LØSNINGSFORSLAG MAT 100 H19

① a) $z = 3-2i \quad , \quad w = 4+5i$

$$zw = (3-2i)(4+5i) = 12 + 15i - 8i - 10i^2$$

$$= 12 + 10 + 7i = \underline{\underline{22 + 7i}}$$

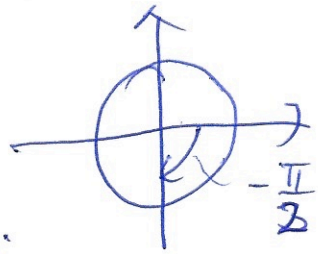
$$\frac{1}{z} = \frac{\bar{z}}{z\bar{z}} = \frac{3+2i}{|z|^2} = \frac{3+2i}{9+4} = \underline{\frac{3}{13} + \frac{2}{13}i}$$

$$\bar{z}^2 = (3+2i)^2 = 3^2 + 2\cdot 3\cdot 2i + 2^2 i^2$$

$$= 9 - 4 + 12i = \underline{\underline{5 + 12i}}$$

b) $z^3 + 8i = 0$

$\Rightarrow z^3 = -8i$



3. RØTTER TIL $-8i = 8e^{-\frac{\pi}{2}i}$

$$z_1 = 8^{\frac{1}{3}} e^{-\frac{\pi}{2\cdot 3}i} = 2e^{-\frac{\pi}{6}i} = 2\cdot\left(\frac{1}{2}\sqrt{3} - \frac{1}{2}i\right)$$

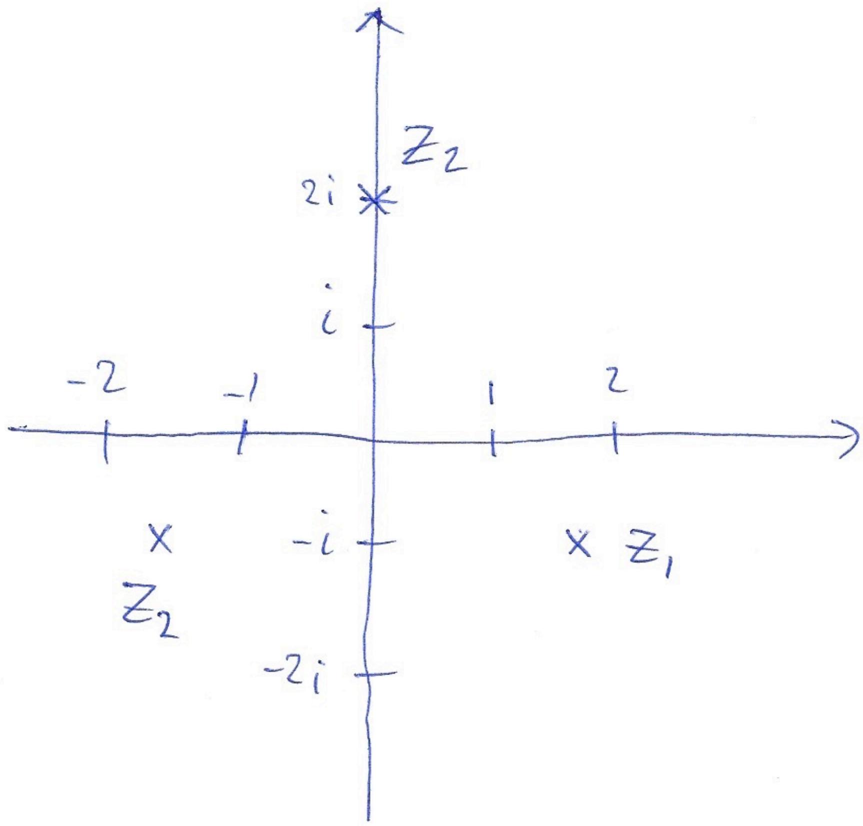
$$= \underline{\sqrt{3} - i}$$

$$z_2 = 8^{\frac{1}{3}} e^{-\frac{\pi}{2\cdot 3}i + \frac{2\pi}{3}i} = 2e^{\frac{3\pi}{6}i} = 2e^{\frac{\pi}{2}i} = \underline{2i}$$

$$z_3 = 8^{\frac{1}{3}} e^{-\frac{\pi}{2\cdot 3}i + \frac{4\pi}{3}i} = 2e^{\frac{7\pi}{6}i} = 2\left(-\frac{1}{2}\sqrt{3} - \frac{1}{2}i\right)$$

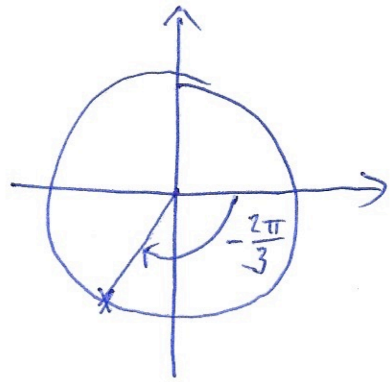
$$= \underline{-\sqrt{3} - i}$$

LØSNINGER: $\underline{\underline{z_1 = \sqrt{3} - i,\ z_2 = 2i,\ z_3 = -\sqrt{3} - i}}$



c) $$\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)^{2019}$$

$$-\frac{1}{2}-\frac{\sqrt{3}}{2}i = e^{-\frac{2\pi}{3}i}$$



$$\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)^{2019} = \left(e^{-\frac{2\pi}{3}i}\right)^{2019}$$

$$= e^{-\frac{2\pi}{3}\cdot 2019 i}$$

2019 ER DELELIG PÅ 3 SIDEN TVERRSUM = 12.

$$\Rightarrow \left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)^{2019} = e^{-2\pi\cdot n} = \underline{\underline{1}}$$ SIDEN $n \in \mathbb{N}$

② a) $\int \left(4x^{5/2} - \sin(2x)\right) dx = 4\cdot\frac{2}{7}x^{7/2} - \left(-\frac{1}{2}\cos 2x\right) + C$

$$= \frac{8}{7}x^{7/2} + \frac{1}{2}\cos 2x + C$$

b) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

$$u = \sqrt{x}$$
$$du = \frac{1}{2\sqrt{x}} dx$$
$$2du = \frac{1}{\sqrt{x}} dx$$

$$= \int e^{u} \cdot 2du = 2e^{u} + C$$

$$= 2e^{\sqrt{x}} + C$$

c) $\int \frac{x-18}{(x-2)(x^2+4)} dx$

DEL BRØKOPPSPALTING:

$$\frac{x-18}{(x-2)(x^2+4)} = \frac{A}{x-2} + \frac{Bx+C}{(x^2+4)} = \frac{A(x^2+4)+(Bx+C)(x-2)}{(x-2)(x^2+4)}$$

MÅ MATCHE!

$$x - 18 = A(x^2+4) + (Bx+C)(x-2)$$

FOR :

$\underline{x=2}$: $2-18 = A\cdot(2^2+4) + (B\cdot 2+C)\cdot 0$

$-16 = 8A \Rightarrow \underline{A=-2}$

$\underline{x=0}$: $-18 = A\cdot(4) + (B\cdot 0 + C)(-2)$

$-18 = 4A - 2C$

$2C = 4A + 18 = 4\cdot(-2) + 18 = 10$

$\underline{C = 5}$

$\underline{x=1}$: $1-18 = A(1^2+4) + (B\cdot 1 + C)(-1)$

$-17 = 5A - B - C$

$B = 5A - C + 17 = 5\cdot(-2) - 5 + 17 = \underline{2}$

$$\int \frac{x-18}{(x-2)(x^2+4)}\,dx = \int\left(\frac{-2}{x-2} + \frac{2x+5}{x^2+4}\right)dx$$

$$= -2\ln|x-2| + \int\left(\frac{2x}{x^2+4} + \frac{5}{x^2+2^2}\right)dx$$

$$= -2\ln|x+2| + \ln(x^2+4) + \frac{5}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$$

d) $\int \frac{x}{\sqrt{2+x}}\,dx = \int \frac{u-2}{\sqrt{u}}\,du$ $\quad u = 2+x$

$du = dx$

$= \int \left(\sqrt{u} - \frac{2}{\sqrt{u}}\right)du = \frac{2}{3}u^{3/2} - 4\sqrt{u} + C$

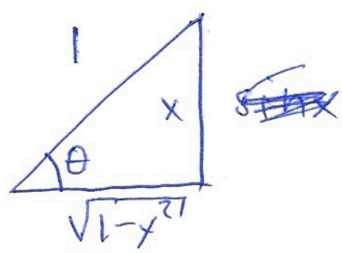
$$= \underline{\underline{\frac{2}{3}(2+x)^{3/2} - 4\sqrt{2+x} + C}}$$

3 a) $\int \sin^2 x\,dx = \int \frac{1}{2}(1-\cos 2x)\,dx$ $\quad \boxed{\sin^2 x = \frac{1}{2}(1-\cos 2x)}$

$$= \underline{\underline{\frac{1}{2}x - \frac{1}{4}\sin 2x + C}}$$

$$= \underline{\underline{\frac{1}{2}(x - \sin x \cos x) + C}}$$

b) $\int_0^1 \frac{x^2}{\sqrt{1-x^2}}\,dx$



$x = \sin\theta$

$dx = \cos\theta\,d\theta$

GJØR UBESTEMTE INT. FØRST:

$$\int \frac{x^2}{\sqrt{1-x^2}}\,dx = \int \frac{\sin^2\theta}{\cos\theta}\cdot\cos\theta\,d\theta = \int \sin^2\theta\,d\theta$$

(FRA a)) $$= \frac{1}{2}(\theta - \sin\theta\cos\theta) + C$$

$$= \underline{\frac{1}{2}\left(\sin^{-1}x - x\sqrt{1-x^2}\right) + C}$$

$$\int_0^1 \frac{x^2}{\sqrt{1-x^2}}\,dx = \left(\frac{1}{2}\sin^{-1}x - x\sqrt{1-x^2}\right)\Bigg|_0^1$$

$$= \frac{1}{2}\sin^{-1}1 - 1\cdot 0 - (\sin^{-1}0 - 0\cdot 1)$$

$$= \frac{1}{2}\cdot\frac{\pi}{2} = \underline{\underline{\frac{\pi}{4}}}$$

4 a)

$$\begin{cases} y' + \frac{4}{x}y = x^3 \\ y(1) = 0 \end{cases}$$

$y' + \frac{4}{x}y = x^3$, 1. ORDENS LINEÆR.

INT. FAKTOR: $e^{\int \frac{4}{x}dx} = e^{4\ln x} = \underline{x^4}$

$$y' + \frac{4}{x} y = x^3 \qquad \Big| \cdot x^4$$

$$x^4 y' + 4x^3 y = x^7$$

$$\frac{d}{dx}(x^4 y) = x^7$$

$$x^4 y = \int x^7 dx = \frac{1}{8} x^8 + C$$

$$y = \frac{\frac{1}{8} x^8 + C}{x^4} = \underline{\frac{1}{8} x^4 + \frac{C}{x^4}}$$

INITIAL BET:

$$y(1) = 0 \Rightarrow \frac{1}{8} \cdot 1^4 + \frac{C}{1^4} = \frac{1}{8} + C = 0$$

$$\Rightarrow \underline{C = -\frac{1}{8}}$$

LØSNING: $y = \frac{1}{8} x^4 - \frac{1}{8x^4} = \underline{\underline{\frac{1}{8}\left(x^4 - \frac{1}{x^4}\right)}}$

b) $y'' - 4y' + 8y = 0$

KAR. LIGN:

$$r^2 - 4r + 8 = 0$$

$$r = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 8}}{2 \cdot 1} = \frac{4 \pm \sqrt{16-32}}{2}$$

$$= \frac{4 \pm \sqrt{-16}}{2} = \frac{4 \pm 4i}{2} = \underline{2 \pm 2i}$$

$$\underline{\underline{y = Ae^{2x}\cos 2x + Be^{2x}\sin 2x}}$$

c) $y'' - 4y' + 8y = x^2 + 1$

y_H: HOMOGEN LØSNING FRA a).

PARTIKULÆR LØSNING: GJETTER PÅ

$y_P = ax^2 + bx + C$

$y_P' = 2ax + b$

$y_P'' = 2a$

$$\Rightarrow y_p'' - 4y_p' + 8y_p = 2a - 4(2ax+b) + 8(ax^2+bx+c)$$

$$= 8ax^2 + (-8a+8b)x + 2a - 4b + 8c$$

$$= x^2 + 1$$

MATCHER KOEFFISIENTENE

x^2: $8a = 1$

x: $-8a + 8b = 0$

1: $2a - 4b + 8c = 1$

$$\underline{a = \frac{1}{8}}$$

$$b = a = \underline{\frac{1}{8}}$$

$$c = \frac{1-2a+4b}{8} = \frac{1 - 2\cdot\frac{1}{8} + \frac{4}{8}}{8}$$

$$= \frac{1 - \frac{1}{4} + \frac{1}{2}}{8} = \frac{4-1+2}{32} = \underline{\frac{5}{32}}$$

LØSNING

$$y = y_h + y_p = Ae^{2x}\cos 2x + Be^{2x}\sin 2x$$

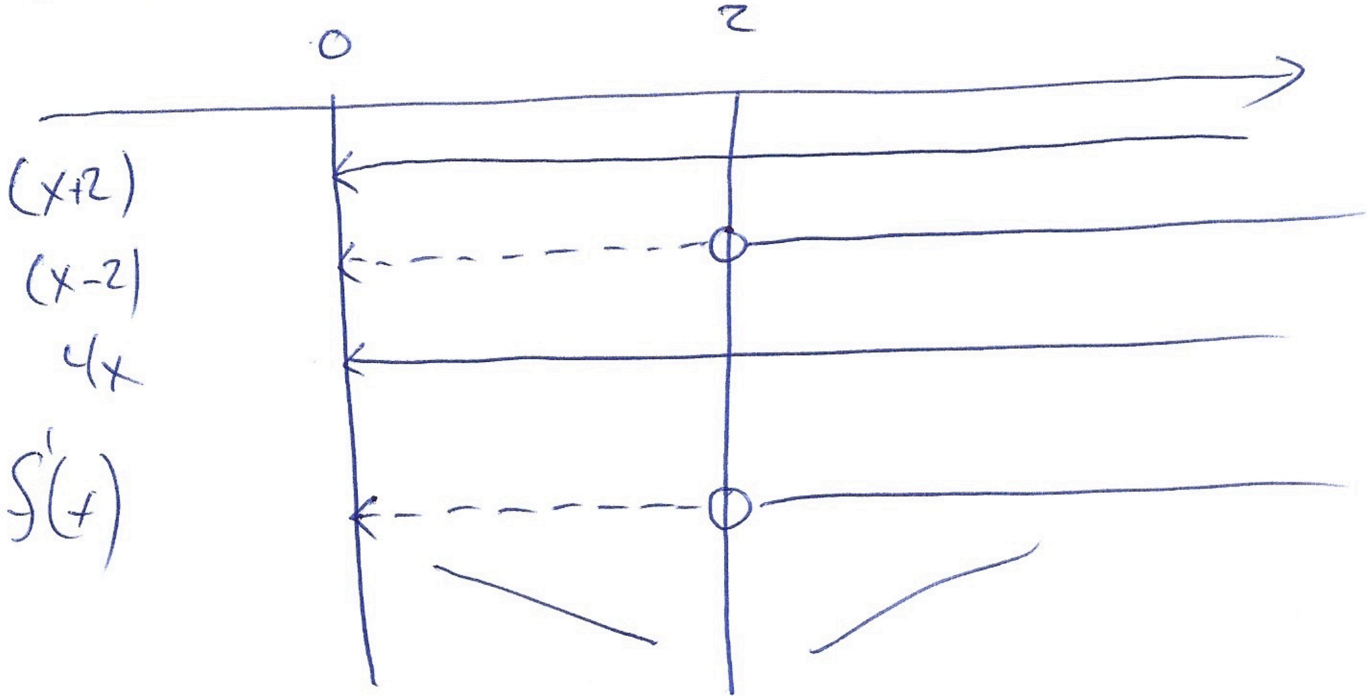
$$\underline{+ \frac{1}{8}x^2 + \frac{1}{8}x + \frac{5}{32}}$$

⑤ $f(x) = \frac{1}{8}x^2 - \ln x, \qquad x > 0$

a) $f'(x) = \frac{1}{4}x - \frac{1}{x} = \frac{1 \cdot x^2}{4x} - \frac{4}{4x}$

$= \frac{x^2 - 4}{4x} = \underline{\frac{(x+2)(x-2)}{4x}}$

FORTEGNSKJEMA



f MINKER PÅ $\langle 0, 2]$

f VOKSENDE PÅ $[2, \infty)$

BUNNPUNKT: $\underline{x = 2}$

$y = \frac{1}{8} \cdot 2^2 - \ln 2 = \underline{\frac{1}{2} - \ln 2}$

$\underline{\underline{\left(2, \frac{1}{2} - \ln 2\right)}}$

b) FOR $\underline{x=1}$, $y=f(1)=\frac{1}{8}\cdot 1^2-\ln 1=\underline{\frac{1}{8}}$

$y=ax+b$

TANGENT: $a=f'(1)=\frac{1^2-4}{4\cdot 1}=-\frac{3}{4}$

GJENNOM $(1,\frac{1}{8})$

$\frac{1}{8}=-\frac{3}{4}\cancel{x}1+b \Rightarrow b=\frac{1}{8}+\frac{3}{4}=\underline{\frac{7}{8}}$

$$\underline{\underline{y=-\frac{3}{4}x+\frac{7}{8}}}$$

NORMALEN: $a=-\frac{1}{f'(1)}=\frac{4}{3}$.

GJENNOM $(1,\frac{1}{8})$

$\frac{1}{8}=\frac{4}{3}\cdot 1+b \Rightarrow b=\frac{1}{8}-\frac{4}{3}=\frac{3-32}{24}=\underline{-\frac{29}{24}}$

$$\underline{\underline{y=\frac{4}{3}x-\frac{29}{24}}}$$

c) BUE LENGDEN:

$$S = \int_1^4 \sqrt{1 + (f'(x))^2}\, dx$$

INTEGRANDEN:

$$\sqrt{1 + (f'(x))^2} = \sqrt{1 + \left(\frac{1}{4}x - \frac{1}{x}\right)^2}$$

$$= \sqrt{1 + \frac{x^2}{16} - 2 \cdot \frac{1}{4} + \frac{1}{x^2}} = \sqrt{\frac{x^2}{16} + \frac{1}{2} + \frac{1}{x^2}}$$

$$= \sqrt{\left(\frac{x}{4} + \frac{1}{x}\right)^2} = \left(\frac{x}{4} + \frac{1}{x}\right)$$

$$S = \int_1^4 \left(\frac{x}{4} + \frac{1}{x}\right) dx = \left(\frac{1}{8}x^2 + \ln x\right)\Big|_1^4$$

$$= \left(\frac{1}{8}4^2 + \ln 4\right) - \left(\frac{1}{8} \cdot 1^2 + \ln 1\right)$$

$$= \underline{\underline{\frac{15}{8} + 2\ln 2}}$$

(6) a) $\frac{dN}{dt} = -kN$

$\Rightarrow N(t) = N_0 e^{-kt}$

$N(0) = N_0 = 2$

$\underline{\underline{N(t) = 2e^{-kt}}}$

b) FINNER k FØRST: (t I ÅR)

$N(5700) = 2e^{-k \cdot 5700} = 1$

$\Rightarrow 2 = e^{5700k}$ (TAR ln)

$\ln 2 = 5700k$

$\underline{k = \frac{1}{5700} \ln 2}$

30%:

$N(t) = 0{,}30 N_0 = 2 \cdot 0{,}30 = 2e^{-kt}$

$0{,}30 = e^{-kt}$

$t = -\frac{\ln 0{,}30}{k} = -5700 \frac{\ln 0{,}30}{\ln 2} \approx \underline{\underline{9900}}$ (ÅR)