Høsten 2015

FYS100 Fysikk
Problems week 38

Have a go at these. And for each, make a little sketch to illustrate the solution.

First some problems from the book:

- 4.12, 4.13, 4.22, 4.25, 4.41, 4.47, 4.51.

Solution 4.12: Use the range equation

$$R = \frac{v^2}{g} \sin(2\theta). \tag{1}$$

Maximal distance is when $\theta = 45°$, and solving for g, one finds $g = v^2/R_{max} = 3^2/15 = 0.6$ m/s^2.

Solution 4.13: This is projectile motion, with initial velocity only in the x-direction, $y_i = 1.22$ m and final distance $d = 1.40$ m. We write a)

$$y_i - \frac{g}{2}t^2 = 0, \tag{2}$$

$$v_i t = d, \tag{3}$$

$$t_{land} = \sqrt{\frac{2y_i}{g}} \to v_i = \frac{d\sqrt{g}}{2y_i} = 2.81 \text{ m/s}. \tag{4}$$

b) The final velocity is then

$$(v_x, v_y) = (v_i, -gt_{land}) = (2.81, -4.89) \tag{5}$$

or $v = 5.64$, and an angle of $\theta = -60.2$ with the horizontal.

Solution 4.22: This is very similar to 4.13, but now with water. Now we have to decide how wide ad high a walkway should be. Let's require that a person of height 180 cm should have 1.00 m wide head space. Then the question is whether the water has dropped below 180 cm, 1 m away from the wall. We calculate at which value of x, the water has dropped to 180 cm, starting from $h = 2.35$ m, that means a drop of $\Delta y = -55$ cm. WIth initial horizontal velocity of $v = 1.70$ m/s:

$$\begin{aligned} -\frac{g}{2}t^2 &= \Delta y, && (6) \\ t_{land} &= \sqrt{\frac{-2\Delta y}{g}}, && (7) \\ vt &= 0.570 \text{ m}. && (8) \end{aligned}$$

Hence, according to our definition of an appropriate height and width of a walkway, there isn't room.

Solution 4.25: Again projectile motion. a) We write

$$v_i \cos\theta t = d \to v_i = \frac{d}{\cos\theta t} = 18.1 \text{ m/s}. \tag{9}$$

(10)

b) Now the height at distance d, minus the height of the wall h is

$$\left(v_i \sin\theta t - \frac{g}{2}t^2\right) - h = 1.13 \text{ m}. \tag{11}$$

c) It lands on the roof, when it reaches a height of $h - 1 = 6$ m. We want to know how far beyond the wall (d) it gets to (let's call it l). We calculate

$$\left(v_i \sin\theta t - \frac{g}{2}t^2\right) = 6.00 \text{ m}, \tag{12}$$

$$l = v_i \cos\theta t - d = 2.79 \text{ m}. \tag{13}$$

Solution 4.41: We need to find the radial and tangential acceleration. Tangential is given by the slowing down from 90 km/h to 50 km/h over 15 s,

$$|a_t| = \frac{(90-50)/3.6}{15} = -0.741 \text{ m/s}^2. \tag{14}$$

The radial acceleration is the centripetal acceleration at the point where it has speed 50 km/h, so

$$|a_c| = \frac{v^2}{r} = \frac{(50/3.6)^2}{150} = 1.29 \text{ m/s}^2. \tag{15}$$

Then the total acceleration has length $\sqrt{a_t^2 + a_c^2} = 1.48$ m/s^2 and direction $\tan^{-1}(a_t/a_c) = 29.9°$ backwards from the radius vector (perpendicular to the track).

Solution 4.47:
a) The relative velocity of the motorist it the difference in speed, and he is getting closer, so $80 - 95 = -15$ km/h. There is an assumption here, that the positive x-direction is in the direction of travel.
b) From the point of view of the motorcycle guy, the police is moving with $95 - 80 = 15$ km/h.
c) With a starting distance of 250 m, it will take $250/(15/3.6) = 60$ s to catch up.

Solution 4.51: Seen from the bank, moving with the stream, the swimmer has speed $c + v$. Going against the stream, $c - v$. Hence going one way takes $t_1 = d/(c + v)$, going the other way takes $t_2 = d/(c - v)$. Adding them up, one gets a)

$$t_1 + t_2 = \frac{2d}{c} \frac{1}{1 - \frac{v^2}{c^2}}. \tag{16}$$

b) If the stream is still, just take $v = 0$, to get $\frac{2d}{c}$ (as one would expect).
c) It is always slower when the stream moves, since the factor

$$\frac{1}{1 - \frac{v^2}{c^2}} \tag{17}$$

is always larger than 1. Note that if $v \to c$ the swimming time goes to infinity. Swimmer gets swept away by the river.

Additional Problem 1 (OblII, 2013)

A submarine targets a battleship at a distance $d = 20.0$ km, in the direction $\theta_1 = 15.0°$ East of North. The battleship is travelling at $v_1 = 30.0$ km/h in a direction $\theta_2 = 40.0°$ degrees East of North. The torpedoes of the submarine can move at a speed of $v_2 = 100$ km/h.

a) In which direction relative to North should the submarine fire its torpedo to hit the battleship?

Solution: The easiest is to rotate the entire problem, so that the y-axis is 15° East of North. Then the battleship is moving at 25.0° to the right of that. Then one just has to aim the torpedo, so that the movement in x (perpendicular to y) is the same as the battleship's movement in x. So

$$v_1 \sin(\theta_2 - \theta_1) = v_2 \sin(\theta_3 - \theta_1),$$
$$\rightarrow \qquad \theta_3 = \theta_1 + \sin^{-1}\left(\frac{v_1}{v_2} \sin(\theta_2 - \theta_1)\right) = 22.3 \text{ degrees},$$

where θ_3 is the direction of the torpedo relative to North.

b) How long will it take the torpedo to reach the target?

Solution: The relative speed in the y-direction is

$$v_{\mathrm{rel,x}} = v_2 \cos(\theta_3 - \theta_1) - v_1 \cos(\theta_2 - \theta_1),$$

so that to cover the initial distance d, it takes

$$\frac{d}{v_{\mathrm{rel,x}}} = 0.278 \text{ hours} = 1000s.$$