Høsten 2019

FYS100 Fysikk: Hand-in III

To be handed in at the latest **Friday 6th September 2019, at 23.59**.
You **must** hand it in by scanning your handwritten solution or compiling your electronic documents into **a single .pdf** file, and uploading it to Canvas in "Week Three Exercises". Bad mobile phone pictures are not acceptable (there are apps that work OK), nor are any other file formats than a single .pdf.

You **must**:

- Put your name and student number on each page.
- Make sketches for all problems, where it makes sense.
- Write in a readable, well-structured way.

Pass is 40% correct (including partial credit). There is no grade. There is **no option** to correct and resubmit.

Good luck!

Problem 1: Drag (Question 6.30 from the book)

A small, spherical piece of Styrofoam packing material is dropped from a height of 2.00m above the ground. Until it reaches terminal speed, the magnitude of its acceleration is given by $a = g - Bv$. After falling 0.500m, the Styrofoam effectively reaches terminal speed and then takes 5.00s more to reach the ground. Answer the following questions, explaining how you found the answer.

a) What is the value of the constant B?

Solution: After the Styrofoam effectively reaches terminal speed, it travels $2.00 - 0.500 = 1.50$m, which takes 5.00s. Thus the terminal speed is $v_T = 1.50/5.00 = 0.300$m/s. At the terminal speed, the acceleration of the particle is zero and we can set $a = 0$ in the formula for the acceleration $a = g - Bv$. Thus we have $B = g/v_T = 9.82/0.300 = 32.7\text{s}^{-1}$.

b) What is the acceleration at $t = 0$?

Solution: Since the Styrofoam is dropped, its initial velocity is zero. In the formula for the acceleration $v = 0$ and the acceleration $a = g = 9.82\text{ms}^{-2}$.

c) What is the acceleration when the speed is 0.150m/s?

Solution: Use $a = g - Bv = 9.82 - 32.7 * 0.150 = 4.91\text{ms}^{-2}$.

Problem 2: Circular motion and centripetal acceleration

The tightest curve on the Sørlandsbanen that connects Stavanger to Oslo by rail has a curvature radius of 243m.

a) If the maximum permitted sidewards acceleration on Norwegian railways is 1.5 ms^{-2}, what is the maximum speed in km/h that a train can pass through this curve at, if the track in the curve is laid down flat?

Solution: The sidewards acceleration on the train is given by $a = v^2/r$. Thus the maximum permitted speed will be $v_{max} = \sqrt{ar} = \sqrt{1.5 * 243} = 19.1$m/s. To covert to km/h we divide by the number of metres in a kilometre and multiply by the number of seconds in a hour. So $v_{max} = 19.1 * 3600/1000 =$ 68km/h to 2 significant figures.

b) If instead the track is constructed with the maximum permitted cant (height of the outer rail over the inner rail, also called *dossering* or *overhøyde* in Norwegian) of 140mm, what speed in km/h can a train pass through such a curve without any friction on the inside of the rail (and a lot of noise)? The distance between the tracks is 1435mm.

Solution: First calculate the angle of the track to the horizontal. This is given by $\theta = \sin^{-1}(140/1435) = 5.60°$. For no friction to be acting, the sidewards acceleration needed to pass through the curve must be given by the horizontal component of the normal force. The only forces acting will be gravity and the normal force. Use Newton's second law and resolve the forces. Horizontally $N \sin\theta = ma$ and vertically $N \cos\theta = mg$. The sidewards acceleration is given by $a = v^2/r$. Combining these three together we have $v = \sqrt{rg\tan\theta} = \sqrt{243 * 9.82 * \tan(5.60)} = 15.3$m/s, which is 55km/h to 2 sig. fig.

c) If a tilting train (*krengetog*) allows an additional 5% tilt in addition to the cant in part (b), what speed in km/h can a tilting train pass through such a curve, without the passengers feeling the discomfort of being pressed to the sides of their seats?

Solution: Now the total angle to the horizontal will be $\theta = 5.6 + 5 = 10.6°$. The same algebraic formula derived in (b) is still valid for the passengers, so $v = \sqrt{rg\tan\theta} = \sqrt{243 * 9.82 * \tan(10.6)} = 21.1$m/s, which is 76km/h.

Problem 3: Areas and circular motion

a) Consider a particle moving at a constant speed in a straight line with no forces acting on it. For an arbitrary observer located at a fixed point O, show mathematically that an imaginary line joining the observer to the particle sweeps out equal areas in equal time intervals.

Solution: The straight line trajectory of the particle can be extended to an infinite straight line and will have a point of closest approach to O. In any interval of time starting at a time t_A and ending at a time t_B the area swept out by the imaginary line joining the particle to O will be a triangle, with the three corners given by the fixed point O, the position of the particle at time t_A and the position at time t_B. The area, A, of this triangle is given by $A = (1/2) * base * height$ where the base is given by the distance travelled between t_A and t_B and the height is given by the distance between the point O and the straight line where the straight line is closest to the point. Since the height remains constant and the base is always the distance travelled in the time interval at a constant speed, the area will always be the same for the same interval of time elapsed.

b) Consider now instead a particle following a circular trajectory with constant angular velocity and an observer at the centre of the circle. Show that the imaginary line that joins the observer to the particle sweeps out equal areas in equal times.

Solution: Since the angular velocity is constant, the particle will move through the same angle in any interval of time of the same length. The area swept out by the imaginary line is given by the area of the circle, $4\pi r^2$, times the ratio of the angle moved through to 2π, the total angle in the circle. Thus the area swept out will always be the same for the same interval of time elapsed.

The formulation of this question echoes Newton's demonstration that his mechanics provided an explanation for Kepler's second law of planetary motion, that planets swept out equal areas in equal times. This was a major achievement of Newtonian mechanics.

Six marks for problem one, two for each part. Six marks for problem two, two for each part. Three marks for problem three, one for the first part and two for the second part. Total marks $6 + 6 + 3 = 15$. 6 needed for approval.