Høsten 2019

FYS100 Fysikk: Hand-in V

To be handed in at the latest **Friday 20th September 2019, at 23.59**. You **must** hand it in by scanning your handwritten solution or compiling your electronic documents into **a single .pdf** file, and uploading it to Canvas in "Week Five Exercises". Bad mobile phone pictures are not acceptable (there are apps that work OK), nor are any other file formats than a single .pdf.

You **must**:

- Put your name and student number on each page.
- Make sketches for all problems, where it makes sense.
- Write in a readable, well-structured way.

Pass is 40% correct (including partial credit). There is no grade. There is **no option** to correct and resubmit.

Good luck!

Problem 1: Power (adapted from Problem 8.37)

For saving energy, bicycling and walking are far more efficient means of transport than is travel by car. For example, when riding at 16 km/h, a cyclist uses food energy at a rate of about 1670 kJ/h above what he would use if merely sitting still. Walking at 5 km/h requires about 940 kJ/h.

It is interesting to compare these values with the energy consumption required for travel by car. Petrol yields about 3.4×10^7J per litre. The Volkswagen Golf VII petrol car achieves a fuel efficiency of around 4.2L/100km. The Hyundai Ioniq electric car achieves 15.7 kWh/100km.

Find the fuel efficiency in kilojoules per kilometre of a) walking, b) cycling, c) the Golf petrol car and d) the Ioniq electric car.

Solution: For the cyclist and walker, energy per distance = power over speed, so a) 940/5=190 kJ/km (2 sig. fig.) b) 1670/16=100 kJ/km. c) The petrol car uses $4.2*3.4*10^7$=$1.4*10^8$J to go 100km. So it uses $1.4*10^3$kJ/km. d) 1kWh is 3,600kJ (last week's calculation), so the electric car requires 15.7*3600=57,000kJ for 100km and thus 570kJ/km. Cycling is the most efficient in energy per kilometre, then walking, then the electric car then the petrol car. Note that the standard way of reporting efficiency for electric cars of kWh/100km, is actually a force, although it is not a Newton.

Problem 2: Power and air resistance

A lorry has a mass of 10 tonnes (one tonne equals 1000kg) and a surface area of 6.3m^2. Its aerodynamic drag coefficient is measured to be 0.530. Assume the drag force is proportional to v^2, given by $F = \frac{1}{2}D\rho Av^2$ and ignore other sources of resistive force. Take the density of air in Norway to be 1.29 kg/m^3.

a) Find the power required to maintain a speed of 60km/h on a flat, straight road.

Solution: Use the formula (also in the book, eqn 6.7) to calculate the drag force, $F = \frac{1}{2}D\rho Av^2$. Then the power needed to maintain a speed v is $P = Fv = \frac{1}{2}D\rho Av^3 = \frac{1}{2} * 0.53 * 1.29 * 6.3 * (60 * 1000/3600)^3 = 10,000$W (2 sig. fig.).

b) Find the power required to maintain a speed of 80km/h on the same flat, straight road.

Solution: Same as above. $P = Fv = \frac{1}{2}D\rho Av^3 = \frac{1}{2} * 0.53 * 1.29 * 6.3 * (80 * 1000/3600)^3 = 24,000$W.

c) If the energy yield of diesel fuel in litres per 100 km is the same at either speed, how much more in percent does it cost to go one kilometre at 80km/h relative to 60km/h?

Solution: Since energy per distance is power divided by speed, the ratio of the energy needed (and hence cost) is the ratio of the powers divided by the ratio of the speeds. So (24000/10000)/(80/60)=1.8. It costs 80% more to go at 80km/h, relative to 60km/h (a 33% increase in speed).

d) Find the power required to maintain a speed of 60 km/h on an incline of 10%.

Solution: The percentage slope here (as used on Norwegian road signs) indicates the ratio of the rise (opposite) over the run (adjacent), so $\tan\theta = $ opposite/adjacent $= 0.1$ which implies $\theta = 5.7°$. To maintain a constant speed going uphill, the engine must not only overcome the drag due to air resistance, but also the component of the weight acting down along the slope, $mg\sin\theta$. So the total force to be applied by the engine is $\frac{1}{2}D\rho Av^2 + mg\sin\theta = \frac{1}{2} * 0.53 * 1.29 * 6.3 * (60 * 1000/3600)^2 + 10000 * 9.82 * \sin(5.7) = 598 + 9753 = 10351\text{N}$ and the power needed is force*velocity=10351*60*1000/3600=170,000W (2 sig. fig.).

Problem 3: Work done in a non-constant gravitational field

At distances far from the Earth's surface, the work done by gravity cannot be approximated by mgh. Consider a cannon, shooting projectiles with speed v_i, straight up in the air from the surface of the Earth. Assume that the Earth is spherical, and has radius R_e. The force of gravity is given by

$$\mathbf{F}_g = -\frac{GMm}{r^2}\hat{\mathbf{r}}. \tag{1}$$

where M is the mass of the Earth 5.97×10^{24}kg, $m = 5.00$kg is the mass of the projectile, and $G = 6.67 \times 10^{-11}\text{m}^3\text{s}^{-2}\text{kg}^{-1}$ is Newton's gravitational constant.

a) Explicitly compute the work done by gravity on the projectile, as it flies from the Earth's surface $r = R_e$, with speed v_i, to some height $r = R_f$, with speed v_f.

Solution: The work done is (using that $d\mathbf{r}$ is along the radial direction):

$$W = \int_{R_e}^{R_f} -\frac{GmM}{r^2}\hat{\mathbf{r}} \cdot d\mathbf{r} = -\frac{GmM}{R_e} + \frac{GmM}{R_f} \tag{2}$$

b) We define the escape velocity (*unnslipningshastighet*) to be the value that v_i should have, such that $v_f \to 0$ as $R_f \to \infty$. What is the escape velocity for the Earth? For the Sun (mass 1.99×10^{30}kg)?

Solution: The total energy of the projectile is conserved. The kinetic energy it has at launch is converted into potential energy by the work done by gravity. Its kinetic energy and potential energy at infinity can formally be taken to be zero. We find

$$\frac{GmM}{R_e} = \frac{1}{2}mv_i^2 \rightarrow v_i = \sqrt{\frac{2GM}{R_e}} \equiv v_e. \tag{3}$$

For the Earth, $v_e = 11.2$ km/s. For the Sun, $v_e = 618$ km/s.

c) The Schwarzschild radius of a gravitating body of mass M is the value R_s, such that had the launch radius been R_s, with the mass all contained within this radius, the escape velocity would have been equal to the speed of light (c=3×10^8m/s). What is R_s for the Earth ignoring atmospheric effects? For the Sun? Gravitating bodies that are smaller than their Schwarzschild radius are called "black holes".

Solution: We find

$$R_s = \frac{2GM}{c^2}, \tag{4}$$

which is 2.95 km for the Sun, and 8.86 mm for the Earth.

d) Going back to part a) and b). What if the projectile was not launched straight up, but at an angle? What would then be the escape velocity?

Solution: It doesn't matter. Because the force only depends on $\hat{\mathbf{r}}$, only the radial component of the path increment $d\mathbf{r}$ matters when computing the work. The work only depends on the starting and end positions, and relates to the magnitude of $\mathbf{v}$, not the direction. So angle doesn't matter...as long as it is not negative and you shoot into the ground, of course.

Four marks for question one, one for each part. Six marks for question two, one each for parts a and b, two each for parts c and d. Five marks for question 3, one each for parts a, b and c and two marks for part b. Total marks 15. 6 marks needed to pass.