Høsten 2019

FYS100 Fysikk: Hand-in VI

To be handed in at the latest **Friday 27th September 2019, at 23.59**. You **must** hand it in by scanning your handwritten solution or compiling your electronic documents into **a single .pdf** file, and uploading it to Canvas in "Week Five Exercises". Bad mobile phone pictures are not acceptable (there are apps that work OK), nor are any other file formats than a single .pdf.

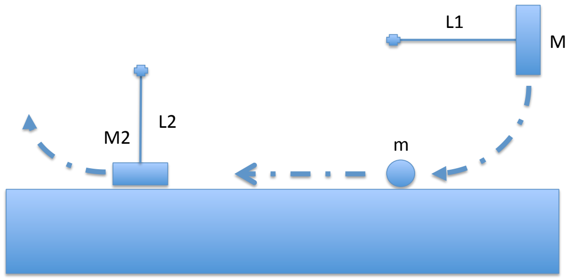
You **must**:

- Put your name and student number on each page.
- Make sketches for all problems, where it makes sense.
- Write in a readable, well-structured way.

Pass is 40% correct (including partial credit). There is no grade. There is **no option** to correct and resubmit.

Good luck!

Problem 1: Conservation of momentum and collisions



A hammer of mass $M_1 = 2.00$ kg on a massless stick of length $L_1 = 30.0$ cm is nailed to a wall, so that it is free to turn around the axis (see the figure). Starting at rest from the horizontal position, it swings down and hits a ball of mass $m = 0.100$ kg in an elastic collision. The ball is initially at rest, and when hit proceeds to slide frictionlessly along the table until it reaches a second hammer of mass $M_2 = 1.50$ kg on a massless stick of length $L_2 = 20.0$ cm. In this collision, the ball attaches itself to the hammer, and the hammer and ball swing up to a height h.

a) What is the speed of the ball as it slides along the table? (Expression and number, please!)

Solution: There is energy conservation as the hammer swings down, so that its speed at the bottom is

$$\frac{1}{2}M_1 v_1^2 = M_1 g L_1 \rightarrow v_1 = \sqrt{2gL_1}. \tag{1}$$

Then it performs an elastic collision with the ball, with the ball initially at rest, so

$$m v_b + M_1 v_h = M_1 v_1, \tag{2}$$
$$\frac{1}{2}m v_b^2 + \frac{1}{2}M_1 v_h^2 = \frac{1}{2}M_1 v_1^2. \tag{3}$$

Solving this gives

$$v_b = \frac{2v_1}{1+\frac{m}{M_1}} = \frac{\sqrt{8gL_1}}{1+\frac{m}{M_1}} = 4.62 \text{ m/s}. \tag{4}$$

b) What is the height h? (Expression and number, please!)

Solution: The second collision is completely inelastic, so that

$$(M_2 + m)v_2 = mv_b \to v_v = \frac{m}{M_2 + m}v_b. \tag{5}$$

Then energy conservation applies as the hammer swings up

$$(M_2 + m)gh = \frac{1}{2}(M_2 + m)v_2^2 \to h = \frac{1}{2g}v_2^2 = 4L_1\frac{m^2}{(M_2 + m)^2}\frac{M_1^2}{(M_1 + m)^2}, \tag{6}$$

which for the numbers given is 4.25 mm.

c) How much of the initial energy in the hammer M_1 ends up in the hammer/ball $(M_2 + m)$ system at the end (as mechanical energy)? (Expression and number, please!)

Solution: Simply compute the ratio of potential energies:

$$\frac{(M_2 + m)gh}{M_1gL_1} = \frac{4m^2M_1}{(M_1 + m)^2(M_2 + m)} = 0.0113 \tag{7}$$

Problem 2: Centre of mass (Problem 9.50 in the book)

A water molecule consists of an oxygen atom (atomic mass 15.999) with two hydrogen atoms (atomic mass 1.0079) bound to it (H_2O). The angle between the two bonds is 106° (this angle is related to the six-fold symmetry of snowflakes). If the bonds between the oxygen atom and the hydrogen atoms are 0.100 nm long, where is the centre of mass of the molecule?

Solution: By symmetry, the centre of mass must lie below the oxygen atom. The centre of mass of the two hydrogen atoms is a distance $0.1\cos(106/2)$nm below the oxygen atom (taking the oxygen atom to be at the top). Relative to the oxygen atom then, the centre of mass of the entire molecule is thus $2 * 1.0079 * 0.1 * \cos 53/(2 * 1.0079 + 15.999) = 0.0067$nm below the oxygen atom.

Problem 3: Rocket equation

The sounding rockets that are launched from Andøya Space Centre in Nordland are typically multi-stage rockets. The first stage of these rockets has a thrust of 516kN and the entire rocket has a weight of 5,500kg.

a) Calculate the acceleration of the rocket when it is first launched.

Solution: The thrust is the force applied by the rocket. Since the rocket is launched from the ground we should not ignore the acceleration due to gravity, although for small heights (suborbital) it can be assumed to be constant. So by Newton's second law, the acceleration is F/m-g=516000/5500-9.8=84m/s^2 (2 sig. fig.).

b) The first stage of the rocket burns fuel at a constant rate of 160kg/s. Calculate the exhaust speed of the first stage and compare it to the speed of sound in air.

Solution: By conservation of momentum (or equivalently equation 9.45 in the book, given in lectures) the exhaust speed is given by the thrust divided by the rate of change of mass. So $v_e = 516000/160 = 3200$m/s. This is approximately 9 or 10 times the speed of sound (v_e=343m/s).

c) The first stage of the rocket burns for only approximately 5.20s. Using the Tsiolkovsky rocket equation, calculate the change in speed of the rocket during this time.

Solution: Burning at a rate of 160kg/s for 5.2s gives a total mass loss of 832kg. The Tsiolkovsky rocket equation (9.44 in the book) $v_f - v_i = v_e ln(m_i/m_f) = 3230 * ln(5500/(5500 - 832) = 530$m/s. This is the speed change due to the mass exhaust. We also need to include the speed change due to gravity, which acts with the same value independently of the mass of the rocket (which is changing). The speed change due to gravity is just g*t=9.8*5.2=51m/s. So total speed change is 530-51=480m/s (2 sig. fig.)

d) Using the answer in part (c), calculate an average acceleration for the rocket during the 5.20s burn-time of the first stage.

Solution: Average acceleration is change in speed over change in time. 480/5.2=92m/s^2.

e) If the rocket is burning fuel at a constant rate and thus providing a constant thrust, why is the average acceleration for the 5.20s burn-time calculated in part (d), not equal to the initial acceleration calculated in part (a) (why is the acceleration changing)?

Solution: The force is constant, but the acceleration is changing because the mass of the rocket is changing as mass is lost through the exhaust

Six marks for question one, two for each part. Four marks for ques-

tion two. Five marks for question 3, one for each part. Total marks 15. 6 marks needed to pass.