Høsten 2015

FYS100 Fysikk
Problems week 37

Have a go at these. And for each, make a little sketch (x-t diagram, or whatever is appropriate) to illustrate the solution.

First some problems from the book:

- 2.3, 2.4, 2.11, 2.20, 2.35, 2.54, 2.85

Solution: 2.3
This is very similar to the going up the hill, and back down example at lecture.
b) The average velocity is zero, since there is no displacement (back where we started). a) The average speed is the distance travelled d divided by the total time t. Let's give things names: Going from A to B is a distance d_1, which takes time t_1 with speed v_1. Going back from B to A is a distance d_2, which takes time t_2 with speed v_2. Then the problem tells us that:

$$d_1 = d_2 = d/2, \qquad v_1 = 3.00 \text{ m/s}, \qquad v_2 = 5.00 \text{ m/s}. \tag{1}$$

$$\rightarrow t_1 = \frac{d_1}{v_1}, \qquad t_2 = \frac{d_2}{v_2}, \tag{2}$$

$$\rightarrow v_{avg} = \frac{d}{t} = \frac{d}{t_1 + t_2} = \frac{2v_1v_2}{v_1 + v_2} = 3.75 \text{ m/s}. \tag{3}$$

Solution: 2.4
The particle position is given by $x = 10t^2$. The average velocity is the displacement divided by the time. So:

$$a) \quad v_{x,avg} = \frac{x(3) - x(2)}{3 - 2} = 50.0 \text{ m/s}. \tag{4}$$

$$b) \quad v_{x,avg} = \frac{x(2.1) - x(2.0)}{2.1 - 2.0} = 41.0 \text{ m/s}. \tag{5}$$

Solution: 2.11
The tortoise gets to the end in a time $t_t = \frac{d}{v_t} = 5000$ s. The hare moves 40

times as fast, but has a long break.
a) The hare resumes from a distance 200 m and moves 40 times as fast as the tortoise; so for them to arrive at the same time, the tortoise must have been 40 times closer to the finish line = 200/40 = 5 m.
b) The hare travels the whole 1000 m in 1000/8 = 125 s. Hence, the break in the middle must have been 5000 − 125 = 4875 s long.

Solution: 2.20
The position is given by $x(t) = 3t^2 - 2t + 3$. It has positive velocity for $x'(t) = 6t - 2 > 0$, which means $t < 1/3$ s. For the rest, velocity is positive, and therefore equal to speed. Then we have

$$a) \quad v_{avg} = \frac{x(3) - x(2)}{3 - 2} = 13 \text{ m/s}, \tag{6}$$

$$b) \quad v_x(t) = x'(t) = 6t - 2 \to v(3) = 16 \text{ m/s}, \quad v(2) = 10 \text{ m/s}, \tag{7}$$

$$c) \quad a_{avg} = \frac{v_x(3) - v_x(2)}{3 - 2} = 6 \text{ m/s}^2, \tag{8}$$

$$d) \quad a_x(t) = v_x'(t) = 6 \text{ m/s}^2, \tag{9}$$

$$e) \quad v_x(t) = 0 \to t = 1/3. \tag{10}$$

Solution: 2.35
Distance to the tree is $x = 62.4$ m, and it takes $t_b = 4.20$ s to get there when braking with $a = -5.6$ m/s^2. Putting the initial position to be $x_i = 0$, we have

$$x = \frac{a}{2}t^2 + v_i t \to v_i = \frac{x - at_b^2/2}{t_b} = 26.6 \text{ m/s}. \tag{11}$$

When it hits the tree, it has speed: $v_i + at_b = 3.10$ m/s.

Solution: 2.54
We have, choosing the $h-$axis to have origin at the height of the throwing hand,

$$a) \quad h = v_i t - \frac{g}{2}t^2 \to v_i = \frac{h + g/2 \times t^2}{t}, \tag{12}$$

$$b) \quad v_h(t) = v_i - gt = \frac{h}{t} - \frac{g}{2}t \tag{13}$$

Note that depending on the numbers, the keys could be caught on the way up or the way down.

Solution: 2.85
Rock falls in time t_1, to a depth $h = g/2 \times t_1^2$. Sound comes up again in a

time $t_2 = h/v_s$, with v_s the speed of sound. Using $t = t_1 + t_2$, we have:

$$h = g/2t_1^2 = v_s(t - t_1) \rightarrow t_1 = 2.32 \text{ s}, \rightarrow h = 26.4 \text{ m}. \quad (14)$$

Without tang into account the speed of sound, one would have written $h = g/2 \times t^2 = 28.2$ m. That would be an error of

$$\frac{28.2 - 26.4}{26.4} \simeq 6.7 \text{ percent} \quad (15)$$

Additional Problem 1 (Prob. 5, Oblig. 1, 2013):
Wile E. Coyote is keen to catch the Road-Runner (check it on Wikipedia, if you don't know the reference. It won't matter for the following). He hides behind a big rock, and as the Road-Runner zooms past at constant speed $v = 15.0$ m/s, Coyote lights up his ACME rocket pack. After waiting $t_0 = 2.00$ s for the rocket fuse to burn down, he accelerates at a constant rate of $a = 5.00$ m/s^2, in pursuit of the Road-Runner.

a) At what time, with what speed and after what distance does he catch the Road-Runner?

b) What if the rocket would stop working after 4 seconds of acceleration and Coyote would continue at constant speed?

Solution: a) Putting $t = 0$ when the RR passes WEC, and defining $x = 0$ at that position, we have

$$x_{RR} = vt, \qquad x_{WEC} = \frac{a}{2}(t - t_0)^2. \quad (16)$$

Setting these equal at the time t_{catch}, we have

$$vt_{catch} = \frac{a}{2}(t_{catch} - t_0)^2 \rightarrow t_{catch} = \text{ or } t_{catch} = (v + at_0)\left(1 \pm \sqrt{\frac{a^2t_0^2}{(at_0 + v)^2}}\right) = 9.58 \text{ s}. \quad (17)$$

The other solution with the minus sign (0.417 s) corresponds to the curves passing after RR passes WEC, but when WEC has not yet started, and so it doesn't apply. Distance one may get from putting the time into either of the equations $\rightarrow$ 144 m. The speed then of WEC is

$$a(t_{catch} - t_0) = 37.9 \text{ m/s}. \quad (18)$$

b) If acceleration stops after 4 s (at time 4+2=6 s), then WEC would have final speed $4 \times a = 20$ m/s. He would therefore still catch up, eventually. At $t = 6$, RR is at a position $x = 6{\times}15 = 90$ m, and WEC is at $x = a/2(6{-}t_0)^2 =$

40 m. So WEC has to catch up 50 m with a speed advantage of $20 - 15 = 5$ m/s. That takes him $50/5 = 10$ s, for a total time of $10 + 6 = 16$ s. By then, they will both be at a distance $x = 90 + 10 \times 15 = 240 = 40 + 20 \times 10$ m from the origin. And WEC will of course have a speed of 20 m/s when he passes the RR.