Høsten 2019

FYS100 Fysikk: Exam/Eksamen

You **must** put your candidate number on every sheet.

There are 4 questions. You need to answer all 4 questions for a full score.

The standard formula sheet for FYS100 Fysikk is part of this question sheet.

Additional written help-material is not allowed.

Standard approved calculators are allowed.

Don't panic! Draw a diagram where relevant. State clearly the relevant physics.

The questions are also attached in Norwegian.

Good luck!

Du **må** legge kandidatnummeret ditt på hvert ark.

Det er 4 spørsmål. Du må svare på alle de fire spørsmålene for en full score.

Standardformelarket for FYS100 Fysikk er en del av dette spørsmålet.

Ekstra skriftlig hjelpemateriell er ikke tillatt.

Standard godkjente kalkulatorer er tillatt.

Ingen panikk! Tegn et diagram der det er relevant. Angi tydelig hvilken fysikk som er relevant.

Spørsmålene er også vedlagt på engelsk.

Lykke til!

Problem 1: An airplane taking off

An airplane of mass 75 tonnes starts from rest on a horizontal, straight runway. The two engines of the airplane provide a maximum thrust of 140kN each.

a) Initially the pilot only uses 50% of the maximum thrust on each engine. What is the initial acceleration of the plane along the runway?

Solution: Use Newtons 2nd law: $\Sigma F = ma$.
The total thrust on the aircraft is 2*(1/2)*140=140kN.

$$a = \frac{F}{m} = \frac{140000}{75000} = 1.9 ms^{-2} (2 sig.fig) \quad (1)$$

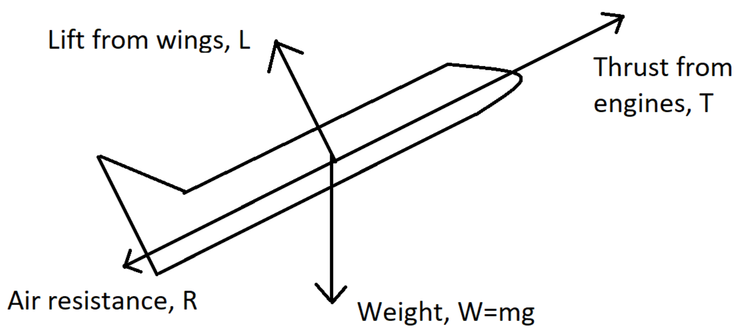
b) The plane needs to reach a speed of 280km/h to take off. Assuming the acceleration along the runway remains constant, how far does the plane need to go to reach take-off speed?

Use a kinematic relation: $v^2 = u^2 + 2as$.
The final speed of the aircraft is 280/3.6= 77.8m/s. Initial speed is zero.

$$s = \frac{v^2}{2a} = \frac{77.8^2}{2 * 1.9} = 1600 m \quad (2)$$

c) The plane now takes off, with an angle to the ground of 15°. Draw a diagram of the main forces on the plane.

Draw a diagram. Weight acts down. Thrust in the direction of travel. Air resistance in the opposite direction to travel. Lift from wings upwards, perpendicular to travel direction.



d) How much lift force do the wings provide perpendicular to the direction of travel?

Use Newton's 2nd law: $\Sigma F = ma$.
The perpendicular lift force must balance the component of the weight perpendicular to the direction of travel, because acceleration in this direction is zero.

$$\mathbf{L = mg\cos\theta = 75000 * 9.82 * \cos 15 = 711,000 = 710}\text{kN(2sig.fig.)} \quad (3)$$

e) The plane climbs to 1200 metres before executing a turn by rolling the wings 30° to the horizontal. At this point the speed of the plane is 300km/h. Assume the speed and altitude of the plane remains constant during the turn. How long does it take to turn the plane around so that it is facing in the opposite direction?

Use Newton's 2nd law: $\Sigma F = ma$.
Decompose the forces into vertical and horizontal components. The vertical acceleration is zero because altitude is constant. The horizontal component provides the centripetal acceleration to turn. The speed of the plane is 300/3.6=83.3m/s.

$$\mathbf{Vertical\ component:\ L\cos\theta - mg = 0} \quad (4)$$

$$\mathbf{Horizontal\ component:\ L\sin\theta = \frac{mv^2}{r}} \quad (5)$$

$$\mathbf{Combine\ together\ to\ get\ the\ radius:\ r = \frac{v^2}{g\tan\theta}} \quad (6)$$

This is the radius of the turning circle and the plane must turn half of such a circle with a speed of v, so the time taken is:

$$\mathbf{t = \frac{\pi r}{v} = \frac{\pi v}{g\tan\theta} = \frac{3.14159 * 83.3}{9.82\tan 30} = 46s} \quad (7)$$

Problem 2: Moving a concrete block on the ground

A concrete block of length 1m, width 0.5m and height 0.25m is lying on the ground and needs to be moved. The density of concrete is 2400kg/m^3. The coefficient of static friction between concrete and the ground is 0.55.

a) A strong man of mass 90kg who is able to generate a force of 1.1kN is brought in the move the block. Show why the man will not be able to move the block horizontally.

Solution: The volume of the block is given by 1*0.5*0.25=0.125m^3 so its mass is 2400*0.125=300kg. To move the block, the man must

overcome the maximum static friction force which is given by $\mu N = \mu mg = 0.55 * 300 * 9.82 = 1620N$. Since the man can only push with 1100N, he does not have enough strength to move the block.

b) What will be the static friction between the block and the ground if the man pushes with all his force on the block?

Solution: If the man pushes with all his force 1100N on the block, the static friction force will resist his pushing as the block will not move and thus the static friction force will be 1100N

c) A tractor is now drafted in to move the block. The weight of the tractor is equally distributed over all four wheels and it pulls on the block via a rope with tension T. The back wheels have a diameter of 1.6m and the front wheels have a diameter of 80cm. The tractor motor generates a torque of τ on each of its back wheels that have radii R_b and masses m_b. The front wheels of the tractor are smaller than the back wheels and have radii R_f and masses m_f. Both sets of wheels can be treated as uniform solid disks with moment of inertia $I = \frac{1}{2}mR^2$. The mass of the tractor body, without the wheels, is M. If the wheels don't slip, show that the tractor can accelerate the block with an acceleration of

$$a = \frac{2\tau - TR_b}{R_b(M + 3m_b + 3m_f)}.$$

Solution: This can be shown is several different ways. Use Newton's 2nd law or conservation of energy. The way suggested in lectures was to break down the tractor into body and wheels components and write seperate force equations for each part and torque equations for the wheels. In that case there will be a force on the tractor body due to each front wheel of F_f and a force on the tractor body due to each back wheel of F_b. There will also be frictional forces on the front wheels of f_f and on the back wheels of f_b. The signs are chosen so that the tractor will accelerate in the positive direction away from the block.

$$\textbf{Horizontal forces on the tractor body}: \quad \mathbf{2F_b - 2F_f - T = Ma} \tag{8}$$

$$\textbf{Horizontal forces on a front wheel}: \quad \mathbf{F_f - f_f = m_f a} \tag{9}$$

$$\textbf{Horizontal forces on a back wheel}: \quad \mathbf{-F_b + f_b = m_b a} \tag{10}$$

$$\textbf{Torques on a front wheel}: \quad \mathbf{-f_f R_f = I_f \alpha_f = \frac{1}{2} m_f R_f^2 \frac{a}{R_f}} \tag{11}$$

$$\textbf{Torques on a back wheel}: \quad \mathbf{\tau + f_b R_b = I_b \alpha_b = \frac{1}{2} m_b R_b^2 \frac{a}{R_b}} \tag{12}$$

Substituting in for F_b, F_f, f_b and f_f in the first equation and rearranging gives the result.

An alternative way to get the result (and check the signs above) is to use energy considerations. The engine supplies work to the tractor through the torque and the tractor does work on the block through the rope tension if it moves it. Some of the energy also goes into increasing the linear kinetic energy of the tractor body and wheels and the rotational kinetic energy of the wheels. Differentiating the relevant form of the work-kinetic energy relation will give the result after some algebra.

d) The tractor has a mass $M = 7100$kg with each of the back wheels weighing additionally $m_b = 200$kg each and the front wheels $m_f = 75$kg each. The back wheels have a diameter of 1.6m and the front wheels have a diameter of 80cm. The tractor's engine can generate a torque of $\tau = 700$Nm on each of its back wheels. The rope will break if the tension in the rope exceeds 1500N. If full power is applied to the tractor in trying to move the block, can the block be moved from rest with an acceleration of 0.1m/s^2 without the rope breaking? Think **carefully** about this and explain your answer.

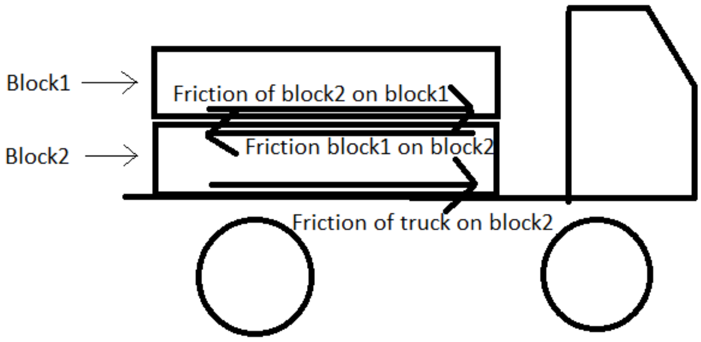
Solution: Here we know from part (a) that a force of 1620N is needed to overcome the static friction of the block and get it moving. So no matter how low the attempted acceleration is, the rope will break before the block starts to move.

Problem 3: Transporting stone blocks on a truck

Two flat stone blocks of masses 100kg each are stacked on top of one another on the back of a truck. The coefficient of static friction between the bottom block and the truck is 0.8 and between the two blocks is 0.7.

a) What is the maximum acceleration the truck can have such that neither of the blocks slide off?

Solution:



It is friction that accelerates the blocks along with the truck via Newton's 2nd law $\Sigma F = ma$. If the acceleration of the truck exceeds that of the maximum static frictional force, then the blocks will slide off. The maximum static friction is given by $f_{s,max} = \mu N$. For blocks moving horizontally the normal force must balace the weight force, so $N = mg$. Thus the maximum acceleration on the top block is:

$$a_{max} = f_{s,max}/m_t = \mu_t g = 0.7 * 9.82 = 6.9 ms^{-2}. \quad (13)$$

For the bottom block, there is both friction from the truck and friction from the top block. These point in opposite directions. From Newton's 2nd law $\Sigma F = ma$ we have $f_{s,b} - f_{s,t} = ma$. The friction from the top block must be able to accelerate the top block without sliding off, so it must be $f_{s,t} = ma$. Then the friction between the bottom block and the truck is $f_{s,b} = 2ma$. This must remain less than the maximum friction between the bottom and the truck which is $f_{s,max} = \mu_b N$ and now the normal force is $N = 2mg$ because there are two blocks on top of one another. So we have:

$$a_{max} = \mu_b g = 0.8 * 9.82 = 7.9 ms^{-2}. \quad (14)$$

Since neither block should slide off, we are limited by the top block and the maximum acceleration is $6.9 ms^{-2}$.

b) If the truck accelerates from rest at this maximum acceleration for 6 seconds, how much more energy will the truck's engine need to output compared to if the stone blocks were not on the truck?

Solution: Use conservation of energy. The truck's engine will need to provide additional kinetic energy to the blocks. Their final speed is given by a kinetic relation $\mathbf{v = u + at}$ so the energy added to the blocks is:

$$\mathbf{E_{added}} = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{2m(at)^2} = \mathbf{100} * (\mathbf{6.9} * \mathbf{6})^{\mathbf{2}} = \mathbf{170kJ}. \tag{15}$$

c) What is the maximum speed that the truck can drive around a curve of minimum curvature radius 100m without either of the blocks sliding off?

Solution: Use Newton's 2nd law $\mathbf{\Sigma F = ma}$. The maximum static friction force from part (a) is now needed to provide the necessary centripetal acceleration to drive around the curve, $\mathbf{f_{s,max} = m\frac{v^2}{r}}$. So we have:

$$\mathbf{v} = \sqrt{\mu_{\mathbf{t}}\mathbf{gr}} = \sqrt{\mathbf{0.7} * \mathbf{9.82} * \mathbf{100}} = \mathbf{26ms^{-1}}(\textbf{which is 94km/h}). \tag{16}$$

d) What is the maximum slope that the truck can drive up at constant speed without either of the blocks sliding off?

Solution: Now there is no acceleration, but the friction must keep the blocks on the truck against gravity. Use Newtons 2nd law: $\mathbf{\Sigma F = ma}$ to resolve the forces both parallel and perpendicular to the direction of motion.

$$\textbf{Forces parallel to the motion :} \mathbf{f_s - mg}\sin\theta = \mathbf{0} \tag{17}$$

$$\textbf{Forces perpendicular to the motion :} \mathbf{N - mg}\cos\theta = \mathbf{0} \tag{18}$$

The maximum value that the static frictional force is $\mathbf{f_{s,max}} = \mu\mathbf{N}$. So the maximum angle is given by:

$$\theta_{\mathbf{max}} = \tan^{-\mathbf{1}}\mu = \tan^{-\mathbf{1}}\mathbf{0.7} = \mathbf{35}^\circ \tag{19}$$

Problem 4: A child on a swing

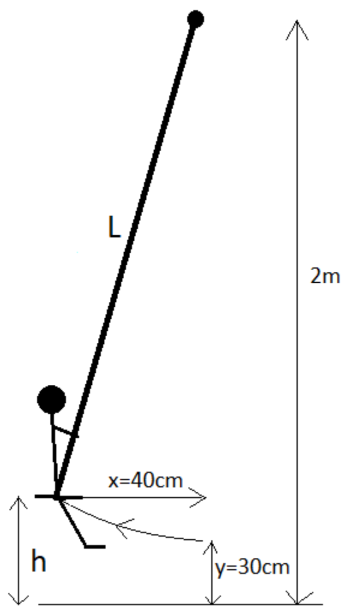
The equation describing damped oscillations is

$$x(t) = Ae^{-\frac{bt}{2m}}\sin(\omega t + \phi).$$

A child of mass 15kg wants to swing on a swing of negligible mass. The seat of the swing is 30cm above the ground and the total vertical height of the swing is 2m above the ground.

a) The child is too young to swing themselves. How much mechanical energy is added to the child if they are pulled back by a parent a distance 40cm from the vertical?

Solution:



Use conservation of energy and geometry. The length of the swing is $L = 2 - 0.3 = 1.7$m. The child has moved from being y=30cm over the ground to being:

$$\mathbf{h = L + y - \sqrt{L^2 - x^2} = 2 - 1.7 * \sqrt{1 - (0.4/1.7)^2}) = 0.3477m.} \qquad (20)$$

over the ground. So the change in the potential energy (which accounts for all the change in the mechanical energy because the kinetic energy remains zero) is:

$$\mathbf{\Delta E = mg(h - y) = 15 * 9.82 * 0.0477 = 7.0J.} \qquad (21)$$

An alternative way to get (approximately) the same answer is to model the swing as an oscillating spring with mechanical energy:

$$\mathbf{E} = \frac{1}{2}\mathbf{kx^2} + \frac{1}{2}\mathbf{mv^2} \tag{22}$$

Here the spring constant is given by k = mg/L and the velocity v = 0 so the mechanical energy is

$$\mathbf{E} = \frac{1}{2}\mathbf{kx^2} = \frac{1}{2}\frac{\mathbf{mgx^2}}{\mathbf{L}} = \frac{1}{2}\frac{15 * 9.82 * 0.4^2}{1.7} = 6.9\mathrm{J} \tag{23}$$

The small difference in value (6.9 versus 7.0) is because we have used the approximation that $\sin\theta \approx \theta$ for small angles, neglecting higher order terms.

b) The child is now released from rest. The distance the child swings decreases by 20cm from the vertical after the child has swung back and forth five times. How much mechanical energy has the child lost?

Solution: Use conservation of energy. The new energy of the child is $\frac{1}{2}\frac{15*9.82*0.2^2}{1.7} = 1.73\mathrm{J}$ so the child has lost $7.0 - 1.7 = 5.3\mathrm{J}$.

c) Is this mechanical energy being lost at a constant rate? Explain your answer.

Solution: The energy is not being lost at a constant rate because the main source of energy loss is air resistance and this depends on the speed of the child which is constantly changing. After each swing the child's maximum speed is lower and the total energy lost in each swing decreases. In fact, it decreases exponentially as a damped harmonic oscillator.

d) If the parent wants to keep the child swinging the same distance, the parent needs to give the child a push. If the parent pushes the child with a constant force of 10N, how far does the parent need to push the child in order to replace the energy that is otherwise lost in the first swing?

Solution: Use conservation of energy. If the energy at the beginning is $\mathbf{E_0} = 7.0\mathrm{J}$, then the energy after five swings is $\mathbf{E_0 e^{-5T/\tau}} = 1.7\mathrm{J}$ where T is the period of one swing and τ is a damping time. Thus the energy after only one swing is:

$$\mathbf{E_1} = \mathbf{E_0 e^{-T/\tau}} = 7.0 * \left(\frac{1.7}{7.0}\right)^{1/5} = 5.3\mathrm{J} \tag{24}$$

Thus the child loses $7.0 - 5.3 = 1.7\mathrm{J}$ in the first swing. Pushing the child the parent will do work on the child given by $\mathbf{W} = \int \mathbf{F}\mathrm{d}\mathbf{x}$ and in order to replace the energy lost the parent therefore needs to push a distance $\mathbf{x} = 1.7/10 = 17\mathrm{cm}$.

FYS100 Physics – Formula sheet

Rotational motion about a fixed axis	**Translational motion**
Angular velocity $\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t}$	Translational velocity $v = \frac{\mathrm{d}x}{\mathrm{d}t}$
Angular acceleration $\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t}$	Translational acceleration $a = \frac{\mathrm{d}v}{\mathrm{d}t}$
Net torque $\sum_k \tau_k = I\,\alpha$	Net force $\sum_k F_k = m\,a$
$\alpha = \text{constant} \begin{cases} \omega_f = \omega_i + \alpha\, t \\ \theta_f = \theta_i + \omega_i\, t + \frac{1}{2}\alpha\, t^2 \\ \omega_f^2 = \omega_i^2 + 2\,\alpha\,(\theta_f - \theta_i) \\ \theta_f = \theta_i + \frac{1}{2}\,(\omega_i + \omega_f)\, t \end{cases}$	$a = \text{constant} \begin{cases} v_f = v_i + a\, t \\ x_f = x_i + v_i\, t + \frac{1}{2} a\, t^2 \\ v_f^2 = v_i^2 + 2\,a\,(x_f - x_i) \\ x_f = x_i + \frac{1}{2}\,(v_i + v_f)\, t \end{cases}$
Work $W = \int_{\theta_i}^{\theta_f} \tau\,\mathrm{d}\theta$	Work $W = \int_{x_i}^{x_f} F\,\mathrm{d}x$
Rotational kinetic energy $K = \frac{1}{2}\,I\,\omega^2$	Kinetic energy $K = \frac{1}{2}\,m\,v^2$
Power $\mathcal{P} = \tau\,\omega$	Power $\mathcal{P} = F\,v$
Angular momentum $L = I\,\omega$	Linear momentum $p = m\,v$
Net torque $\sum_k \tau_k = \frac{\mathrm{d}L}{\mathrm{d}t}$	Net force $\sum_k F_k = \frac{\mathrm{d}p}{\mathrm{d}t}$

General formulas	
Motion with constant acceleration	$\begin{cases} \vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}\, t \\ \vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i\, t + \frac{1}{2}\,\vec{\mathbf{a}}\, t^2 \end{cases}$
Newton's second law	$\sum_k \vec{\mathbf{F}}_k = m\,\vec{\mathbf{a}}$
Work	$W = \int \vec{\mathbf{F}} \cdot \mathrm{d}\vec{\mathbf{r}}$
Work-kinetic energy theorem	$\Delta K = W$
Linear momentum	$\vec{\mathbf{p}} = m\,\vec{\mathbf{v}}$
Newton's second law	$\sum_k \vec{\mathbf{F}}_k = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}$
Impulse	$\vec{\mathbf{I}} = \int \vec{\mathbf{F}}\,\mathrm{d}t$
Impulse-momentum theorem	$\Delta\vec{\mathbf{p}} = \vec{\mathbf{I}}$
Center of mass	$\vec{\mathbf{r}}_{\text{CM}} = \frac{1}{M}\int \vec{\mathbf{r}}\,\mathrm{d}m$
Moment of inertia	$I = \int r^2\,\mathrm{d}m$
Parallel-axis theorem	$I = I_{\text{CM}} + M\,D^2$
Torque	$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$
Angular momentum	$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}$
Net torque	$\sum_k \vec{\boldsymbol{\tau}}_k = \frac{\mathrm{d}\vec{\mathbf{L}}}{\mathrm{d}t}$
Rotational motion	$s = r\,\theta,\ v = r\,\omega,\ a_c = r\,\omega^2,\ a_t = r\,\alpha$
Harmonic oscillator	$\frac{d^2x}{dt^2} = -\omega^2 x,\ x(t) = A\cos(\omega t + \phi)$

Mathematical rules

Vector relations	
Scalar product	$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = \lvert\vec{\mathbf{A}}\rvert\,\lvert\vec{\mathbf{B}}\rvert\,\cos\phi$
Magnitude of vector product	$\lvert\vec{\mathbf{A}} \times \vec{\mathbf{B}}\rvert = \lvert\vec{\mathbf{A}}\rvert\,\lvert\vec{\mathbf{B}}\rvert\,\sin\phi$

Trigonometry	
Definitions	$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$
Identities	$\sin^2\alpha + \cos^2\alpha = 1$
	$\sin 2\alpha = 2\,\sin\alpha\,\cos\alpha$
	$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\,\cos^2\alpha - 1 = 1 - 2\,\sin^2\alpha$
	$\sin(\alpha \pm \beta) = \sin\alpha\,\cos\beta \pm \cos\alpha\,\sin\beta$
	$\cos(\alpha \pm \beta) = \cos\alpha\,\cos\beta \mp \sin\alpha\,\sin\beta$
	$\frac{\sin\alpha}{a} = \frac{\sin\beta}{B} = \frac{\sin\gamma}{C}$
	$a^2 + b^2 - c^2 = 2ab\cos\gamma$
Derivatives	$\frac{\mathrm{d}\sin\alpha}{\mathrm{d}\alpha} = \cos\alpha$
	$\frac{\mathrm{d}\cos\alpha}{\mathrm{d}\alpha} = -\sin\alpha$

Quadratic equations	
Equation	$a\,t^2 + b\,t + c = 0$
Solution	$t = \dfrac{-b \pm \sqrt{b^2 - 4\,a\,c}}{2\,a}$

Equation of a straight line	
Two points on the line are given	$y = y_1 + \dfrac{y_2 - y_1}{x_2 - x_1}\,(x - x_1)$

Oppgave 1: Et fly letter

Et fly med masse 75 tonn starter fra hvile på en horisontal, rett rullebane. De to motorene i flyet gir en maksimal skyvekraft på 140kN hver.

a) Opprinnelig bruker piloten bare 50% av maksimal skyvkraft på hver motor. Hva er den initiale akselerasjonen av flyet langs rullebanen?

b) Flyet må nå en fart på 280km/h for å lette. Forutsatt at akselerasjonen langs rullebanen forblir konstant, hvor langt trenger flyet å gå for å nå startfarten?

c) Flyet letter nå, med en vinkel på bakken av 15°. Tegn et diagram av hovedkreftene på flyet.

d) Hvor meget løftekraft gir vingene vinkelrett på flyretningen?

e) Flyet stiger til 1200 meter før det utfører en sving ved å rulle vingene 30° til horisontalen. På dette tidspunktet er flyets hastighet 300km/h. Anta at farten og høyden til flyet forblir konstant under svingen. Hvor lang tid tar det å snu flyet slik at det vender tilbake i motsatt retning?

Oppgave 2: Flytte en betongblokk på bakken

En betongblokk med lengde 1m, bredde 0,5m og høyde 0,25m ligger på bakken og må flyttes. Tettheten av betong er 2400kg/m^3. Den statiske friksjonskoeffisienten mellom betong og bakken er 0,55.

a) En sterk mann med en masse på 90kg som er i stand til å generere en kraft på 1,1kN blir brakt inn for å flytte blokken. Vis hvorfor mannen ikke greier å flytte blokken horisontalt.

b) Hva blir den statiske friksjonen mellom blokken og bakken hvis mannen skyver med all sin kraft på blokken?

c) En traktor blir så brakt inn for å flytte blokken. Vekten til traktoren er likt fordelt på alle fire hjul og den trekker blokken med et tau med spenningen T. Traktormotoren genererer et dreiemoment τ på hvert av bakhjulene som begge har radien R_b og massen m_b. Bakhjulene har en diameter på 1,6m og forhjulene har en diameter på 80cm. Begge hjulsettene kan behandles som identiske faste skiver med treghetsmoment $I = \frac{1}{2}mR^2$. Massen til traktoren, uten hjul, er M. Hvis hjulene ikke glir, vis at traktoren kan akselerere blokken med en akselerasjon på

$$a = \frac{2\tau - TR_b}{R_b(M + 3m_b + 3m_f)}.$$

d) Traktoren har en masse $M = 7100$kg med hvert av bakhjulene som veier i tillegg $m_b = 200$kg hver og forhjulene $m_f = 75$kg hver. Bakhjulene har en diameter på 1,6m og forhjulene har en diameter på 80cm. Traktorens motor kan generere et dreiemoment på $\tau = 700$Nm på hvert av bakhjulene. Tauet vil gå i stykker hvis spenningen i tauet overstiger 1500N. Hvis full kraft brukes til traktoren til å flytte blokken, kan den flyttes fra hvile med akselerasjonen 0,1m/s^2 uten at tauet går i stykker? Tenk **nøye** gjennom dette og forklar svaret.

Oppgave 3: Transport av steinblokker på en lastebil

To flate steinblokker med masser på 100kg hver er stablet oppå hverandre på lasteplanet av en lastebil. Den statiske friksjonskoeffisienten mellom den underste blokken og lastbilen er 0,8 og mellom de to blokkene er 0,7.

a) Hva er den maksimale akselerasjonen lastbilen kan ha uten at noen av blokkene glir av?

b) Hvis lastbilen akselererer fra hvile med denne maksimale akselerasjonen i 6 sekunder, hvor mye mer energi trenger lastbilens motor å produserer i forhold til om steinblokkene ikke var på lastbilen?

c) Hva er den maksimale hastigheten som lastbilen kan kjøre rundt en kurve med minimum krumningsradius 100m uten at noen av blokkene glir av?

d) Hva er den maksimale helningen som lastbilen kan kjøre oppover med konstant fart uten at noen av blokkene glir av?

Oppgave 4: Et barn på en huske

Likningen som beskriver dempede svingninger er

$$x(t) = Ae^{-\frac{bt}{2m}} \sin(\omega t + \phi).$$

Et barn på 15kg ønsker å svinge på en huske med ubetydelig masse. Husken er hengt op 2m over bakken og huskens sete er 30cm over bakken.

a) Barnet er for ung til å svinge selv. Hvor mye mekaniske energi blir tilføjet til barnet når det blir trukket bakover av moren et stykke på 40cm fra vertikalen?

b) Barnet blir nå slippet til å svinge. Avstanden barnet svinger reduseres med 20cm fra vertikalen etter at barnet har svingt frem og tilbake fem ganger. Hvor mye mekanisk energi har barnet mistet?

c) Mister barnet like mye mekaniske energien per tidsenhet? Forklar svaret ditt.

d) Hvis moren ønsker å få barnet til å svinge samme avstand, må hun gi barnet en dytt. Hvis hun skyver barnet med en konstant kraft på 10N, over hvilken avstand må hun skyver barnet for å erstatte den energien som ellers går tapt i den første svingen?

FYS100 Fysikk – formelark

Rotasjon om en fast akse	**Éndimensjonal bevegelse**
Vinkelhastighet $\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t}$	Hastighet $v = \frac{\mathrm{d}x}{\mathrm{d}t}$
Vinkelakselerasjon $\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t}$	Akselerasjon $a = \frac{\mathrm{d}v}{\mathrm{d}t}$
Resultantmoment $I\,\alpha = \sum_k \tau_k$	Resultantkraft $m\,a = \sum_k F_k$
$\alpha = \text{konstant} \begin{cases} \omega_f = \omega_i + \alpha\,t \\ \theta_f = \theta_i + \omega_i\,t + \frac{1}{2}\alpha\,t^2 \\ \omega_f^2 = \omega_i^2 + 2\,\alpha\,(\theta_f - \theta_i) \\ \theta_f = \theta_i + \frac{1}{2}\,(\omega_i + \omega_f)\,t \end{cases}$	$a = \text{konstant} \begin{cases} v_f = v_i + a\,t \\ x_f = x_i + v_i\,t + \frac{1}{2}a\,t^2 \\ v_f^2 = v_i^2 + 2\,a\,(x_f - x_i) \\ x_f = x_i + \frac{1}{2}\,(v_i + v_f)\,t \end{cases}$
Arbeid $W = \int_{\theta_i}^{\theta_f} \tau\,\mathrm{d}\theta$	Arbeid $W = \int_{x_i}^{x_f} F\,\mathrm{d}x$
Kinetisk energi $K = \frac{1}{2}\,I\,\omega^2$	Kinetisk energi $K = \frac{1}{2}\,m\,v^2$
Effekt $\mathcal{P} = \tau\,\omega$	Effekt $\mathcal{P} = F\,v$
Spinn $L = I\,\omega$	Bevegelsesmengde $p = m\,v$
Spinnsatsen $\frac{\mathrm{d}L}{\mathrm{d}t} = \sum_k \tau_k$	Newtons 2. lov $\frac{\mathrm{d}p}{\mathrm{d}t} = \sum_k F_k$

Generelle sammenhenger	
Bevegelse med konstant akselerasjon	$\begin{cases} \vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}\,t \\ \vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i\,t + \frac{1}{2}\,\vec{\mathbf{a}}\,t^2 \end{cases}$
Newtons 2. lov	$m\,\vec{\mathbf{a}} = \sum_k \vec{\mathbf{F}}_k$
Arbeid	$W = \int \vec{\mathbf{F}} \cdot \mathrm{d}\vec{\mathbf{r}}$
Arbeid-kinetisk energi teoremet	$\Delta K = W$
Bevegelsesmengde	$\vec{\mathbf{p}} = m\,\vec{\mathbf{v}}$
Newtons 2. lov	$\frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t} = \sum_k \vec{\mathbf{F}}_k$
Impuls	$\vec{\mathbf{I}} = \int \vec{\mathbf{F}}\,\mathrm{d}t$
Impuls-bevegelsesmengde teoremet	$\Delta\vec{\mathbf{p}} = \vec{\mathbf{I}}$
Massesenter	$\vec{\mathbf{r}}_{\text{CM}} = \frac{1}{M}\int \vec{\mathbf{r}}\,\mathrm{d}m$
Treghetsmoment	$I = \int r^2\,\mathrm{d}m$
Steiners sats (parallellakseteoremet)	$I = I_{\text{CM}} + M\,D^2$
Kraftmoment	$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$
Spinn	$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}$
Spinnsatsen	$\frac{\mathrm{d}\vec{\mathbf{L}}}{\mathrm{d}t} = \sum_k \vec{\boldsymbol{\tau}}_k$
Sirkelbevegelse	$s = r\,\theta,\ v = r\,\omega,\ a_c = r\,\omega^2,\ a_t = r\,\alpha$
Harmonisk oscillator	$\frac{d^2x}{dt^2} = -\omega^2 x,\ x(t) = A\cos(\omega t + \phi)$

Matematiske sammenhenger

Vektorrelasjoner	
Prikkprodukt	$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = \lvert\vec{\mathbf{A}}\rvert\,\lvert\vec{\mathbf{B}}\rvert \cos\phi$
Absoluttverdi av kryssprodukt	$\lvert\vec{\mathbf{A}} \times \vec{\mathbf{B}}\rvert = \lvert\vec{\mathbf{A}}\rvert\,\lvert\vec{\mathbf{B}}\rvert \sin\phi$

Trigonometri	
Definisjoner	$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$
Identiteter	$\sin^2\alpha + \cos^2\alpha = 1$
	$\sin 2\alpha = 2\,\sin\alpha\,\cos\alpha$
	$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\,\cos^2\alpha - 1 = 1 - 2\,\sin^2\alpha$
	$\sin(\alpha \pm \beta) = \sin\alpha\,\cos\beta \pm \cos\alpha\,\sin\beta$
	$\cos(\alpha \pm \beta) = \cos\alpha\,\cos\beta \mp \sin\alpha\,\sin\beta$
	$\frac{\sin\alpha}{a} = \frac{\sin\beta}{B} = \frac{\sin\gamma}{C}$
	$a^2 + b^2 - c^2 = 2ab\cos\gamma$
Deriverte	$\frac{\mathrm{d}\sin\alpha}{\mathrm{d}\alpha} = \cos\alpha$
	$\frac{\mathrm{d}\cos\alpha}{\mathrm{d}\alpha} = -\sin\alpha$

2. grads ligning	
Ligning	$a\,t^2 + b\,t + c = 0$
Løsning	$t = \dfrac{-b \pm \sqrt{b^2 - 4\,a\,c}}{2\,a}$

Ligningen for en rett linje	
Gitt to punkter på linjen	$y = y_1 + \dfrac{y_2 - y_1}{x_2 - x_1}\,(x - x_1)$