Høsten 2015

FYS100 Fysikk
Problems week 34

Have a go at these. Don't use a calculator except perhaps for the last problem.

Problem 1: Compute the sums:

a) $\frac{1}{2}+\frac{1}{3}$. $\quad \frac{1}{2}+\frac{1}{4}$. $\quad \frac{1}{2}+\frac{1}{5}$. $\quad \frac{1}{2}+\frac{1}{6}$.

b) $\frac{1}{3}+\frac{1}{4}$. $\quad \frac{1}{3}+\frac{1}{5}$. $\quad \frac{1}{3}+\frac{1}{6}$.

c) $\frac{1}{4}+\frac{1}{5}$. $\quad \frac{1}{4}+\frac{1}{6}$.

d) $\frac{1}{5}+\frac{1}{6}$.

e) $\frac{1}{m}+\frac{1}{n}$, for n, m integer.

f) $\frac{r}{m}+\frac{s}{n}$, for n, m, r, s integer.

g) Do the numbers have to be integer?

Solutions: a) $\frac{5}{6}$; $\frac{3}{4}$; $\frac{7}{10}$; $\frac{2}{3}$; b) $\frac{7}{12}$; $\frac{8}{15}$; $\frac{1}{2}$; c) $\frac{9}{20}$; $\frac{5}{12}$; d) $\frac{11}{30}$; e) $\frac{n+m}{nm}$; f) $\frac{rn+sm}{nm}$; g) No, it works for all numbers.

Problem 2: Solve the following quadratic equations:

a) $x^2 - 3x + 4 = 0$.

b) $-6y^2 + 8y = 0$.

c) $2v^2 + 4v - 5 = 0$.

Solutions:
a) $x = \frac{3\pm\sqrt{9-16}}{2} = \frac{3}{2} \pm i\frac{\sqrt{7}}{2}$ (no real solutions).
b) $y = \frac{-8\pm\sqrt{64}}{-12} = 0$ and $\frac{4}{3}$
c) $v = \frac{-4\pm\sqrt{16+40}}{4} = -1 \pm \frac{\sqrt{14}}{2}$

Problem 3: Find all the solutions for θ to the following equations, expressed in both radians and degrees. Draw the solutions on a unit circle:

a) $\cos\theta = 1$. **Solution:** $\theta = 0$ (radians or degrees) as well as $2\pi \times n$ (or $360 \times n$), for any integer n (positive or negative).

b) $\sin\theta = 0.4$. **Solution:** $\theta = 0.412$ (radians, 23.6 degrees). But also $\pi - 0.412$ (180-23.6) and adding $2\pi \times n$ (or $360 \times n$).

c) $\tan\theta = -2$. **Solution:** $\theta = -1.11$ (-63.4 degrees). But also $\pi - 1.11$ (180-63.4) and adding $2\pi \times n$ (or $360 \times n$).

d) $\cos\theta + \sin\theta = 0$. **Solution:** This is the same as $\tan\theta = -1$, $\theta = -\pi/4$ (-45 degrees). But also $\pi - \pi/4$ (180-45) and adding $2\pi \times n$ (or $360 \times n$).

e) $\cos\theta + 3\sin\theta = 0$. **Solution:** This is the same as $\tan\theta = -1/3$, $\theta = -0.322$ (-18.4 degrees). But also $\pi - 0.322$ (180-18.4) and adding $2\pi \times n$ (or $360 \times n$).

Problem 4: Consider a right triangle with sides a and b, and hypotenuse c. Expressed in a, b, c, what is:

a) The sine, cosine and tangent of the angle opposite a? **Solution:** $\sin A = \frac{a}{c}$, $\cos A = \frac{b}{c}$, $\tan A = \frac{a}{b}$.

b) The sine, cosine and tangent of the angle opposite b? **Solution:** $\sin B = \frac{b}{c}$, $\cos B = \frac{a}{c}$, $\tan B = \frac{b}{a}$.

c) The sine, cosine and tangent of the angle opposite c? **Solution:** It's the right angle. $\sin C = 1$, $\cos C = 0$, $\tan C = \infty$.

Problem 5: You have two glasses of wine: One is red wine, the other is white wine. There is exactly the same amount of red and white wine. Now you take a spoonful of wine from the red wine glass and transfer to the white wine glass. You mix. Then you take an equal spoonful of the mixture and put it back into the red wine glass.

Is the red wine now more diluted by white wine? Or the white wine more diluted by red wine? Why?

Solution: They are equally diluted. Two ways to see it: Since you start with equal amounts of red and white, you must end with equal amounts of red and white. And since your start with equal amounts in each glass, and you take a spoonful back and forth, you also end up with equal amount in each glass. This is only possible if as much red has gone into the white as white has gone into the red. The second way is to calculate it (try it): Assume that there is x of red in the red glass and x of white in the white glass. Then you move d (a spoonful) red wine from red to white glass. After this operation, you have

$$\begin{aligned} \text{Red glass :} \quad & x - d \text{ red,} \quad && 0 \text{ white,} && (1) \\ \text{White glass :} \quad & d \text{ red,} \quad && x \text{ white,} && (2) \\ & && && (3) \end{aligned}$$

Then you take d of the mixture back, in which there is the appropriate fraction of red and white respectively. Afterwards, you have

$$\begin{aligned} \text{Red glass :} \quad & x - d + d\tfrac{d}{x+d} \text{ red,} \quad && d\frac{x}{x+d} \text{ white,} && (4) \\ \text{White glass :} \quad & d - d\tfrac{d}{x+d} \text{ red,} \quad && x - d\frac{x}{x+d} \text{ white,} && (5) \end{aligned}$$

If you simplify the expressions, you indeed find

$$\begin{aligned} \text{Red glass :} \quad & \tfrac{x^2}{x+d} \text{ red,} \quad && \frac{dx}{x+d} \text{ white,} && (6) \\ \text{White glass :} \quad & \tfrac{dx}{x+d} \text{ red,} \quad && \frac{x^2}{x+d} \text{ white,} && (7) \end{aligned}$$

Problem 6: There are 8 people at a party. As they get up to leave, everybody shakes the hand of everybody else.

How many handshakes are exchanged, all in all?

Solution: Think of it as one leaving at a time. First guy shakes hands with 7; next guy with 6, then 5 etc. In total $7 + 6 + 5 + 4 + 3 + 2 + 1 = 28$ handshakes. One could also say that each of the 8 persons shakes hand with 7 people, but that it takes two people to shake hands. $8 * 7/2 = 28$.

What if there would have been n persons there (n integer)?

Solution: $n(n-1)/2$ or $\sum_{j=0}^{n-1} j$, which must then be equal.

Problem 7: Consider a perfectly spherical planet of radius $R = 6378$ km, and density $\rho = 7800$ kg/m^3.

a) How big is its diameter? **Solution:** $D = 2R = 12756$ km.

b) How big is its surface? **Solution:** $A = 4\pi R^2 = 5.11 \times 10^8 \text{km}^2$.

c) How big is its volume? **Solution:** $V = \frac{4\pi}{3} R^3 = 1.09 \times 10^{12} \text{km}^3$.

d) What is its mass? **Solution:** $M = M \times \rho = 8.48 \times 10^{24}$ kg. (Note conversion to m from km).

e) Which way is "up"? You may choose your coordinate system as you wish, and then it would be natural to define "up"as along the increasing positive direction of, say the z-coordinate. Then you have to orient the planet in some way of choice. Or you may define "up" to be the direction away from the centre of the planet, in any direction. This would then be the opposite of the pull of gravity.

f) What colour is the planet? Hard to tell. The density suggests iron, and so it would be metallic or maybe black... but it will definitely not be water-coloured, unless painted that way. Density is too high (there could be a thin layer of water around a solid iron planet, I suppose).