Høsten 2019

FYS100 Fysikk: Hand-in VII

To be handed in at the latest **Friday 4th October 2019, at 23.59**.
You **must** hand it in by scanning your handwritten solution or compiling your electronic documents into **a single .pdf** file, and uploading it to Canvas in "Week Five Exercises". Bad mobile phone pictures are not acceptable (there are apps that work OK), nor are any other file formats than a single .pdf.

You **must**:

- Put your name and student number on each page.
- Make sketches for all problems, where it makes sense.
- Write in a readable, well-structured way.

Pass is 40% correct (including partial credit). There is no grade. There is **no option** to correct and resubmit.

Good luck!

Problem 1: Angular speed
In this problem, assume the Earth to be a uniform sphere of radius 6.37×10^6m, that makes one revolution every 24 hours.

a) Find the angular speed of a point on the Equator (at latitude 0°N) .

Solution: It takes 24 hours to turn 2π radians at a constant rate, so the angular speed $\omega = 2\pi/(24 * 60 * 60) = 7.27 \times 10^{-5}$ s^{-1}.

b) What is the tangential speed of this point on the Equator?

Solution: Tangential speed $v = \omega r = 7.27 * 6.37 * 10 = 463$m/s

c) What is the centripetal acceleration of this point on the Equator? Compare this value to the acceleration due to gravity on the Equator (9.78m/s^2). Why is it smaller or larger?

Solution: Centripetal acceleration $a = v^2/r = 463^2/6.37 \times 10^6 = 0.0337$ms^{-2} This is much smaller than the acceleration due to gravity. Objects still fall to the ground at the Equator. The Earth is a solid object, not a gravitationally bound system of free particles.

d) Find the angular speed of a point in Stavanger (latitude 59.0°N). Compare this to the answer in part (a).

Solution: Since the time taken to go around is the same (24 hours), the angular speed is the same (this is why angular speed is useful) 7.27×10^{-5} s^{-1}.

e) Find the tangential speed and centripetal acceleration of the same point in Stavanger.

Solution: We need the radius of rotation at Stavanger (it is not the radius of the Earth). Trigonometry shows $r_S = r_E \cos 59 = 3.28 \times 10^6$m. Then the tangential speed is $v = \omega r = 7.27*3.28*10 = 238$m/s. The tangential acceleration is $a = v^2/r = 238^2/3280000 = 0.0173$ms^{-2}.

f) Using the results of part (c) and part (e), compare the acceleration due to gravity at the Equator and the acceleration due to gravity in Stavanger (9.82m/s^2). What accounts for the difference? Are all the assumptions valid?

Solution: The difference in acceleration due to gravity means that objects are not held as tightly to the Earth at the Equator as they are in Stavanger. The difference in centripetal acceleration accounts for part of the difference (around 40%) but the rest is due to the fact the Earth is not a perfect sphere and Stavanger is closer to the centre than the equator is. In this question we assumed the Earth is a perfect sphere, which it is not at the level of three significant figures.

Problem 2: Rotational energy:

A smooth solid cube of mass m and edge length r slides with speed v on a horizontal surface with negligible friction. The cube then moves up a smooth incline that makes an angle θ with the horizontal.

Additionally, a solid cylinder of mass m and radius r rolls horizontally without slipping with its centre of mass moving with speed v and encounters an incline of the same angle of inclination but with sufficient friction that the cylinder continues to roll up the incline without slipping.

a) Which object will go a greater distance up the incline before stopping?

b) Explain what accounts for this difference in distance travelled.

c) Find an algebraic expression for the difference between the maximum distances the objects travel up the incline.

Solution:

a) and b) The rotational motion of the cylinder provides additional initial energy, that may be transferred to gravitational potential energy. Hence it can go higher up the incline.

c) For the cube, energy conservation determines that

$$\frac{1}{2}mv^2 = mgh = mgd\sin\theta \to d = \frac{v^2}{2g\sin\theta}. \tag{1}$$

For the cylinder, we have

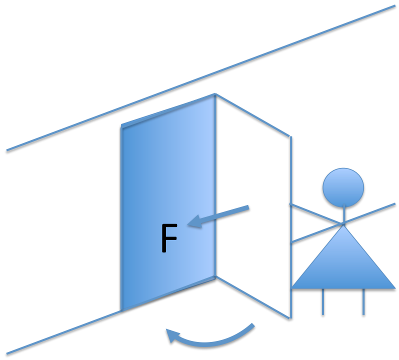
$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgd\sin\theta. \tag{2}$$

With the moment of inertia of a cylinder $I = mr^2/2$, we ge

$$\frac{3}{4}mv^2 = mgd\sin\theta \to d = \frac{3v^2}{4g\sin\theta}. \tag{3}$$

So the cylinder goes $3/2 = 1.5$ times as far up the incline.

Problem 3: Moment of inertia, torque and angular acceleration



A door of width $l = 1.00$ m and mass $M = 15.0$ kg is attached to a door frame by two hinges. For this problem, you may ignore gravity, as we are interested in rotational motion around the vertical axis. There is no friction of any kind. A student slams the door shut, by pushing at the middle of the door ($l/2$ from the hinges) with a force of $F = 100N$, lasting a time $\Delta t = 0.200$ s. The door is initially not rotating. The door can be taken to be a uniform rod, for the purpose of this exercise.

a) What is the moment of inertia of the door?

Solution: $I = Ml^2/3 = 5\text{kg m}^2$ for a uniform rod around its end

b) What torque does the student apply to the door when pushing it?

Solution: $\tau = Fl/2 = 50\text{Nm}$

c) What is the angular acceleration of the door while it is being pushed?

Solution: $\alpha = \tau/I$, $\tau = Fl/2$, and $I = Ml^2/3$ for a uniform rod around its end. Hence $\alpha = 3F(/2Ml) = 10/s^2$.

d) What is the resulting angular velocity, angular momentum and rotational kinetic energy from this push?

Solution: $\omega_f = \alpha\Delta t = 2/s$. $L = I\omega_f = Ml^2/3\alpha\Delta t = 10kgm^2/s$. $E_{kin} = I\omega_f^2/2 = 10J$.

e) Assuming that she lets go of the door (leaving it to slam shut) at the moment in the motion when the door is perpendicular to the wall, how long does it take for the door to close, ignoring air resistance?

Solution: The door has to turn an angle of $\pi/2$, with angular velocity ω_f, so $t_{shut} = \pi/(2\omega_f) = 0.8s$.