Høsten 2015

FYS100 Fysikk
Problems week 42

Have a go at these. And for each, make a little sketch to illustrate the solution.

Some problems from the book:

- 8.4, 8.7, 8.11, 8.12, 8.15, 8.25, 8.38.

Solution, 8.4: a) The vertical shot gets the ball to a maximum height given by conservation of energy (velocity is zero)

$$\frac{m}{2}v^2 = mgh \rightarrow h = \frac{v^2}{2g} = 51.0 \text{ km}. \quad (1)$$

At an angle we instead have that even at the maximum height, the speed in the y-direction is unchanged, so

$$\frac{m}{2}[(v\sin\theta)^2 + (v\cos\theta)^2] = mgh + \frac{m}{2}(v\sin\theta)^2 \rightarrow h = \frac{(v\sin\theta)^2}{2g} = 18.5 \text{ km}. \quad (2)$$

b) The total energy of the system is constant and the same for both shots $= mv^2/2 = 10^4$ kJ.

Solution, 8.7: The system starts out with potential energy $E_{pot} = m_1 gh$. Then m_1 goes down and m_2 goes up and immediately before m_1 hits the table, $\Delta E_{kin} = (m_1 - m_2)gh$ of potential energy has been turned into kinetic energy. Since the two masses are connected, they have the same speed, so

$$(m_1 - m_2)gh = \frac{1}{2}(m_1 + m_2)v^2 \rightarrow v^2 = 2gh\frac{m_1 - m_2}{m_1 + m_2} = (4.43 \text{ m/s})^2. \quad (3)$$

Then m_1 hits the ground, and its kinetic energy goes to internal energy. But m_2 continues up as a projectile, until its maximal height (given by the height h plus whatever the projectile motion gives)

$$m_2 g(h_{max} - h) = \frac{1}{2}m_2 v^2 \rightarrow h_{max} = h\left(1 + \frac{m_1 - m_2}{m_1 + m_2}\right) = 5.00 \text{ m}. \quad (4)$$

Solution, 8.11: First let us put the y-axis to be positive upwards. Then note that a shift in the position of B of x_B corresponds to a shift in A of

$x_A = -2x_B$. Similarly $v_A = -v_B$, $a_A = -2a_B$. The point when the separation between the two blocks is h is then when $x_A - x_B = \pm h$ which means that $x_B = \pm h/3$ and $x_A = \mp 2h/3$. In the first case, we have from energy conservation that

$$\frac{m}{2}(v_A^2 + v_B^2) + mg(x_A + x_B) = 0 \to \frac{5m}{2}v_B^2 = \mp mg\frac{h}{3}. \tag{5}$$

This is only possible in the case where A goes up and B goes down. Otherwise the RHS is negative. Continuing with this, we find $v_A = 2v_B = \sqrt{8gh/15}$.

Solution, 8.12: Initial kinetic energy is $m/2v^2$. Force of friction is $\mu_k|\overrightarrow{\mathbf{n}}| = \mu_k mg$. Then to transfer everything into internal energy, we have

$$\frac{m}{2}v^2 = \mu_k mgd \to d = \frac{v^2}{2\mu_k g} = 2.04 \text{ m}. \tag{6}$$

Solution, 8.15: We start out with no kinetic energy, but spring potential energy $k/2(\Delta x)^2$. If there is no friction a) this is all converted to kinetic energy at the equilibrium point, so

$$\frac{m}{2}v^2 = \frac{k}{2}(\Delta x)^2 \to v = \sqrt{\frac{k}{m}}\Delta x = 0.791 \text{ m/s}. \tag{7}$$

If there is friction b), some of the energy goes into internal energy. The friction force is $\mu_k|\overrightarrow{\mathbf{n}}| = \mu_k mg$, and so we have

$$\frac{m}{2}v^2 = \frac{k}{2}(\Delta x)^2 - \mu_k mg\Delta x \to v = \sqrt{\frac{k}{m}(\Delta x)^2 - 2\mu_k g\Delta x} = 0.531 \text{ m/s}. \tag{8}$$

Solution, 8.25: Similar stuff, but now there are two types of potential energy. a) Initially, in the spring $k/2(\Delta x)^2$ and then if moves along an incline, for a distance $L = h/\sin\theta$, where h is the vertical height that it reaches. This is given by energy conservation

$$mgh = \frac{k}{2}(\Delta x)^2 \to L = \frac{k}{2mg\sin\theta}(\Delta x)^2 = 4.12 \text{ m}. \tag{9}$$

b) If there is friction, this works for the entire distance L, with a friction force $\mu_k|n| = \mu_k mg\cos\theta$. Hence

$$mgh = \frac{k}{2}(\Delta x)^2 - L\mu_k mg\cos\theta \to L = \frac{k}{2mg(\sin\theta + \mu_k\cos\theta)}(\Delta x)^2 = 3.35 \text{ m}. \tag{10}$$

Solution, 8.38: The power is $P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$, starting from rest. a) Then the average P is the total work done divided by the time it took. Work has gone both into kinetic energy and potential energy, so

$$W = \frac{m}{2} v_{final}^2 + v_{avg} \Delta t m g, \tag{11}$$

$$v_{avg} = \frac{v_{final} - v_{initial}}{2}, \tag{12}$$

$$\rightarrow \frac{W}{\Delta t} = \frac{\frac{m}{2} v_{final}^2}{\Delta t} + \frac{m g v_{final}}{2} = 5.91 \text{ kW}. \tag{13}$$

b) The instantaneous work at cruising speed then involves only a force compensating for gravity, so

$$\vec{\mathbf{F}} \cdot \vec{\mathbf{v}} = m g v_{final} = 11.1 \text{ kW}, \tag{14}$$