Høsten 2015

FYS100 Fysikk
Problems week 41

Have a go at these. And for each, make a little sketch to illustrate the solution.

Some problems from the book:

- 7.17, 7.18, 7.31, 7.41, 7.44, 7.54

Solution, 7.17: a) Since the gravitational force is proportional to the mass and the force of the spring is proportional to its stretching, hanging 1.5 kg instead of 4 gives a stretching of $2.5 \times 1.5/4 = 0.938$ cm.
b) To stretch a spring, one must provide work equal to

$$W = \frac{k}{2}(x_f^2 - x_i^2). \tag{1}$$

The spring constant follows from question a), since $mg = kx$, where $m = 4$ kg and $x = 0.025$ m. Then $k = 1568$ N/m. Now the work required in stretching from $x = 0$ to $x = 4$ cm is 1.25 J.

Solution, 7.18: a) We compute the string constant k from the information that when 7.5 kg hang from it, the spring is stretched from 35 to 41.5 cm, i.e. an extension of $dx = 6.5$ cm. Using $mg = kx$, we find $k = 1131$ N/m.
b) If two people pull in opposite ends with 190 N, it is tempting to think that the spring is stretched as if for twice that. But imagine them pulling, and then attaching one of the ends to the wall. Now the spring is pulling the wall with 190 N, and the remaining person with 190 N, but it has the same extension as before. So the amount of stretching follows from $kx = 190$ N, so $x = 16.8$ cm.

Solution, 7.31: The kinetic energy when having a velocity of $(6, -2)$ m/s is

$$\frac{m}{2}v^2 = 60J. \tag{2}$$

When accelerating to a velocity of $(8, 4)$, the kinetic energy has increased to 120 J, and so 60 J of work must have been provided.

Solution, 7.41: The origin of the coordinate system (the zero of the potential energy) is placed at the top edge of the well. Then before and after the drop, the potential energy is

$$E_b = mgh_b = 2.55J, \tag{3}$$
$$E_a = mgh_a = -9.8J. \tag{4}$$

The difference between the two is 12.3 J.

Solution, 7.44: a) Take the integral

$$W = \int_i^f \overrightarrow{\mathbf{F}} \cdot d\overrightarrow{\mathbf{r}}. \tag{5}$$

If the force is constant, I can take it outside the integral

$$W = \overrightarrow{\mathbf{F}} \cdot \int_i^f d\overrightarrow{\mathbf{r}}, \tag{6}$$

and the integral is simply $\overrightarrow{\mathbf{r}}_f - \overrightarrow{\mathbf{r}}_i$, independently of the path. Hence so is the inner product, and the force is by definition conservative. b). The force is now $\overrightarrow{\mathbf{F}} = (3, 4)$, and the three paths considerd go from $(0, 0)$ to $(5, 5)$. Then the work is independent of the paths, so

$$W = \overrightarrow{\mathbf{F}} \cdot \int_i^f d\overrightarrow{\mathbf{r}} = (3, 4) \cdot (5, 5) = 35J. \tag{7}$$

Solution, 7.54: The force is minus the derivative of the potential. a) In this case

$$F_x = 3x^2 - 4x - 3. \tag{8}$$

b) This is zero for $x = -0.535$ and $x = 1.87$.
c) The question of equilibrium depends on the second derivative of the potential, which is $-6x + 4$. This is positive for $x = -0.535$, which is therefore a minimum and stable; and it is negative for $x = 1.87$, which is therefore a maximum and unstable. The inflexion point (second derivative is zero) is at $x = 2/3$.

And then have a go at these:

Additional problem 1 (Obl 2013-3)
A block is lying on a table and attached to a spring in the horizontal direction, and the spring is in turn attached to a wall. We take the x-axis to be positive in the direction of stretching the spring, and the equilibrium position of the spring is at $x = 0$. The block has mass $m = 1.00$ kg, the spring constant is

$k = 100$ N/m and there is a coefficient of kinetic friction with the table of $\mu_k = 0.15$. There is gravity.

a) If the block is let go from a position $-x_0 = 10.0$ cm (compressed), how far does it stretch $+x_1$ on the other side?

Solution (7 percent): Change in potential energy equals loss to internal energy $k/2(x_0^2 - x_1^2) = (x_0 + x_1)\mu_k mg$, which means that

$$x_1 = x_0 - \frac{2\mu_k mg}{k} = 7.06 \text{ cm.} \tag{9}$$

b) And how far does it go when returning to the squeezing side $-x_2$? And the next stretching x_3?

Solution (7 percent): Same computation, replacing x_0 by x_1 and x_1 by x_2 gives $x_2 = 4.12$, and then $x_3 = 0.118$.

c) Make a plot of the compression/stretching points x_0, x_1, ... How many times does the spring oscillate before coming to a stop?

Solution (6 percent): As we can see, it is a straight line with intercept 10.0 cm and slope -2.94 **(and a typo here)** cm per crossing. It goes to zero on its way from x_3 to x_4.

Additional problem 2 (Obl 2014-3)

A man of mass 80.0 kg jumps out from a bridge, with a lightweight rubber rope attached to his feet. He falls 50.0 m before turning around and coming back up, which is 30.0 m further than the equilibrium point of the rubber band (without man), which we take to be like a spring when stretched (but not when squeezed; no spring force on the man until he has passed the equilibrium point).

In the following questions, you must provide algebraic equations as well as final numbers. You must also draw some relevant sketches, illustrating the problem. You may find energy bar diagrams useful in this problem.

Solution: (3) Numbers; lightweight rope; equilibrium point 20.0m down;

a) What is the effective spring constant of the rope?

Solution: (7) At the bottom, initial gravitational potential energy has turned into spring potential energy, so that

$$mgh = \frac{1}{2}kx^2 \rightarrow k = \frac{2mgh}{x^2} = 87.1\ N/m, \tag{10}$$

where $h = 50$ m and $x = 30$ m.

b) How fast is the man going when passing the equilibrium point on his way up again?
Solution: (5) Going up or down doesn't matter, it's all conservative forces. Either way, having dropped 20 m, the initial potential energy has partly been turned into kinetic energy. And the spring is at its equilibrium point, so has no potential energy.

$$\frac{1}{2}mv^2 = mg(h-x) \rightarrow v = \sqrt{2g(h-x)} = 19.8\ m/s, \tag{11}$$

where $h - x = 20$ m.