? We assume that the car starts from rest.*

Solution: This is simple 1-D motion, constant acceleration

$$x = \frac{a}{2}t^2, \qquad v = at. \tag{9}$$

Solve for t and insert, to find

$$t = 26.5 \text{ s}, \qquad x = \frac{v^2}{2a} = 368 \text{ m} \tag{10}$$

c) *The car now goes up an incline, sloping at an angle of $\theta = 15.0°$. What should the torque on each of the front wheels be, so that the car continues up the hill at constant speed?*

Solution: Do the same thing as in a), but with the set of equations including gravity of car and wheels

$$\begin{aligned} a &= R\alpha, && (11)\\ I &= mR^2/2, && (12)\\ Ma &= 2f_1 - 2f_2 - Mg\sin\theta, && (13)\\ ma &= f_{1,s} - f_1 - mg\sin\theta, && (14)\\ ma &= f_2 - f_{2,s} - mg\sin\theta, && (15)\\ I\alpha &= \tau - f_{1,s}R, && (16)\\ I\alpha &= f_{2,s}R. && (17) \end{aligned}$$

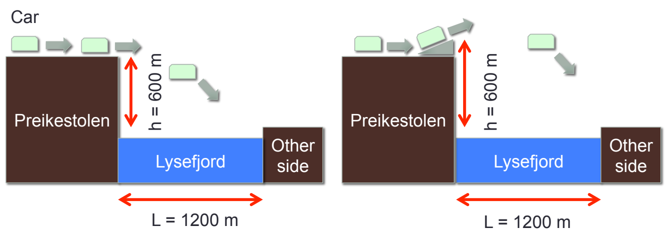
to find

$$a = \frac{2\tau/R - (M + 4m)g\sin\theta}{M + 6m}, \tag{18}$$

which is zero when $\tau = (M + 4m)gR\sin\theta/2 = 471$ Nm.

Typical Mistakes:
Not splitting up into 3 systems and writing down the force equations. Assuming that the force is just τ/R (which gives a numerical result close, but not quite right; moments of inertia don't come into play.). Using the wrong mass (no wheels). Not including the torque equations on the wheels. Not being able to eliminate the friction forces. Getting the 1-D kinematics mixed up. Computational errors.

Problem 2: Jump! (17)



Figur 2: a) No ramp, b) With ramp.

During the car chase, the robbers suddenly find themselves on Preikestolen, with the police in close pursuit. In order to shake them off, the robbers contemplate jumping across Lysefjorden. There is gravity, $g = 9.80$ km/s^2. The car still has mass $M = 1000$ kg, not including the wheels of mass $m = 15.0$ kg each.

a) *Given that Preikestolen is $h = 600$ m higher than the opposite side of the fjord, and that Lysefjorden at that point is about $L = 1200$ m wide, is it possible to get across with an initial horizontal speed of $v = 100$ km/h? What is the minimum speed required to succeed?*

Solution: Simple projectile motion with only an initial horizontal speed. The horizontal distance flown is $x = vt$. The time available is given by $h = gt^2/2$, so that

$$x = v\sqrt{\frac{2h}{g}} = 307 \text{ m}. \tag{19}$$

which is much less than 1200 m. In order to get all the way, we require

$$v = L\sqrt{\frac{g}{2h}} = 390 \text{ km/h} \quad (108 \text{ m/s}). \tag{20}$$

b) *They now build a ramp out of rocks and tourists, capable of launching the car at an angle of θ with horizontal. What is the distance, that the car can fly, as a function of v and θ (and h and g)? Will that get them across the fjord for any value of θ if the speed is $v = 100$ km/h (do this by inserting some different values of θ and making a plot)?*

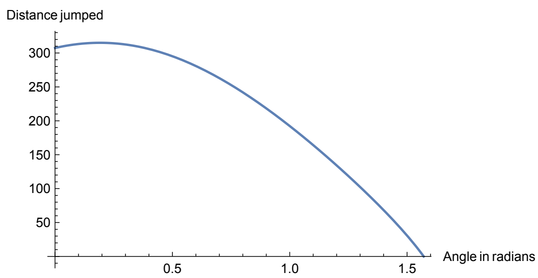
Solution: General projectile motion. Use

$$x = v\cos\theta t, \qquad y = h + v\cos\theta t - \frac{g}{2}t^2. \tag{21}$$

Solving for $y = 0$ and reinserting t in x gives

$$x = \frac{v^2}{g}\sin\theta\cos\theta\left(1 \pm \sqrt{1 + \frac{2hg}{v^2\sin^2\theta}}\right). \tag{22}$$

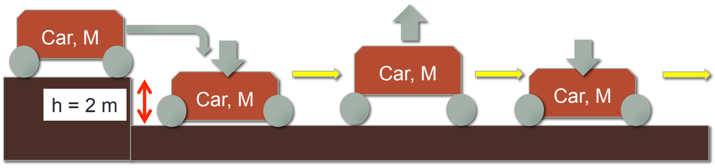
Inserting the speed and plotting the curves gives the following: No way, one



gets to 1200 m for any angle!

Typical Mistakes:
Not using projectile motion in 2 D, just 1D kinematics. Using the range equation (ignoring the height difference h). Using energy conservation. Having gravity included in both the x and y direction equation. Computational mistakes.

Problem 3: Plan B: Spring, sprang, sprung. (33)



Figur 3: a) + b) Jumps and lands. c) + d) + e) Damped harmonic oscillation.

Realising that jumping Lysefjorden is an impossible mission, the bank robbers decide on a different escape plan. They double back along the road, and as the police cars approach, make a hard right turn, over the edge of the cliff, dropping $h = 2.00$ m straight down. The car still has mass $M = 1000$ kg, not including the wheels of mass $m = 15.0$ kg each. There is gravity, $g = 9.80$ km/s^2.

a) *What is the vertical speed of the car as it lands?*

Solution: Energy conservation $(M + 4m)gh = \frac{1}{2}(M + 4m)v^2$ so that $v = \sqrt{2gh} = 6.26$ m/s.

The car suspension is composed of four springs, each with spring constant $k = 4.00 \times 10^5$ N/m, but we will think of them as a single spring with spring constant $4k$. The wheels are assumed to not deform during landing, and the equilibrium position of the springs is taken to be when the wheels first touch the ground.

b) *As the car lands, how far down from the equilibrium position do the springs squeeze? Neglect friction for now.*

Solution: The wheels just land immediately, losing all the energy to internal energy. But the car of mass M has kinetic energy that needs to be transferred to the springs. At the same time, the spring is squeezed by x. Note that it is only the car M that weighs down on the spring, not the wheels.

$$Mg(h + x) = \frac{4k}{2}x^2 \rightarrow x = \frac{Mg}{4k}\left(1 + \sqrt{1 + \frac{8kh}{Mg}}\right) = 0.168 \text{ m.} \tag{23}$$

With the springs getting squeezed on impact, the car now starts to perform a damped harmonic oscillation (we now bring friction into play). We can think of the damping as due to a resistive force of the form

$$\mathbf{R} = -b\mathbf{v}, \tag{24}$$

where $b = 4000$ kg/s.

c) *Write down the differential equation governing the oscillating motion. We choose the "amount of squeezing", $x(t)$, to be positive downwards.*

Solution: There is spring and resistive force, as well as gravity

$$Ma = -4kx - bv + Mg \rightarrow \frac{d^2x}{dt^2} + \frac{b}{M}\frac{dx}{dt} + \frac{4k}{M}x - g = 0. \tag{25}$$

d) *Show by insertion that*

$$x(t) = Ae^{-t/\tau}\cos(\omega t) + x_0, \tag{26}$$

is a solution. $t = 0$ *corresponds to when the car is at the lowest point for the first time. Find expressions for* τ, x_0 *and* ω *in terms of* b, M *and* k *and compute their values. (Hint: Insert and consider the coefficients of* $\cos\omega t$, $\sin\omega t$ *and* 1 *separately, setting them to zero and solving for* τ, x_0 *and* ω*).*

Solution: By insertion, one finds

$$0 = -Ae^{-t/\tau}\left[\left(\omega^2 - \frac{1}{\tau^2} - 4\frac{k}{M} + \frac{b}{M\tau}\right)\cos(\omega t) + \omega\left(-\frac{b}{M} + \frac{2}{\tau}\right)\sin(\omega t)\right] + \frac{4k}{m}x_0 - g \tag{27}$$

Each of the terms must be zero, and so

$$\tau = \frac{2M}{b}, \qquad \omega^2 = \frac{4k}{M} - \left(\frac{b}{2M}\right)^2, \qquad x_0 = \frac{gM}{4k}. \tag{28}$$

Inserting numbers, we find $\tau = 0.50$ s, $\omega = 40.0$ and $x_0 = 0.613$ cm.

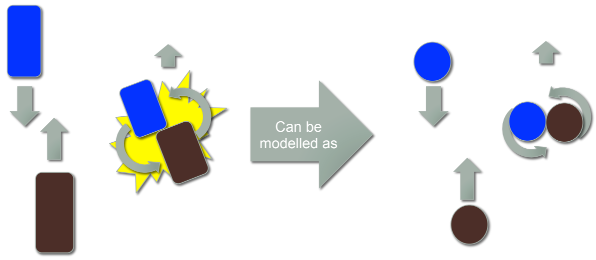
e) *How long does it take for the amplitude of oscillation to reduce to 5% of the initial one?*

Solution: That's when the exponential factor is 0.05, so

$$e^{-t/\tau} = 1/20 \rightarrow t/\tau = \log 20 = 3.00 \rightarrow t = 1.50 \text{ s} \tag{29}$$

Typical mistakes:
Including horizontal speed in the energy conservation in a). Forgetting that the height drop includes the squeezing of the spring (still partial credit, though). Writing down the differential equation for a particle in liquid rather than for spring. Forgetting gravity. Computational mistakes.

Problem 4: Chicken! (25)



The clever manoeuvre in problem 3 did not succeed, and faced with no other option, the robbers drive at the approaching police car to force it to let them pass. But neither give way, and the two cars collide. The collision is not straight on, but can be described as two uniform discs of radius $r = 2.00$ *m sticking together as they pass each other (see figure). The robbers collide with a speed of* $v = 100$ *km/h, in their car of mass* $M = 1000$ *kg, not including four wheels of mass* $m = 15.0$ *kg each. The police car has a total mass of* $M_p = 1500$ *kg, and its speed at impact is* 80.0 *km/h. We imagine that as soon as they have collided, they slide frictionlessly for a short while.*

a) *What is the final linear speed of the entangled cars, after the collision?*

Solution : Inelastic collision, momentum conservation (will use $v_p = -80$ km/h, since the cars go in opposite directions to start with)

$$M_p v_p + (M + 4m)v = (M_p + M + 4m)v_f \rightarrow \tag{30}$$

$$v_f = \frac{M_p v_p + (M + 4m)v}{M_p + M + 4m} = -1.52 \text{ m/s} = -5.47 \text{ km/h}. \tag{31}$$

b) *Where is the combined center of mass and what is the moment of inertia around this center of mass after they stick together?*

Solution : Let us put the origin of the coordinate system at the center of the police disc. Then the CM is at

$$r_{\text{CM}} = \frac{(M + 4m)2r}{M_p + M + 4m} = 1.66 \text{ m}. \tag{32}$$

Using the parallel axis theorem on the two discs, moving to the CM, we find

$$I_{\text{tot}} = \frac{M_p}{2}r^2 + M_p r_{\text{CM}}^2 + \frac{M + 4m}{2}r^2 + (M + 4m)(2r - r_{\text{CM}})^2 = 15.1 \times 10^3 \text{ kgm}^2. \tag{33}$$

c) *What is the angular speed of the rotation around the combined center of mass, after the collision?*

Solution : Angular momentum is conserved. Initially, around the CM, it is (now, $v_p = +80$ km/h, since angular momentum of both cars acts in the same direction)

$$|L| = M_p v_p r_{\rm CM} + (M + 4m)v(2r - rcm) = 124 \times 10^3 \text{ kg m}^2/\text{s}. \tag{34}$$

This must be equal afterwards to $I\omega$, so that

$$\omega = \frac{|L|}{I} = 8.25 \text{ s}^{-1}. \tag{35}$$

Typical mistakes:
Forgetting that the speeds are opposite, and so one of the cars has $-v$ when doing momentum conservation (Very, very common!). Not remembering the moment of inertia of a disc. Using 1/3 instead of 1/2. Not using the parallel axis theorem correctly. Getting signs wrong in computing the CM (of the positions vectors relative tot he choice of coordinate system). Using energy conservation in an inelastic collision. Using energy conservation instead of angular momentum conservation. Computational errors.