Høsten 2019

FYS100 Fysikk: Hand-in VIII

To be handed in at the latest **Friday 11th October 2019, at 23.59**.
You **must** hand it in by scanning your handwritten solution or compiling your electronic documents into **a single .pdf** file, and uploading it to Canvas in "Week Eight Exercises". Bad mobile phone pictures are not acceptable (there are apps that work OK), nor are any other file formats than a single .pdf.

You **must**:

- Put your name and student number on each page.
- Make sketches for all problems, where it makes sense.
- Write in a readable, well-structured way.

Pass is 40% correct (including partial credit). There is no grade. There is **no option** to correct and resubmit.

Good luck!

Six marks for problem three, two for each part. Four marks for question two, one for each part. Five marks for question one, three for part (a) and one each for parts (b) and (c). 15 marks in total. Six needed for pass.

Problem 1: Torques and acceleration

A car accelerates from rest on a horizontal surface. The engine provides a torque of $\tau = 200$ Nm on each of the two front wheels. Each of the four wheels on the car weigh $m = 15$ kg, have radius $R = 35.0$ cm and can be considered solid, uniform discs. The rest of the car (not the wheels) has a mass of $M = 1000$ kg.

a) What is the acceleration of the car?

Solution: Write down the relations for front (1) and back (2) wheels and for the middle bit. Assume that the wheels interact with the middle bit with forces F_1 and F_2.

$$2F_1 - 2F_2 = Ma, \tag{1}$$
$$-F_1 + f_s^1 = ma, \tag{2}$$
$$F_2 - f_s^2 = ma, \tag{3}$$
$$\tau - f_s^1 R = I\alpha, \tag{4}$$
$$f_s^2 R = I\alpha. \tag{5}$$

Adding them up, one gets

$$2\frac{\tau}{R} = (M + 4m + \frac{I}{R^2})a \rightarrow a = \frac{2\tau/R}{M + 6m} = 1.05 \text{ m/s}^2. \tag{6}$$

The car now starts going up an incline of angle $\theta = 15°$.

b) If the engine is turned off, what is now the acceleration of the car?

Solution: Same type of problem, but now there are gravity components, no torque and some of the forces push/pull the other way.

$$2F_1 - 2F_2 - Mg\sin\theta = Ma, \tag{7}$$
$$-F_1 + f_s^1 - mg\sin\theta = ma, \tag{8}$$
$$F_2 - f_s^2 - mg\sin\theta = ma, \tag{9}$$
$$-f_s^1 R = I\alpha, \tag{10}$$
$$f_s^2 R = I\alpha. \tag{11}$$

Adding them up, one gets

$$2\frac{\tau}{R} = (M + 4m + \frac{I}{R^2})a \to a = \frac{-(M+4m)}{M+6m} g \sin\theta = 2.47 \text{ m/s}^2. \tag{12}$$

c) What should the torque on each of the front wheels be, so that the car continues up the hill at constant speed?

Solution: Again the same, but restore the torque, and set equal to zero.

$$\begin{aligned} 2F_1 - 2F_2 - Mg\sin\theta &= Ma, & (13)\\ -F_1 + f_s^1 - mg\sin\theta &= ma, & (14)\\ F_2 - f_s^2 - mg\sin\theta &= ma, & (15)\\ -f_s^1 R &= I\alpha, & (16)\\ f_s^2 R &= I\alpha. & (17) \end{aligned}$$

Adding them up, one gets

$$2\frac{\tau}{R} = (M + 4m + \frac{I}{R^2})a \to a = \frac{2\tau/R - (M+4m)g\sin\theta}{M+6m} = 0, \tag{18}$$

$$\to \tau = \frac{(M+4m)gR\sin\theta}{2} = 471 \text{ Nm}. \tag{19}$$

Problem 2: Conservation of momentum and kinetic energy
(Question in Norwegian)

En ishockeypuck med hastighet $v_1 = 40$m/s treffer en annen ishockeypuck som ligger i ro på et glatt underlag. De to puckene har samme masse. Etter kollisjonen observerer man at den ene pucken kommer ut i en vinkel $\alpha = 30°$ og den andre i en vinkel $\beta = 45°$ i forhold til den opprinnelige innfallsretningen.

a) Hvor stor er hastigheten til de to puckene like etter kollisjonen?

Solution: Momentum is conserved, both the component parallel to the original motion and perpendicular to the original motion. Parallelly we have

$$mv_1 = mv_a \cos 30 + mv_b \cos 45. \tag{20}$$

Perpendicularly we have

$$0 = mv_a \sin 30 - mv_b \sin 45. \tag{21}$$

This system of simultaneous equations can be solved to give

$$v_a = v_1 \tan 45/(\cos 30 \tan 45 + \sin 30) = 2v_1/(\sqrt{3}+1) = 29\text{m/s}, \quad (22)$$
$$v_b = v_1 \tan 30/(\cos 45 \tan 30 + \sin 45) = \sqrt{2}v_1/(1+\sqrt{3}) = 21\text{m/s}. \quad (23)$$

since $\tan 30 = 1/\sqrt{3}$, $\sin 30 = 1/2$, $\cos 30 = \sqrt{3}/2$, $\sin 45 = \cos 45 = 1/\sqrt{2}$ and $\tan 45 = 1$

b) Hvor stor brøkdel av den kinetiske energien går tapt i kollisjonen?

Solution: The relative change in kinetic energy is (after cancelling factors of two and the mass) $(v_a^2 + v_b^2 - v_1^2)/v_1^2 = -19\%$. So 19% is lost.

c) Er kollisjonen elastisk eller uelastisk?

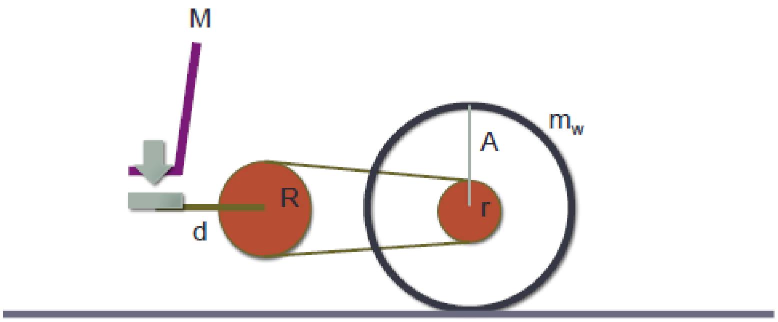
Solution: The collison is inelastic (uelastisk) because kinetic energy is not conserved.

d) Er energi konservert i kollisjonen? Hva skjer der med den tapte kinetiske energi?

Solution: Total energy is conserved, but kinetic energy is not conserved. The lost kinetic energy goes into sound energy, thermal energy and vibrational energy.

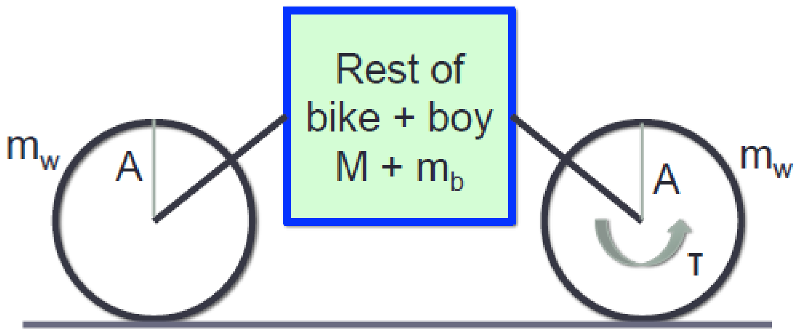
Problem 3: Torques and acceleration (from 2016 Konte-exam)

See next page



A boy of mass M is on his bike going along a horizontal road, and puts all his weight on the pedal to force the wheel round. The pedal is attached by a rod of length d to the center of a cog wheel of radius R. Which is in turn connected by a chain to a smaller cog wheel of radius r, which is attached to the back wheel (as on a standard bike). All rods, chains and cog wheels can be taken to be massless. There is no kinetic friction, and everything is turning without sliding. There is gravity g.

a) If the boy stands on the pedal when it is at "9 o'clock" (see figure), what is the torque provided to the back wheel? The bike starts from rest.



The bike itself is taken to have mass m_b plus the mass of its two wheels, each providing m_w, plus M for the boy. The wheels have radius A and the moment of inertia of a ring $I = m_w A^2$.

b) Assuming that the back wheel and front wheel roll without sliding on the ground, what is the acceleration of the bike + boy? Hint: Consider the back wheel, front wheel and middle part (bike+boy) as sub-systems.

c) Assuming he started at rest, how fast is he going by the time he gets the pedal to position "6 o'clock"? Ignore all sources of kinetic friction.

For each question, provide an algebraic expression and a sketch.