Høsten 2019

FYS100 Fysikk: Hand-in XII

To be handed in at the latest **Friday 8th November 2019, at 23.59**.
You **must** hand it in by scanning your handwritten solution or compiling your electronic documents into **a single .pdf** file, and uploading it to Canvas in "Week Twelve Exercises". Bad mobile phone pictures are not acceptable (there are apps that work OK), nor are any other file formats than a single .pdf.

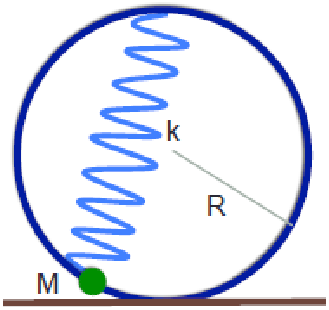
You **must**:

- Put your name and student number on each page.
- Make sketches for all problems, where it makes sense.
- Write in a readable, well-structured way.

Pass is 40% correct (including partial credit). There is no grade. There is **no option** to correct and resubmit.

Good luck!

Problem 1: Springs and equilibirum (2016 Konte-exam)



A bead of mass M is sliding on a ring of radius R, the ring standing on its side. A spring is attached to the top of the ring at one end, and to the bead at the other end. The spring has spring constant k and we will assume that the equilibrium point of the spring is when it is completely unstretched. There is gravity, g.

a) If the bead starts from rest (almost) exactly at the bottom (at 6 o'clock), what is its speed when it reaches the top (at 12 o'clock)?

b) Find a requirement on k, M, g and R for the bead getting to the top at all.

c) Find the potential energy function $E(\theta)$, as a function of the angle along the ring, including both gravitational and spring potential energy. We take $\theta = 0$ when the bead is at the bottom of the ring. Find the stable equilibrium point, depending on whether the criterion in b) is fulfilled or not.

For each question, provide an algebraic expression and a sketch.

Problem 2: Simple harmonic motion (adapted from problem 15.14 in the book)

A ball dropped from a height of 4.00m makes an elastic collision with the ground in a vacuum chamber.

a) What is the difference between the kinetic energy of the ball immediately before the collision and the kinetic energy of the ball immediately after the collision?

Solution: Zero. The kinetic energy is conserved in an elastic collision.

b) Is the kinetic energy of the ball conserved in its subsequent motion after the collision?

Solution: No, kinetic energy is transformed into potential energy.

c) What is the difference between the momentum of the ball immediately before the collision and the momentum of the ball immediately after the collision?

Solution: The difference is 2mv, since the ball changes direction. The velocity change is $2\sqrt{2as} = 2\sqrt{8g}$.

d) Is the total momentum of the system conserved, and if so, what other parts of the system must be included other than the ball?

Solution: The total momentum of the system is conserved, if one includes the whole Earth in the system.

b) Identify an equilibrium point for the entire system. Explain why the motion around this equlibirum point is periodic and determine the period of the motion.

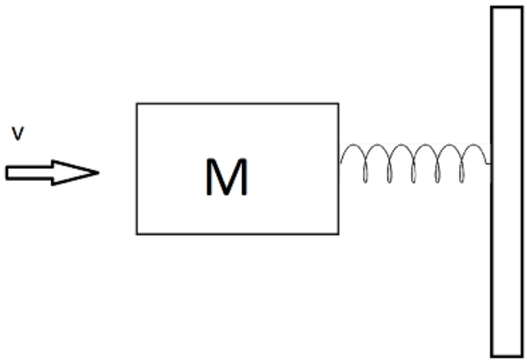
Solution: The ball-Earth system is in equilibrium when the ball is resting on the surface of the Earth. The motion around this point is periodic because the ball bounces back to the distance from which it was dropped (4m) and then falls back down and bounces again. The time taken to fall from 4m to the ground is $\sqrt{2s/a} = \sqrt{2 * 4/9.82} = 0.9$s and since it takes the same time to bounce back up, then total period is 2*0.9=1.8s.

c) Is the motion simple harmonic motion? Explain.

Solution: Although the motion is periodic and oscillatory, it is not simple harmonic motion, because the restorative force (gravity) is not strictly linear in the separation. The equilibrium point (on the ground) is not a zero of the restorative force (gravity), but a point where another force (a normal force) acts, that causes the ball to bounce back. This normal force does not act except when the ball is in contact with the ground.

More technically, while it is possible to linearised the gravitational force for small displacements from the surface of the Earth, the resulting equilibrium point of this linearisation will be halfway to the Earth's centre, where the linearisation approximation will break down. This makes the bouncing ball situation different from the swinging ball on a string pendulum, even though both are masses moving under gravity.

Problem 3: Harmonisk svingning



En kloss med masse M som er i ro på et friksjonsfritt underlag er festet til en vegg med en fjær med fjærkonstant k. En geværkule med masse m og hastighet v treffer klossen som vist på figuren. Kulen blir sittende fast i klossen. Bestem amplituden og frekvensken i den harmoniske svingningen som oppstå, og gi resultatet uttrykt ved størrelsene m, M,v og k.

Solution: Momentum is conserved (even if kinetic energy is not) and so the block will move with a speed $v_i = m\text{v}/(m+M)$ after the bullet comes to a rest in the block. We can assume that the bullet is initially travelling much faster than the block will be ($M \gg m$) and the process of the bullet coming to a rest is much faster than the period of oscillation of the block against the spring.

Then the block and spring will move with simple harmonic motion, with an initial displacement of 0 (the block starts approximately at the equilibrium point of the spring) but an initial velocity of $v_i = m\text{v}/(m+M)$. The angular frequency of the motion will be $\omega = \sqrt{k/(M+m)}$, so the frequency will be $f = \omega/2\pi = \sqrt{k/(M+m)}/2\pi$. The amplitude of the motion can be found from energy considerations, or from the fact that the intial velocity of the block at $t = 0$ will be $v_i = A\omega$, so $A = m\text{v}/(m+M)/\sqrt{k/(M+m)} = m\text{v}/\sqrt{k(m+M)}$.