Høsten 2015

# FYS100 Fysikk
# Problems week 36

Have a go at these:

First some problems from the book:

- 3.7, 3.8, 3.10 (ignore the word *graphically*).
- 3.23, 3.36

**Solution:** 3.7
Trigonometry tells you that

$$\frac{l}{d} = \tan\theta \to l = d\tan\theta = 70.0 \text{ m}. \tag{1}$$

**Solution:** 3.8
Vector $\vec{\mathbf{A}} = (0, 29)$, and vector $\vec{\mathbf{A}} + \vec{\mathbf{B}} = (0, -14)$. Then $\vec{\mathbf{B}} = (\vec{\mathbf{A}} + \vec{\mathbf{B}}) - \vec{\mathbf{A}} = (0, -43)$.

**Solution:** 3.10
The information given correspond to $\vec{\mathbf{F}}_1 = (6\cos 30°, 6\sin 30°) = (5.20, 3.00)$ and $\vec{\mathbf{F}}_2 = (0.00, 5.00)$. Hence the sum is

$$\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = (5.20, 8.00) \tag{2}$$

**Solution:** 3.23
Given $\vec{\mathbf{A}} = (3, -2)$ and $\vec{\mathbf{B}} = (-1, -4)$, we find

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} = (2, -6), \qquad \vec{\mathbf{A}} - \vec{\mathbf{B}} = (4, 2), \qquad |\vec{\mathbf{A}} + \vec{\mathbf{B}}| = \sqrt{40}, \qquad |\vec{\mathbf{A}} - \vec{\mathbf{B}}| = \sqrt{20}. \tag{3}$$

and

$$\theta_{\vec{\mathbf{A}}+\vec{\mathbf{B}}} = \tan^{-1}\frac{-6}{2} = -71.6°, \qquad \theta_{\vec{\mathbf{A}}-\vec{\mathbf{B}}} = \tan^{-1}\frac{2}{4} = 26.6°. \tag{4}$$

**Solution:** 3.36
Given $\vec{\mathbf{A}} = (3, -4, 4)$ and $\vec{\mathbf{B}} = (2, 3, -7)$, we find

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} = (5, -1, -3), \qquad 2\vec{\mathbf{A}} - \vec{\mathbf{B}} = (4, -11, 15), \tag{5}$$

with magnitudes $\sqrt{35}$ and $\sqrt{362}$, respectively.

**Additional Problem 1:** Consider the vectors (in cartesian coordinates, given some basis and coordinate system),

$$\overrightarrow{\mathbf{A}} = (-1, 4), \qquad \overrightarrow{\mathbf{B}} = (1, 2), \qquad \overrightarrow{\mathbf{C}} = (2, 1). \tag{6}$$

Compute

- The projection of $\overrightarrow{\mathbf{A}}$ onto $\overrightarrow{\mathbf{B}}$.
- The projection of $\overrightarrow{\mathbf{A}}$ onto $\overrightarrow{\mathbf{C}}$.

Is the sum of the projections equal to the original vector $\overrightarrow{\mathbf{A}}$? Why?

**Solution:** The projection is given by

$$\overrightarrow{\mathbf{A}}_{\overrightarrow{\mathbf{B}}} = \frac{\overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{B}}}{\overrightarrow{\mathbf{B}} \cdot \overrightarrow{\mathbf{B}}} \overrightarrow{\mathbf{B}}. \tag{7}$$

Then we find

$$\overrightarrow{\mathbf{A}}_{\overrightarrow{\mathbf{B}}} = \frac{7}{5}(1, 2) = \left(\frac{7}{5}, \frac{14}{5}\right), \tag{8}$$

$$\overrightarrow{\mathbf{A}}_{\overrightarrow{\mathbf{C}}} = \frac{2}{5}(2, 1) = \left(\frac{4}{5}, \frac{2}{5}\right). \tag{9}$$

The sum is $\overrightarrow{\mathbf{A}}_{\overrightarrow{\mathbf{B}}} + \overrightarrow{\mathbf{A}}_{\overrightarrow{\mathbf{C}}} = (11/5, 16/5) \neq (-1, 4)$. This is because the vectors $\overrightarrow{\mathbf{B}}$, $\overrightarrow{\mathbf{C}}$ are not orthogonal.

Find the decomposition of $\overrightarrow{\mathbf{A}}$ onto $\overrightarrow{\mathbf{B}}$ and $\overrightarrow{\mathbf{C}}$. Use whatever method you find simplest.

**Solution:** To find the decomposition, one may use the complicated procedure sketched at lecture. But may also just introduce numbers $c_{1,2}$ and write

$$\overrightarrow{\mathbf{A}} = c_1 \overrightarrow{\mathbf{B}} + c_2 \overrightarrow{\mathbf{C}} \rightarrow (-1, 4) = (c_1 + 2c_2, 2c_1 + c_2) \rightarrow c_1 = 3, \quad c_2 = -2 \tag{10}$$

**Additional Problem 2:** Consider the vectors in 3-D

$$\overrightarrow{\mathbf{A}} = (1, 2, 1), \qquad \overrightarrow{\mathbf{B}} = (2, 1, 2). \tag{11}$$

Compute

- $\overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}}$. **Solution:** (3,0,-3)
- $\overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{B}}$. **Solution:** 6

Find the relative angle between the vectors, using either the scalar or the vector product. Do they agree?

**Solution:**

$$\cos\theta = \frac{\overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{B}}}{|\overrightarrow{\mathbf{A}}||\overrightarrow{\mathbf{B}}|} \to \theta = 35.3^\circ, \tag{12}$$

$$\sin\theta = \frac{|\overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}}|}{|\overrightarrow{\mathbf{A}}||\overrightarrow{\mathbf{B}}|} \to \theta = 35.3^\circ, \tag{13}$$

In fact, we have the identity

$$\left(\frac{\overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{B}}}{|\overrightarrow{\mathbf{A}}||\overrightarrow{\mathbf{B}}|}\right)^2 + \left(\frac{|\overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}}|}{|\overrightarrow{\mathbf{A}}||\overrightarrow{\mathbf{B}}|}\right)^2 = \frac{(\overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{B}})^2 + (\overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}}) \cdot (\overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}})}{(\overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{A}})(\overrightarrow{\mathbf{B}} \cdot \overrightarrow{\mathbf{B}})} = 1. \tag{14}$$