Høsten 2015

FYS100 Fysikk
Problems week 40

Have a go at these. And for each, make a little sketch to illustrate the solution.

Some problems from the book:

- 6.6, 6.7, 6.20, 6.21, 6.31, 6.41, 6.65.

Solution, 6.6:
a) Since we are told that it goes with uniform speed, that speed is $l/t = 235/36 = 6.53$ m/s. The radius of the circle is $r = 235/(\pi/2) = 149.6$ m, since the arc is a quarter circle. Then the acceleration is the centripetal one,

$$a_c = \frac{v^2}{r} = \frac{l^2}{t^2} \times \frac{\pi}{2l} = \frac{\pi l}{2t^2} = 0.285 \text{ m/s}^2. \tag{1}$$

We are asked to provide it at an angle of 35.0°. Lets have the x-axis towards East, y-axis towards North. Since the centripetal acceleration is towards the centre, along the radius vector, it is simply $(-a_c \cos\theta, a_c \sin\theta) = (0.233, 0.163)$ m/s^2.

b) The car's average speed is the one found above, 6.53 m/s.

Solution, 6.7: The fake gravity is the normal force resisting the centrifugal force (in the rotating frame)...or the normal force providing the centripetal force for the rotation (in the inertial frame). It has magnitude $mv^2/r = mr\omega^2$. We want the corresponding acceleration to be 3 m/s^2, and $r = 60$ m. So we have

$$\omega = \sqrt{\frac{a_c}{r}} = 0.224 \text{ s}^{-1}. \tag{2}$$

One revolution of 2π takes $2\pi/\omega = 28.1$ s. It therefore does 2.14 revolutions per minute.

Solution, 6.20: a) From the point of view of the inertial frame on the ground, the spring is providing an acceleration of a, corresponding to a force of ma. If that force is 18 N, then the acceleration is $18/5 = 3.6$ m/s^2.

b) If the car moves with constant velocity, there is no acceleration, and the spring shows 0 N.

c+d) Seen from the inertial frame, there is a spring force accelerating the mass. Seen from the non-inertial car frame, there is a fictitious force pulling the mass backwards, thereby stretching the spring.

Solution, 6.21: This is exactly like Example 6.7 (it's a truck instead of a train...). So we have a)

$$\tan\theta = \frac{a}{g} \rightarrow \theta = \tan^{-1}\frac{3}{9.8} = 17.0^\circ. \tag{3}$$

and b)

$$T = \frac{mg}{\cos\theta} = 5.12N. \tag{4}$$

Solution, 6.31: a) The terminal speed is $v_T = mg/b$, and so

$$b = \frac{mg}{v_T} = 1.47. \tag{5}$$

b) The solution to the evolution is $v(t) = v_T(1 - e^{-t/\tau})$, where $\tau = m/b$, which is equal to 0.632τ when

$$1 - e^{-t/\tau} = 0.632 \rightarrow t = -Ln(0.368)\tau = \tau = 0.0204s \tag{6}$$

c) The terminal speed is never exactly reached, but the resistive force goes asymptotically to exactly cancel the force of gravity $mg = 0.0294$ N.

Solution, 6.41: a) The road exerts a normal force equal tot he force of gravity, minus the force required to keep the car in uniform circular motion with radius R and speed v, hence

$$|\overrightarrow{\mathbf{n}}| = mg - \frac{mv^2}{r} = mg\left(1 - \frac{v^2}{rg}\right). \tag{7}$$

The car leaves contact with the road (and normal force goes to zero), when $v^2 = rg$.

Solution, 6.65: We know the relation

$$v(t) = v_T(1 - e^{-t/\tau}), \qquad \tau = \frac{m}{b}, \qquad v_T = g\tau. \tag{8}$$

a) If $v = v_T/2$, then $e^{-t/\tau} = 1/2$, and $t/\tau = \log(2)$, so $v_T = g\tau = gt/L\log(2) =$ 78.3 m/s.
b) If $v = 3v_T/4$ then $e^{-t\tau} = 1/4 = (1/2)^2 = (e^{-5.54/\tau})^2 = e^{-2\times 5.54/\tau}$. So that

happens after 11.1s (twice as long).
c) Integrating the speed equation, we find

$$x(t) = v_T(t + \tau(e^{-t/\tau} - 1)) = v_T(5.54 - 5.54/\log(2) \times 1/2) = 121m. \quad (9)$$

And then have a go at these:

Additional problem 1 (Obl 2014-2)
A child is at a playground, and chooses to try the *spinning disc*. The radius of the disc is 2.00 m, and the coefficient of static friction between child-surface and disc-surface is $\mu_s = 0.350$.
In the following questions, you must provide algebraic equations as well as final numbers. You must also draw some relevant sketches, illustrating the problem.

a) If the child sits 1.5 m from the centre, how fast can the spinning disc turn, without her slipping off?

Solution: Static friction must provide the centripetal acceleration, so

$$mr\omega^2 = \mu_s|\bar{n}| = \mu_s mg \to \omega_{max} = \sqrt{\frac{g\mu_s}{r}} = 1.51 \text{ rad/s}. \quad (10)$$

b) The evil big brother now spins the disc with $\omega = 2.00$ rad/s. At what distance from the centre of the disc should the kid sit, to avoid falling off?

Solution: The other way around, so that

$$r = \frac{g\mu_s}{\omega^2} = 0.858 \ m \quad (11)$$

is the maximal radius. Kid should sit inside that radius.

Having safely moved to the radius 0.500 m, the child starts complaining, because she cannot get off the ride. The evil big brother decides to stop the rotating disc, by providing a constant tangential force.

c) How quickly can he stop the disc, without her sliding in any direction (neither forwards, backwards, inwards or outwards)?

Solution: If the force is constant, the tangential acceleration is constant, too. The largest total acceleration is just as he starts braking, since then there is both tangential and the maximal radial acceleration, corresponding to $\omega = 2$ rad/s to be provided by the static friction. So

$$a_{max} = \mu_s g = \sqrt{a_c^2 + a_t^2} \to a_t^{max} = \sqrt{(\mu_s g)^2 - (r\omega^2)^2} = 2.79 \ m/s^2. \quad (12)$$

So the maximal (negative) tangential acceleration allowed at $r = 0.5$ m is 2.79 m/s^2, and since the kid is rotating at $\omega r = 1$ m/s, it takes a minimum of $1/2.79 = 0.359$ s to stop the disc.

Additional problem 2

Consider a small ball dropping under the effect of gravity in a fluid. It experiences a resistive force proportional to the velocity,

$$\overrightarrow{\mathbf{R}} = -b_1 \overrightarrow{\mathbf{v}}. \tag{13}$$

a) If the ball is initially at rest, what is its speed as a function of time? Choose the y-axis to be positive downwards.

Solution: You can find this from the book, or compute it yourselves. One finds

$$v(t) = v_T + (v_i - v_T)e^{-t/\tau}, \qquad v_T = \frac{mg}{b_1}, \qquad \tau = \frac{m}{b_1}. \tag{14}$$

For the present case, $v_i = 0$.

b) What is the position as a function of time, choosing the initial position to have $y = 0$.

Solution: If we know the speed, we simply have to integrate it to get the position. So we have

$$y(t) = \int_0^t dt' \, v(t') = v_T t - \tau(v_i - v_T)(e^{-t/\tau} - 1). \tag{15}$$