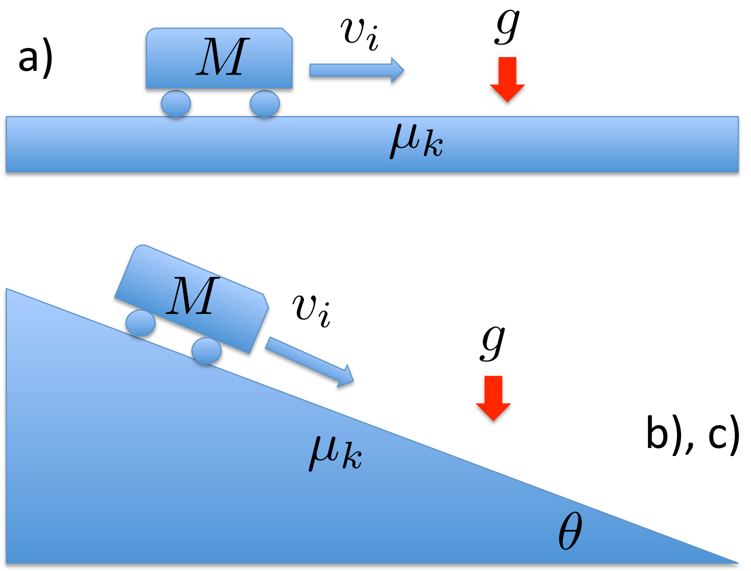
Høsten 2018

FYS100 Fysikk
Konte-Eksamen/Re-Exam
Solution

Problem 1: Hi, ho, hi, ho, we're sliding down the street... (25 points)



A car of mass $M = 1.00 \times 10^3$ kg drives along a horizontal road. Suddenly, the driver remembers having again forgotten to pick up his small, innocent children at the barnehage, and he slams on the brakes. In the following we will assume that when braking, the wheels stop turning completely, and the car just slides. $g = 9.80\ \mathrm{m/s^2}$.

a) If his initial speed is $v_i = 50.0$ km/h and the coefficient of kinetic friction between road and car is $\mu_k = 0.800$, how long does it take the car to stop, and how far does it slide before coming to rest?

Solution: 1-D kinematics means that $t = v_i/a$ and $s = v_i t + at^2/2$. The friction force is $F = Mg\mu_k$, and the resulting acceleration $a = -g\mu_k$. As a result, we have

$$t = \frac{v_i}{\mu_k g} = 1.77\,\mathrm{s}, \tag{1}$$

$$s = \frac{v_i^2}{2g\mu_k} = 12.3\ \mathrm{m}. \tag{2}$$

b) Now answer the same questions, in the case where the car is originally driving down an incline at $\theta = 15.0°$ with horizontal.

Solution: Now the normal force is different, and a component of gravity pulls the car forwards, against the force of friction. We have

$$Ma = Mg(\sin\theta - \mu_k \cos\theta), \tag{3}$$

so that

$$t = \frac{v_i}{g(\mu_k \cos\theta - \sin\theta)} = 2.76\,\text{s}, \tag{4}$$

$$s = \frac{v_i^2}{2g(\mu_k \cos\theta - \sin\theta)} = 19.2 \text{ m}. \tag{5}$$

c) How steep would the incline have to be (in terms of θ), in order for him not to be able to stop at all? Would it be possible for him to park his car on an incline with some angle larger than that? Why/why not?

Solution: The expression for the time and distance go to infinity when

$$\mu_k \cos\theta = \sin\theta \rightarrow \tan\theta = 0.8 \rightarrow \theta = 38.7°. \tag{6}$$

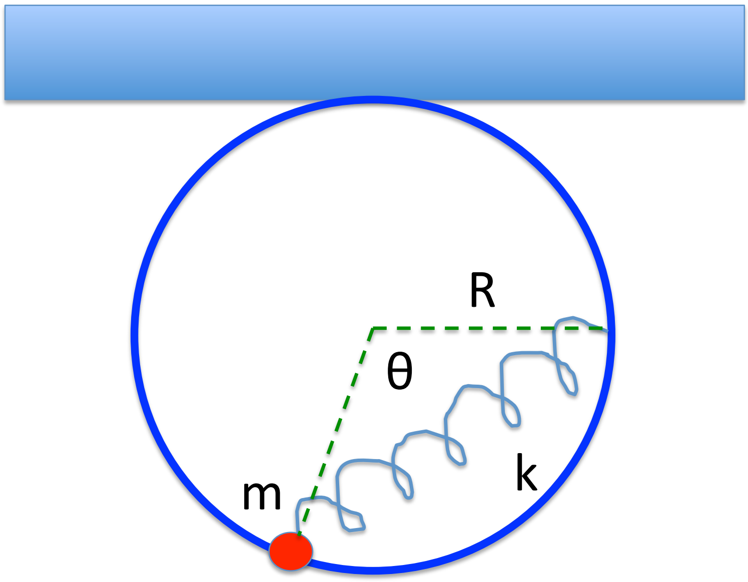
Usually, the coefficient of static friction (relevant for parking) is larger than the coefficient of kinetic friction. So there is a range of angles larger than 38.7°, where the car can be parked, but he would not be able to brake a moving car.

d) What expressions for the time and sliding distance would you have expected from dimensional analysis?

Solution: Dimensional analysis just gives $t = v_i/g$ and $s = v_i^2/g$. But no mention of friction coefficients and angle, because they are dimensionless.

Typical mistakes: Writing down wrong force equation (including v_i, say). Doing kinematics wrong. On the incline, forgetting the gravity component along the incline and/or along the normal force. Calculational mistakes. Getting the tangent upside down in c). Not knowing what dimensional analysis is and/or works.

Problem 2: A bead on a spring on a ring... (20 points)



A bead of mass m is made to slide around on a ring of radius R standing on its side. The bead is attached to one end of a spring with string constant k, and the other end of the spring is attached to one point on the ring, halfway up one side (see figure). There is gravity, g, and you may ignore friction. You may assume that the length of the unstretched spring (when there is no tension) is $\simeq 0$.

a) What is the combined potential function of the system, as a function of the angle θ (see figure)?

Solution: The gravity contribution is

$$U_g = mgR(1 - \sin\theta), \tag{7}$$

while for the spring, we have, using Pythagoras

$$U_s = kR^2(1 - \cos\theta). \tag{8}$$

b) What are the equilibrium points of the system, and are they stable or unstable?

Solution: Take the derivative of the total potential, to find

$$\frac{dU}{d\theta} = -mgR\cos\theta + kR^2\sin\theta. = 0 \rightarrow \tan\theta = \frac{mg}{kR}. \tag{9}$$

Since all constants are positive, that gives two solutions, one in the first quadrant (bottom right part of the ring in the picture), one in the third (top left part of the ring). Note that without gravity ($g = 0$), we get the angles 0 and 180 degrees. And without the spring, we get angles 90 and 270 degrees.

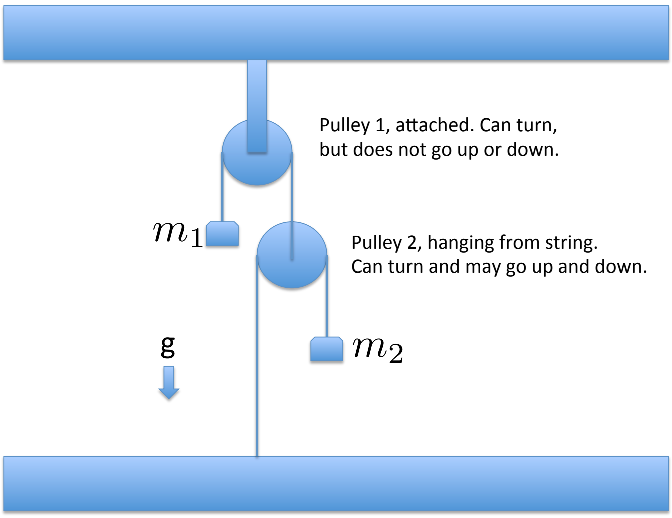
Stability follows from the second derivative

$$\frac{d^2U}{d\theta^2} = mgR\sin\theta + kR^2\cos\theta = \frac{kR^2}{\cos\theta}, \tag{10}$$

which is positive for the first quadrant solution (stable, bottom right of ring) and negative for the third quadrant solution (unstable, top left of ring).

Typical mistakes: Forgetting the gravity component or the spring component. Getting the geometry wrong (cos and sin). Thinking that equilibrium is when the potential function is zero. Calculational mistakes. Not knowing what stable and unstable is (second derivative positive and negative).

Problem 3: Masses and pulleys. (25 points)



A pulley hangs from the ceiling. Over it, a light string hangs with a mass m_1 at one end and a second pulley at the other. Over the second pulley hangs a second light string, with a mass m_2 at one end, and with the other end attached to the floor. There is gravity g, and in the following, you may ignore kinetic friction and assume that the pulleys are massless. The pulleys turn with the strings without sliding.

a) What is the relation between the two accelerations of the two masses m_1 and m_2? Between the two string tensions T_1 and T_2?

Solution: System geometry show that $a_2 = 2a_1$. Since the pulleys are massless, $T_1 = 2T_2$ to cancel out at pulley 2.

b) What are the accelerations of the masses? What are the string tensions?

Solution: Write down equations

$$m_2 a_2 = m_2 g - T_2, \tag{11}$$
$$m_1 a_1 = T_1 - m_1 g, \tag{12}$$

combining with the constraints, one finds

$$a_2 = 2a_1 = 2\frac{2m_2 - m_1}{m_1 + 4m_2}g, \tag{13}$$
$$T_1 = 2T_2 = \frac{6m_1 m_2}{m_1 + 4m_2}g. \tag{14}$$

c)What if the pulleys had not been massless, but with masses M_1 and M_2, and both of radius R? What are then the accelerations of the masses m_1, m_2? For the purpose of the moments of inertia, you may consider the pulleys to be discs, $I = MR^2/2$. Carefully reconsider the relations between the accelerations and angular accelerations (or speeds and angular speeds) of masses and pulleys, and the motion of the pulleys.

Solution: The string tensions on either side of the pulleys is now different, and the second pulley also accelerates.

$$m_2 a_2 = m_2 g - T_2', \tag{15}$$
$$m_1 a_1 = T_1 - m_1 g, \tag{16}$$
$$M_1 a_p = M_1 g + T_2 + T_2' - T_1' \tag{17}$$
$$(T_1' - T_1)R = I_1 \alpha_1, \tag{18}$$
$$(T_2' - T_2)R = I_2 \alpha_2, \tag{19}$$
$$a_1 = a_p = a_2/2 = R\alpha_1 = R\alpha_2. \tag{20}$$

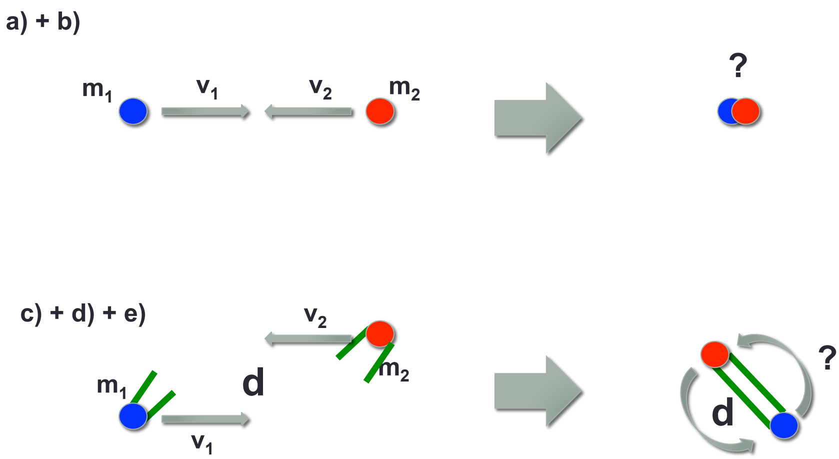
These can be solved to find

$$a_2 = 2a_1 = 2\frac{2m_2 - m_1 + M_1}{m_1 + 4m_2 + M_1 + I_1/R^2 + I_2/R^2}g = 2\frac{2m_2 - m_1 + M_1}{m_1 + 4m_2 + 3M_1/2 + M_2/2}g \tag{21}$$

Energy considerations will give the same (obviously...).

Typical mistakes: Getting relation between accelerations upside down. Getting tension relation upside down. Calculational mistakes. Forgetting gravity. Getting that the directions of the accelerations positive in an inconsistent way (coordinate systems wrong). Forgetting to solve for the string tensions. In c) forgetting that the string tensions on opposite sides of the pulleys is different.

Problem 4: Frost'y Collisions (30 points)



Elsa and Anna are skating on an icy lake, and approach each other from opposite directions with speeds v_1 and v_2. They have masses m_1 and m_2, and collide head-on, in such a way that their collision can be thought of as perfectly inelastic (see Figure, top line). The surface of the lake can be taken as frictionless.

a) What is their final speed v_f after the collision?

Solution: Momentum is conserved, and we find

$$m_1 v_1 - m_2 v_2 = (m_1 + m_2) v_f \rightarrow v_f = \frac{m_1 v_1 - m_2 v_2}{m_1 + m_2}. \tag{22}$$

Whether one defines v_2 to be positive or negative, will determine the sign in the numerator.

b) Is kinetic energy conserved (calculate it!)? If not, how much did the kinetic energy change, and where did it go/come from?

Solution: Kinetic energy is not conserved, and explicitly

$$\begin{aligned}\Delta K &= \frac{1}{2}(m_1 + m_2) v_f^2 - \frac{1}{2} m_1 v_1^2 - \frac{1}{2} m_2 v_2^2 \\ &= -\frac{m_1 m_2}{2(m_1 + m_2)} (v_1 + v_2)^2,\end{aligned} \tag{23}$$

which is lost to internal energy.

The girls now skate apart again, turn around and perform a trick where they pass at a distance d, but so that they just manage to grab hold of each others hands (se Figure, bottom line). As a result, they start spinning around.

c) Where is their joint center of mass, as they collide (so when they are right "next to" each other, and their hands connect)? What is their combined moment of inertia around the center of mass I_{CM}? (Think of Elsa and Anna as point particles, their arms are massless).

Solution: Put the origin at the initial position of m_1 and define the initial position of m_2 to be (x_0, d). Then the CM at any time is at

$$r_{\text{CM}} = \frac{m_2}{m_1 + m_2}(x_0, d) + \frac{m_1 v_1 - m_2 v_2}{m_1 + m_2} t(1, 0). \quad (24)$$

At "collision", it is just the y-component $r_{\text{CM}} = m_2 d/(m_1 + m_2)$. One could have chosen the origin of the coordinate system to be halfway between the girls, shifting the y-coordinate by $d/2$. That's ok.
The moment of inertia relative to this CM is in any case

$$I_{\text{CM}} = m_1 r_{\text{CM}}^2 + m_2 (d - r_{\text{CM}})^2 = \frac{m_1 m_2}{m_1 + m_2} d^2. \quad (25)$$

d) After the collision, what is the speed of the center of mass v_{CM}?

Solution: The speed of the CM follows from momentum conservation or just by differentiation of r_{CM}. Either way, it is as before

$$m_1 v_1 - m_2 v_2 = (m_1 + m_2) v_f \rightarrow v_f = \frac{m_1 v_1 - m_2 v_2}{m_1 + m_2}. \quad (26)$$

e) What is the angular momentum around the CM, just before the collision? What is the angular speed of rotation ω around the center of mass after the collision?

Solution: Angular momentum conservation around the CM (for instance in the CM frame) gives

$$m_1 (v_1 - v_{\text{CM}}) r_{\text{CM}} + m_2 (v_2 + v_{\text{CM}})(d - r_{\text{CM}}) = \frac{m_1 m_2}{(m_1 + m_2)} d(v_1 + v_2) = I_{\text{CM}} \omega. (27)$$

That means that $\omega = (v_1 + v_2)/d$.

Typical mistakes: Using energy conservation in a). Not doing the calculation as required in b). Thinking that energy is conserved. Calculational mistakes. Confusion when choosing origin of CM.