LØSNINGSFORSLAG V19 MAT100

① a) $zw = (1+2i)(-2-3i) = -2-3i+2i\cdot(-2)+2i(-3i)$

$$= -2-3i-4i+6 = \underline{\underline{4-7i}}$$

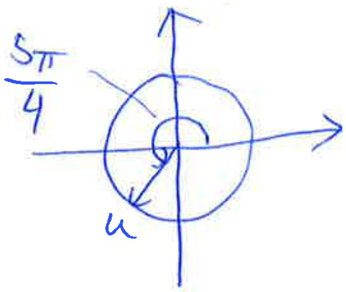
$$\frac{1}{w} = \frac{\overline{w}}{w\overline{w}} = \frac{-2+3i}{(-2)^2+(-3)^2} = \frac{-2+3i}{13}$$

$$= \underline{\underline{-\frac{2}{13}+\frac{3}{13}i}}$$

$$|z|^2 = 1^2+2^2 = \underline{\underline{5}}$$

b) $u = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$

$$|u| = \sqrt{\left(-\frac{\sqrt{2}}{2}\right)^2+\left(-\frac{\sqrt{2}}{2}\right)^2} = \sqrt{\frac{1}{2}+\frac{1}{2}} = 1$$



$$u = \underline{1\cdot e^{\frac{5\pi}{4}i}} = \underline{\underline{e^{\frac{5\pi}{4}i}}}$$

$$u^4 = \left(e^{\frac{5\pi}{4}i}\right)^4 = e^{\frac{5\pi}{4}i\cdot 4} = e^{5\pi i} = \underline{-1}$$

$\Rightarrow$ u ER EI $\underline{\underline{\text{4-ROT AV } -1}}$

c) $-1 = e^{\pi i}$

4. RØTTER:

$$z_1 = e^{\frac{\pi i}{4}} = \underline{\underline{\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i}}$$

$$z_2 = e^{\frac{\pi i}{4} + \frac{2\pi i}{4}} = e^{\frac{3\pi i}{4}} = \underline{\underline{-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i}}$$

$$z_3 = e^{\frac{\pi i}{4} + \frac{4\pi i}{4}} = e^{\frac{5\pi}{4} i} = \underline{\underline{-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i}}$$

$$z_4 = e^{\frac{\pi}{4} i + \frac{6\pi i}{4}} = e^{\frac{7\pi}{4} i} = \underline{\underline{\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i}}$$

$$z^5 + z = 0$$

$$z \cdot (z^4 + 1) = 0$$

$$\underline{z = 0} \quad \vee \quad z^4 + 1 = 0$$

$$\Downarrow$$

$$z^4 = -1$$

$\hookrightarrow$ $\underline{z = \text{4. RØTTER TIL } -1}$

2 a) $\int (2x^{3/2} - \sin 2x)\,dx$

$$= 2 \cdot \frac{2}{5} x^{\frac{5}{2}} + \frac{1}{2}\cos 2x + C$$

$$= \underline{\underline{\frac{4}{5} x^{\frac{5}{2}} + \frac{1}{2}\cos 2x + C}}$$

b) $\int (\ln x)^2\,dx \overset{\text{DECUIS INT.}}{=} x(\ln x)^2 - \int x \cdot 2\ln x \cdot \frac{1}{x}\,dx$

$$= x(\ln x)^2 - 2\int \ln x\,dx \overset{\text{DECUIS INT.}}{=} \dots =$$

$$= x(\ln x)^2 - 2(x\ln x - x) + C$$

$$= \underline{\underline{x(\ln x)^2 - 2x\ln x + 2x + C}}$$

c) $\int (\ln x + 1) e^{x \ln x}\,dx$

$$\boxed{\begin{aligned} u &= x\ln x \\ du &= (\ln x + x\cdot\tfrac{1}{x})dx \\ du &= (\ln x + 1)dx \end{aligned}}$$

$$= \int e^{x\ln x}\,du = \int e^u\,du$$

$$= e^u + C = \underline{\underline{e^{x\ln x} + C}}$$

d) $\int \frac{x^2 - x}{(x^2+4)(x+4)}\,dx$

DELBRØK OPPSPALTING:

$$\frac{x^2 - x}{(x^2+4)(x+4)} = \frac{Ax+B}{x^2+4} + \frac{C}{x+4}$$

$$= \frac{(Ax+B)(x+4) + C(x^2+4)}{(x^2+4)(x+4)}$$

$$x^2 - x = (Ax+B)(x+4) + C(x^2+4)$$

$\underline{x=-4}$: $16-(-4) = (Ax+B)\cdot 0 + C\cdot(16+4)$

$$20 = 20\cdot C \Rightarrow \underline{C=1}$$

$\underline{x=0}$: $0 = B\cdot 4 + C\cdot 4$

$$B = -C = \underline{-1}$$

$x=1$: $0 = (A+B)\cdot 5 + C\cdot 5$

$$0 = 5A + \underbrace{5B + 5C}_{0}$$

$$\Rightarrow \underline{A=0}$$

$$\int \frac{x^2-x}{(x^2+4)(x+4)}dx = \int\left(\frac{-1}{x^2+4} + \frac{1}{x+4}\right)dx$$

$$= -\int \frac{1}{x^2+2^2}dx + \ln|x+4|$$

$$= -\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + \ln|x+4| + C$$

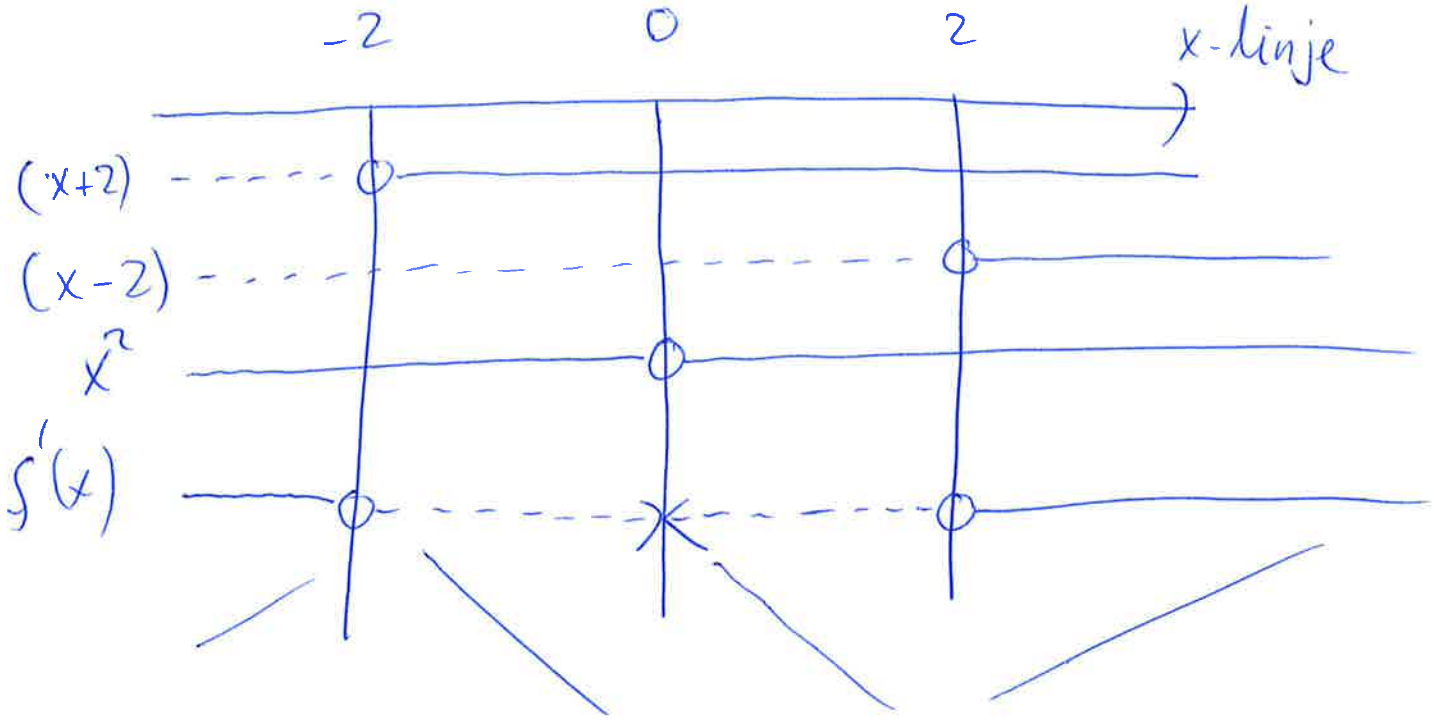
e) $\int \sin^2 x \, dx$ $\qquad \sin^2 x = \frac{1}{2}(1-\cos 2x)$

$$= \int \frac{1}{2}(1-\cos 2x)dx = \frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) + C$$

$$= \frac{1}{2}(x - \sin x \cos x) + C$$

③ $f(x) = 2 + x + \frac{4}{x}$

a) $f'(x) = 1 - \frac{4}{x^2} = \frac{x^2 - 4}{x^2} = \frac{(x+2)(x-2)}{x^2}$



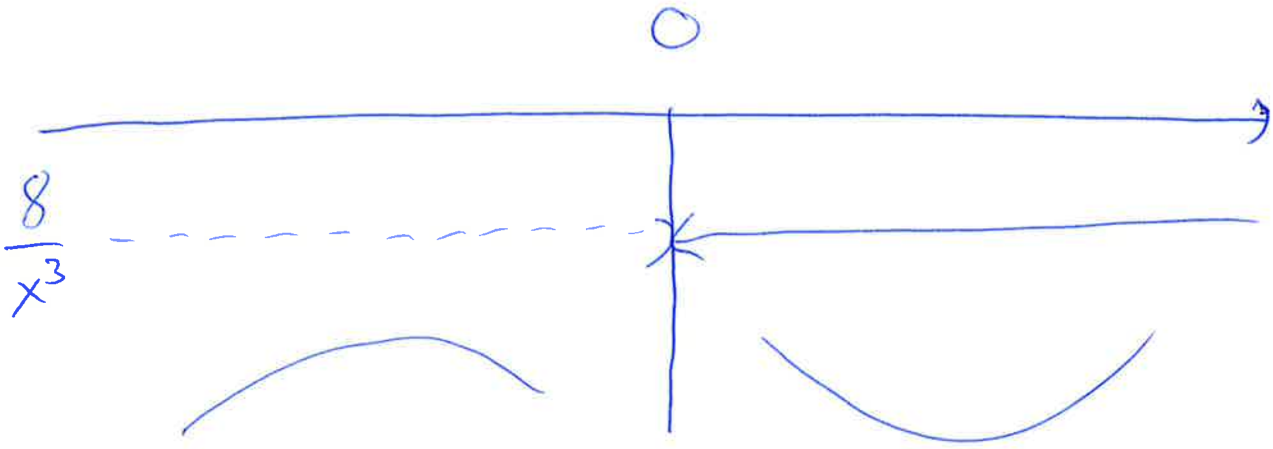
f(x) ER STIGENDE PÅ $\langle \leftarrow, -2]$ OG PÅ $[2, \rightarrow\rangle$

f(x) ER MINKENDE PÅ $[-2, 0\rangle$ OG PÅ $\langle 0, 2]$

MAKS. FOR $x = -2$. $y = f(-2) = 2 - 2 + \frac{4}{-2} = -2$

MIN FOR $x = 2$. $y = f(2) = 2 + 2 + \frac{4}{2} = 6$

b) $f''(x) = -(-2)\cdot\frac{4}{x^3} = \underline{\frac{8}{x^3}}$



KRUMMER OPP PÅ $\underline{\underline{\langle 0, \rightarrow)}}$

KRUMMER NED PÅ $\underline{\underline{\langle \leftarrow, 0\rangle}}$

<u>INGEN VENDEPUNKTER</u>

c) VERTIKAL ASYMPTOTE: $\underline{\underline{x = 0}}$ (NEVNER=0)

SKRÅ ASYMPTOTE: $\underline{\underline{y = 2+x}}$

FORDI: $\lim\limits_{x\to\pm\infty}\left(f(x)-(2+x)\right) = \lim\limits_{x\to\pm\infty}\frac{4}{x} = \underline{0}$

4 a) $$\frac{dy}{dx}+\frac{y}{x}=1$$

$$\mu=\int\frac{1}{x}dx=\ln|x|$$

$$e^{\mu}=e^{\ln|x|}=|x|$$

KAN TA VEKK |·|, SER AT x ER INT. FAKTOR.

$$x\frac{dy}{dx}+y=x$$

$$\frac{d}{dx}(xy)=x$$

$$xy=\int x\,dx=\frac{1}{2}x^2+C$$

$$y=\frac{\frac{1}{2}x^2+C}{x}=\underline{\underline{\frac{1}{2}x+\frac{C}{x}}}$$

b) OMDREINING OM y-AKSEN:

$$V=2\pi\int_0^{\pi}x\cdot f(x)\,dx=2\pi\int_0^{\pi}x(x+\sin x)\,dx$$

$$=2\pi\int_0^{\pi}(x^2+x\sin x)\,dx=$$

④ b) FORTS.

$$V = 2\pi \int_0^{\pi} (x^2 + x\sin x)\,dx = 2\pi \left(\frac{1}{3}x^3 - x\cos x + \sin x\right)\Bigg|_0^{\pi}$$

$$= 2\pi\left(\frac{1}{3}\pi^3 - \pi\cdot\cos\pi + \underbrace{\sin\pi}_{=0} - (0 - 0 + 0)\right)$$

$$= 2\pi\left(\frac{1}{3}\pi^3 + \pi\right) = \underline{\underline{\frac{2}{3}\pi^4 + 2\pi^2}}$$

⑤ a) $\lim_{x\to 1} \frac{\ln x}{x-1}$ $\quad$ "$\frac{0}{0}$"

$$\overset{l'H}{=} \lim_{x\to 1} \frac{\frac{1}{x}}{1} = \underline{\underline{1}}$$

b) $\lim_{x\to\infty} \frac{x\ln x - x}{x^2 - 5x + 6}$ $\quad$ "$\frac{\infty}{\infty}$"

$$\overset{l'H}{=} \lim_{x\to\infty} \frac{\ln x + 1 - 1}{2x - 5} \quad \text{"}\frac{\infty}{\infty}\text{"}$$

$$\overset{l'H}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{2} = \underline{\underline{0}}$$

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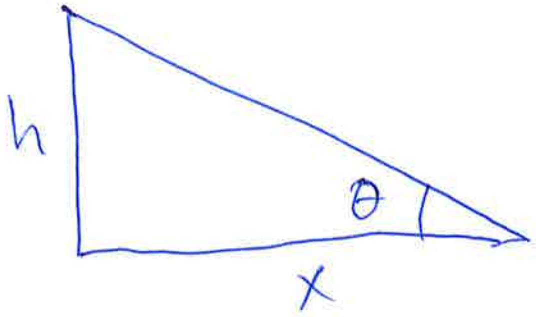

a) $\tan\theta = \frac{h}{x}$

$$\frac{d}{dt}(\tan\theta) = \frac{d}{dt}\left(\frac{h}{x}\right)$$

$$\frac{1}{\cos^2\theta}\frac{d\theta}{dt} = -\frac{h}{x^2}\frac{dx}{dt}$$

$$\Rightarrow \frac{dx}{dt} = -\frac{x^2}{h\cdot\cos^2\theta}\frac{d\theta}{dt} = -\frac{x^2}{8\cos^2\theta}\frac{d\theta}{dt}$$

b) $\frac{d\theta}{dt} = -\frac{\pi}{12}$, $\theta = \frac{\pi}{6}$

$$\tan\frac{\pi}{6} = \frac{8}{x} \Rightarrow x = \frac{8}{\tan\frac{\pi}{6}} = \frac{8}{\frac{1}{\sqrt{3}}} = 8\sqrt{3}$$

$$\left.\frac{dx}{dt}\right|_{\theta=\frac{\pi}{6}} = -\frac{(8\sqrt{3})^2}{8\cos^2\frac{\pi}{6}}\left(-\frac{\pi}{12}\right) = +\frac{8^2\cdot 3}{8\cdot\left(\frac{1}{2}\sqrt{3}\right)^2}\frac{\pi}{12}$$

$$= \frac{8\pi}{\frac{1}{4}\cdot 12} = \frac{8\pi}{3} \simeq 8,38$$