Høsten 2015

FYS100 Fysikk
Problems week 39

Have a go at these. And for each, make a little sketch to illustrate the solution.

First some problems from the book:

- 5.27, 5.28, 5.29, 5.30, 5.36, 5.46, 5.65.

Solution 5.27: Since the boat has constant speed, there is no acceleration, and so the forces must cancel out.
a) x is East, y is North. Then $\vec{\mathbf{n}} = (-|\vec{\mathbf{n}}|, 0)$, $\vec{\mathbf{P}} = (|\vec{\mathbf{P}}|\cos\theta, |\vec{\mathbf{P}}|\sin\theta)$ and $F_{\text{drag}} = (0, -220N)$. We then have

$$|\vec{\mathbf{P}}|\sin\theta = 220N \rightarrow |\vec{\mathbf{P}}| = \frac{220N}{\sin 40^\circ}, \quad (1)$$

$$|\vec{\mathbf{P}}|\cos\theta = |\vec{\mathbf{n}}| = \frac{220N}{\tan 40^\circ}, \quad (2)$$

and so $\vec{\mathbf{n}} = (-262N, 0)$ and $\vec{\mathbf{P}} = (262N, 220N)$, length 342 N.

b) Now use axes along $\vec{\mathbf{P}}$ and orthogonal to it. Then instead one finds

$$|\vec{\mathbf{P}}| = 220N\sin\theta + |\vec{\mathbf{n}}\cos\theta, \quad (3)$$

$$|\vec{\mathbf{n}}|\sin\theta = 220N\cos\theta, \quad (4)$$

which of course gives the same result. Probably a) should be considered the simplest.

Solution 5.28: Think carefully about this one. First b) Clearly, the string/spring is carrying $5 \times 9.8 = 49$ N. Now consider a): it is essentially twice b) and so the reading must be the same, 49 N. (Imagine in b) that you would reach down and grab the rope with both hands and cut it between your hands. Now you are holding both weights, in the same way as the wall in a). It would be quite weird if as you did this, the spring would suddenly be stretched more!). In c), the Newtonmeter is carrying both weights, to 98 N. And in d) it is opposing the component of gravity along the incline, $g\sin 30^\circ$ so a weight of $24.5N$.

Solution 5.29: Draw a picture of the situation. Let's call the masses $m_{1,2,3}$ according to their weight. And let's split the system up in to system 1,2,3 accordingly. Let us put the positive x-axis in the direction of the external force F. The string tension is T, the forces between m_1 and m_2 are $F_{12} = -F_{21}$. All the blocks accelerate at the same rate a.

System 1: $m_3 a = F - T$,
System 2: $m_2 a = F_{12}$,
System 3: $m_1 a = T - F_{12}$.

Add them up and solve for a to get

$$a = \frac{F}{m_1 + m_2 + m_3} = 7.0 \text{ m/s}^2 \tag{5}$$

(not a great surprise!). Reinserting this, one finds

$$\begin{aligned} T &= F - m_3 a = 21N, & (6) \\ F_{12} &= m_2 a = 14N. & (7) \end{aligned}$$

Solution 5.30: It's frictionless, so one needs only the force of gravity along the incline. The normal force compensates for the gravity component orthogonal to the incline, and that's all we need to know. We therefore have

$$mg \sin\theta = ma \to a = g \sin\theta = 2.54 \text{ m/s}^2. \tag{8}$$

Travelling the 2 m down, we have

$$d = \frac{a}{2} t^2, \qquad v = at \to v = \sqrt{2dg \sin\theta} = 3.19 \text{ m/s}. \tag{9}$$

Solution 5.36: a) This is very similar to the Example presented in the lecture. We revisit this here: Put the x-axis horizontally and the y-axis vertically. Let the mass of the weight be m. Let the two angles be $\theta_{1,2}$. Then T_3 holds the whole weight, and therefore has tension mg. Considering the point where the three strings are attached to each other, we have

$$\begin{aligned} T_3 = T_1 \sin\theta_1 + T_2 \sin\theta_2, & \quad (10) \\ T_1 \cos\theta_1 = T_2 \cos\theta. & \quad (11) \end{aligned}$$

Solving for T_2 and reinserting, we find

$$\begin{aligned} T_3 = T_1 \left(\sin\theta_1 + \frac{\cos\theta_1}{\cos\theta_2} \sin\theta_2 \right) \to T_1 &= \frac{T_3 \cos\theta_2}{\sin(\theta_1 + \theta_2)}, & (12) \\ T_2 &= \frac{T_3 \cos\theta_1}{\sin(\theta_1 + \theta_2)}, & (13) \end{aligned}$$

In this particular case, $\theta_1 + \theta_2 = 90°$, and so the denominator is just 1. We then find $T_1 = 31.5$ N and $T_2 = 37.5$ N.
b) is the case where $\theta_1 = 60°$ and $\theta_2 = 0$, and the mass is 10 kg. We find $T_1 = 113$ N and $T_2 = 56.6$ N.

Solution 5.46: a) The first important thing to notice is that if m_1 moves a distance d, P_2 also moves a distance d, but m_2 moves $2d$. As a result (differentiating twice), $a_2 = 2a_1$. The second is that the tension of the string attached to the wall (T_2) and attached at the other end to m_2 together with the tension in the first string (T_1) provide acceleration for the second pulley, which has mass 0. Splitting up into 3 systems (m_1, m_2 and pulley), we have, putting the positive x-axis downwards:

$$m_1 a_1 = m_1 g - T_1, \tag{14}$$

$$m_2 a_2 = T_2, \tag{15}$$

$$m_{\text{pulley}} a_3 = T_1 - 2T_2 = 0, \tag{16}$$

Inserting $a_2 = 2a_1$, we find

$$a_1 = \frac{a_2}{2} = g\frac{m_1}{m_1 + 4m_2}, \tag{17}$$

$$T_1 = g\frac{4m_1 m_2}{m_1 + 4m_2}, \tag{18}$$

$$T_2 = g\frac{2m_1 m_2}{m_1 + 4m_2}. \tag{19}$$

Solution 5.65: Splitting up in to 2 systems, and putting the positive x-axis in the direction for he external force, we have:

System 1: x: $T - \mu_k|\overrightarrow{\mathbf{n}}|_1 = m_1 a$, y: $m_1 g = |\overrightarrow{\mathbf{n}}_1|$,
System 2: x: $F - T - \mu_k|\overrightarrow{\mathbf{n}}|_2 = m_2 a$, y: $m_2 g = |\overrightarrow{\mathbf{n}}_2|$.

Adding up, we have

$$F - \mu_k(m_1 + m_2)g = (m_1 + m_2)a \rightarrow a = \frac{F - \mu_k(m_1 + m_2)g}{m_1 + m_2}, \tag{20}$$

$$T = \frac{m_1}{m_1 + m_2}F. \tag{21}$$