

Høsten 2015

# FYS100 Fysikk

## Solutions week 35

Have a go at these!

First some problems from the book:

- 1.9, 1.10, 1.12
- 1.35, 1.38

**Solution:** 1.9. Badly posed problem, since there is no specification of what  $a, x, y, v$  are. Make the point! Of course, they mean that  $x, y$  are lengths,  $a$  is an acceleration and  $v$  are speed. In this case a) is wrong, because the unit of  $ax$  is  $m^2/s^2$  rather than  $m/s$ . And  $b$  is correct, since  $y$  and "2 m" have the same unit, and  $kx$  inside the  $\cos$  has no unit. Worth noting that stuff inside functions ( $e^x$ ,  $\cos(x)$ ,  $\sin(x)$ ,  $\log(x)$ ) must be unit-free!

**Solution:** 1.10. Check the units.  $h$  and  $r$  are lengths, and so b) must be a circumference (d); c) must be a volume (e); and a) must be an area (f). If you feel like it, you can always prove that the expressions are also correct, but it is not required.

**Solution:** 1.12. One may match units as follows:

$$[F] = kg \frac{m}{s^2} = [G] \frac{kg^2}{m^2} \rightarrow [G] = \frac{m^3}{s^2 kg} \text{ or } \frac{Nm^2}{kg^2}. \quad (1)$$

**Solution:** 1.35.  $(21.3 \pm 0.2) \times (9.8 \pm 0.1) = 208.74 \pm 4.09 \simeq (209 \pm 4) \text{cm}^2$

**Solution:** 1.38. a)  $756 + 37.2 + 0.83 + 2 = 796.03 \simeq 796$ .

b)  $0.0032 \times 356.3 = 1.14016 \simeq 1.1$ .

c)  $5.620 \times \pi = 17.6558... \simeq 17.66$ .

**Additional Problem 1:** How many significant digits and decimals are there in the following numbers:

- 1 **solution:** 1 significant, 0 decimals
- 0.12 **solution:** 2 significant, 2 decimals
- 0.0034 **solution:** 2 significant, 4 decimals

- 117.8 **solution:** 4 significant, 1 decimal
- $\pi$  **solution:** infinite significant, infinite decimals

**Additional Problem 2:** Assume that you have measured two quantities  $a = 0.4$  and  $b = 0.2$  with an error of  $\Delta a = 0.1$  and  $\Delta b = 0.03$ , respectively. Here are some functions  $f(a, b)$  of  $a$  and  $b$ :

- $f(a, b) =: a + b; a - b; a/b; b/a; ab.$
- $f(a, b) =: a^2; a^3; a^2b.$
- $f(a, b) =: \log(a)$

Compute their value, and compute an estimate for the error in three ways:

1. Computing "worst case scenarios" (the extreme cases, where the errors on the input are the largest possible). Then use as the estimated error half the difference between the largest and smallest possible values of the output.
2. Computing (you may want to compare this with B.8 of the book)

$$\Delta f(a, b) = \frac{df}{da} \Delta a + \frac{df}{db} \Delta b. \quad (2)$$

3. Computing

$$\Delta f(a, b) = \sqrt{\left(\frac{df}{da} \Delta a\right)^2 + \left(\frac{df}{db} \Delta b\right)^2}. \quad (3)$$

**Solution:** One gets, in order Value, Error 1, Error 2, Error 3 (note that I am not following the rounding-off conventions here; this is to illustrate the difference between the methods, and so I need to keep significant digits far enough that I see that difference!):

0.6, 0.13, 0.13, 0.104;

0.6, 0.13, 0.13, 0.104;

2, 0.82, 0.8, 0.583;

0.5, 0.213, 0.2, 0.146.

0.08, 0.032, 0.0233.

0.16, 0.08, 0.08;

0.064, 0.049, 0.048, 0.048.

0.032, 0.0211, 0.0208, 0.0167.  
−0.916, 0.255, 0.25, 0.25.

Since we are not going to perform full error propagation in this course, we will stick to the rule of thumb: **As many significant digits in the output as the lowest number in the input. This will typically be 3. Unless it is just addition/subtraction: then it is the smallest number of decimals that count.**