Høsten 2019

FYS100 Fysikk: Hand-in IX

To be handed in at the latest **Friday 18th October 2019, at 23.59**.
You **must** hand it in by scanning your handwritten solution or compiling your electronic documents into **a single .pdf** file, and uploading it to Canvas in "Week Nine Exercises". Bad mobile phone pictures are not acceptable (there are apps that work OK), nor are any other file formats than a single .pdf.

You **must**:

- Put your name and student number on each page.
- Make sketches for all problems, where it makes sense.
- Write in a readable, well-structured way.

Pass is 40% correct (including partial credit). There is no grade. There is **no option** to correct and resubmit.

Good luck!

Problem 1: Torque, springs and angular acceleration

Some seesaws in the playgrounds of Stavanger have two springs attached to them, as shown in the photograph above. Using the physics of FYS100, we can analyse why these springs are attached.

a) Write an expression for the force exerted by a spring when displaced a distance Δx from its equilibrium position (this is in chapter 7 (or even chapter 15) of the text book).

Solution: This is Hooke's law. The students need to recognise this as it will be used in lectures in coming weeks. $\mathbf{F} = -k\mathbf{\Delta x}$. They need to show that the force acts back towards the equilibrium position, so the minus sign is important.

b) Assume that the springs are identical and at equilibrium when the seesaw is horizontal. A father and daughter now sit on the seesaw at each end and the seesaw is seen to settle with an angle θ to the horizontal. Draw a diagram of the relevant forces acting on the seesaw when it is dispaced by this angle, taking care to indicate the correct direction of each force.

Solution: The weight forces should be acting directing downwards, even though the seesaw should be at an angle to the horizontal. The compressed spring force should be acting upwards and the stretched spring should be acting downwards (both back towards the equilibrium position). There is also the weight of the seesaw itself, acting downwards from the centre of mass and a normal force at the pivot point acting upwards.

c) Derive an algebraic expression for the net torque around the pivot point of the seesaw as a function of the weights of the father and daughter, the forces from the two springs, the distances along the seesaw from the pivot point that these forces act and the angle θ that the seesaw is displaced fom the horizontal.

Solution: Torque is given by $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$. The forces are not acting perpendicularly to the line that joins them to the pivot point. The angle between the forces and the pivot arm is $90 - \theta$ so there are factors of $\cos\theta$ in the torque magnitudes. The torque however still act perpendicularly to the plane of the seesaw. The displacement of the springs is given by $\Delta x = r_s \sin\theta$ and the torques of the two springs add together as they both torque to seesaw in the same direction. In terms of components then, the total torque is $\tau_{net} = r_f m_f g \cos\theta - 2r_s^2 k \cos\theta \sin\theta - r_d m_d g \cos\theta$.

The distance of the father r_f and the distance of the daughter r_d can be taken to be the same and a trignometirc identity can be applied to the spring torque term $\sin 2\theta = 2 \sin\theta \cos\theta$. The weight of the seesaw and the normal force do not generate torques around the pivot point, because they act at the pivot point. At the equilibrium point, the net torque should be zero.

d) Suppose now that the father has a mass of 80kg, the daughter has a mass of 15kg and the angle at which the seesaw settles is 5°. The humans sit a distance 1m from the pivot point and the springs acts at a distance 0.2m. Find the value of the spring constant.

Solution: From the above we have $k = (r_f(m_f - m_d)g)/(2r_s^2 \sin\theta = (1 * (80 - 15) * 9.82)/(2 * 0.2^2 * \sin 5°) = 92000\text{N/m}$

e) If the seesaw is returned to horizontal (for example, by the father pushing on the ground with his legs), in what direction will the net torque point when the father lifts his legs off the ground?

Solution: If the father is seen sitting on the left of the seesaw, the net torque will point in the same direction as the father's torque and will point towards the observer (right hand rule). If the father sits on the right hand side, the net torque will point in the opposite direction (compare to the diagram drawn in part (b)).

f) After lifting his legs off the ground, the seesaw swings back to 5°. When the seesaw reaches this point, does it continue turning in the same direction, start turning in the opposite direction or immediately stop turning? Explain your answer in terms of the torque and angular velocity.

Solution: The seesaw will continue turning in the same direction because of rotational inertia and conservation of angular momentum. The net torque is zero at this point but the angular velocity is non zero. The angular momentum and angular velocity will only change when there is a net torque acting.

g) If the seesaw is displaced 10° from the horizontal, in what direction does the net torque point? Compare to your answer in part (e).

Solution: The net torque will point in the opposite direction as in part (e) because now it will be dominated by the force from the springs.

h) How is this behaviour different from a seesaw with no springs attached to it? What is the benefit of having the springs attached?

Solution: The springs provide a restorative force that keeps the seesaw oscialating around an equilibrium point. With unequal masses on the seesaw (a father and daughter) this equilibrium point is not on the ground. Without the springs, the seesaw would just swing until the father was on the ground. The springs thus allow people of different masses to play on the seesaw without too much effort (it is even possible to play alone). The springs also provide a safety buffer that stops someone being slammed into the ground if the other person suddenly jumps off the seesaw.

Problem 2: Rotational motion and kinetic energy (question in Norwegian)

En sirkulær skive i et horisontalt plan roterer fritt og uten friksjon om en vertikal akse gjennom sentrum. Vinkelhastigheten er $\omega = 0.1\text{s}^{-1}$. En likedan skive, som til å begynne med ikke roterer, plasseres oppå den første slik at den akkurat dekker.

a) De to skivene begynner å rotere med samme vinkelhastighet. Finn den felles vinkelhastigheten.

Solution: Angular momentum $L = I\omega$ for the system is conserved (because there is no easy way to dissipate it). The two disks are identical and so have the same moment of inertia. Thus the final moment of inertia is double the moment of inertia of one disk. Thus the final common angular velocity must be half the initial angular velocity of the spinning disk $\omega_f = \omega/2 = 0.05\text{s}^{-1}$.

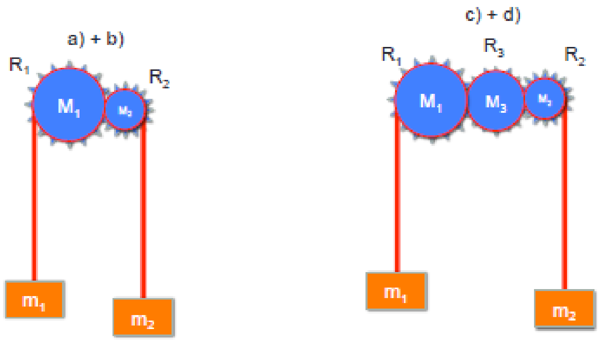
b) Finn den eventuelle relative forandringen i total kinetisk energi.

Solution: Angular kinetic energy is $\frac{1}{2}I\omega^2$. Since the final spinning body has double the moment of inertia of one of the initial disks and the angular velocity has been halved, the total kinetic energy is also halved. The final kinetic energy is 50% less than the initial kinetic energy.

c) Hvis energi går tapt fra rotasjonen, hvor går den til? Hvis energi er vunnet, hvor kommer den fra?

Solution: Rotational kinetic energy is lost. It is primarily lost to friction, because friction must act between the two disks when they are placed together to bring them to the same speed.

Problem 3 (see next page): Cog wheels (2016 exam)



Two cog wheels connect with each other, and each have a rope wound around them. Two blocks of masses $m_1 = 4.00$ kg and $m_2 = 3.00$ kg hang at the end of each rope. The cog wheels/pulleys have mass $M_1 = 2.00$ kg and radius $R_1 = 0.400$ m and mass $M_2 = 5.00$ kg and radius $R_2 = 0.300$ m, respectively, and can be thought of as uniform discs, $I^i_{\mathrm{CM}} = M_i R_i^2/2$. The ropes unwind without sliding. There is gravity $g = 9.80$ m/s^2. See Figure (left).

a) Draw a sketch with all the relevant forces and torques of the problem. What are the relations between the various accelerations and angular accelerations in the problem (the no-slipping conditions)? Think carefully about the force between the two cog wheels/pulleys.

b) What is the acceleration of the blocks? What is the angular acceleration of each cog wheel/pulley?

We now add a third cog wheel between the first two, of mass $M_3 = 1.00$ kg and radius $R_3 = 0.350$ m. It is also a uniform disc. See Figure (right).

c) What is now the acceleration of the blocks?

d) Now we imagine that the third wheel is turned by an engine, providing a torque of 10.0 Nm, counterclockwise. What is now the acceleration of the blocks?

For each question, provide a sketch, an algebraic expression, as well as the numerical result.