Høsten 2015

FYS100 Fysikk
Problems week 43

Have a go at these. And for each, make a little sketch to illustrate the solution.

Some problems from the book:

- 9.2, 9.6+7, 9.16, 9.24, 9.29, 9.47, 9.63.

Solution: 9.2 $E_{kin} = mv^2/2$ and $p = mv$. Then on readily finds that

$$v = \frac{2E_{kin}}{p} = 2 \times 275/25 = 22 \text{ m/s}, \tag{1}$$

$$m = \frac{p^2}{2E_{kin}} = 1.14 \text{ kg}. \tag{2}$$

Solution: 9.6+7 Note that it is the relative speed, we are given. Call v_g and v_p the speed of the two relative to the ice, and $v_r = v_g - v_p$ the relative speed. Then momentum conservation (starts at zero) gives

$$m_g v_g + m_p v_p = 0 \to v_g = \frac{m_p v_r}{m_g + m_p}, \qquad v_p = .\frac{-m_p}{m_g + m_p} v_r, \tag{3}$$

which when inserting the nmbers gives $v_g = 1.15$ m/s, $v_p = 0.346$ m/s.

Solution: 9.16 a) The initial momentum in the $x - y$ plane is $\overrightarrow{\mathbf{p}}_i = (-v_i \cos\theta_i, -v_i \sin\theta_i)$, where $v_i = 15$ m/s and $\theta_i = 45°$. And after the hit, the final momentum is $\overrightarrow{\mathbf{p}}_f = (v_f \cos\theta_f, v_f \sin\theta_f)$, where $v_f = 40$ m/s and $\theta_f = 30°$. Then the impulse is given as

$$\overrightarrow{\mathbf{I}} = \overrightarrow{\mathbf{p}}_f - \overrightarrow{\mathbf{p}}_i = (9.04, 6.12) \text{ kg m/s}. \tag{4}$$

b) Integrating $\overrightarrow{\mathbf{F}}dt$ for the described development of the force gives $\overrightarrow{\mathbf{I}} = 24 \text{ ms}\overrightarrow{\mathbf{F}}_{max}$, where $\overrightarrow{\mathbf{F}}_{max}$ is the maximal force. We find

$$\overrightarrow{\mathbf{F}}_{max} = \frac{\overrightarrow{\mathbf{I}}}{0.024 \text{ s}} = (0.378, 0.256) \times 10^3 \text{ N}. \tag{5}$$

Solution: 9.24 a) Momentum conservation tells us that in this perfectly inelastic collisions,

$$mv_1 + (2m)v_2 = (3m)v_f \to v_f = \frac{v_1 + 2v_2}{3}. \tag{6}$$

b) The change in kinetic energy is

$$\Delta E_{kin} = \frac{1}{2}(3m)v_f^2 - \frac{1}{2}mv_1^2 - \frac{1}{2}(2m)v_2^2 = -\frac{1}{3}m(v_1 - v_2)^2. \tag{7}$$

Solution: 9.29 a) After dropping a height h, the speed of both of the balls is $\sqrt{2gh} = 4.85$ m/s, since energy conservation requires $mgh = mv^2/2$. Then the basketball velocity flips and it immediately colide with the tennis ball. At what height, we don't know but it is the diameter of the basketball. b) Now they collide elastically, and so the tennis ball gets a speed

$$v_t^f = \frac{m_t - m_b}{m_t + m_b}(-\sqrt{2gh}) + \frac{2m_b}{m_t + m_b}\sqrt{2gh} = \frac{3m_b - m_t}{m_t + m_b}\sqrt{2gh} = 12.8 \text{ m/s}. \tag{8}$$

Using again energy conservation, it gets to a height $h = v^2/(2g) = 8.41$ m above the collision point, so 7.21 m above here it started.

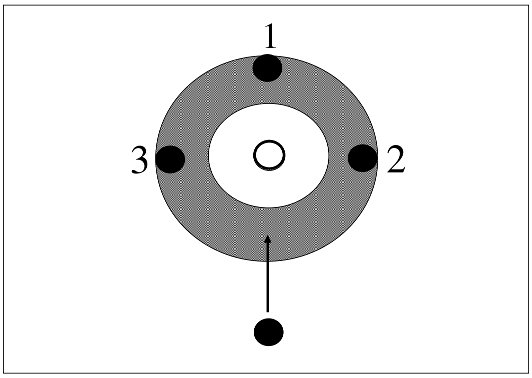
Solution: 9.47 In other words, find the centre of mass of that object. Since it is equally fat", we don't need to care about the 3.6 m side. It is also clear that the CM in in the middle, at 32.4 m from either end. The only thing remaining is the height, y_{CM}. Again, it is enough to calculate it for one half of the object. The two halves are symmetric, so will have the same CM height; and hence the sum of the two will again have the same CM height. From example 9.12, we know that that point for a right triangle is 2/3 along the side (closest to the right angle). That of course applies to both the a- and b-sides of the triangle in example 9.12. Hence, $y_{CM} = 1/3 \times 15.7 \text{ m} = 5.23$ m. Then the total potential energy of the whole construction, relative to the rocks being all spread out on the ground, is $Mgy_{CM} = 3.57 \times 10^8$ J. For this we needed to compute the total mass $15.7 \times 3.6 \times 32.4 \times 3800 = 6.96 \times 10^6$ kg.

Solution: 9.63 a) Using the rocket equation, we have

$$v_f = v_e \ln\left(\frac{M_i}{M_f}\right) \rightarrow M_i - M_f = M_f\left(e^{v_f/v_e} - 1\right), \tag{9}$$

where M_f is the mass of the rocket at the end (so what we need to propel, 3 tonnes); v_f is the required final speed, 10 km/s; v_e is the exhaust speed 2 km/s; and $M_i - M_f = M_{fuel}$ is the weight of the fuel. $M_{fuel} = 442$ tonnes. b) If instead $v_e = 5$ km/s, we would have $M_{fuel} = 19.2$ tonnes. c) If one would throw the entire fuel mass off in one go, one would need 2.5 times more fuel with the slow v_e. All the exhausted fuel moves with v_e relative to the initial rest frame. But with the rocket, the exhaust speed is relative to the current speed. And the fuel emitted later moves with small speed relative to the initial rest frame, even though it always has speed $-v_e$ relative to the instantaneous rest frame.

Additional problem 1: (Konte-exam, Feb. 2015)



In the game of curling, massive stones are sent sliding over (essentially) frictionless ice towards a *target circle* at the far end of the pitch (the grey ring in the figure). The radius of the target circle is 2.00 m. Two teams take turns sliding stones, and the winner is whoever gets a stone (or several stones) to stop closest to the centre. All the stones have the same mass of $M = 20.0$ kg, their radius is 15.0 cm, and all collisions are taken to be elastic. There is gravity, $g = 9.80$ m/s^2.
We join the game as the second team (black stones) is ready to throw their last stone, and the configuration is as shown in the figure; the stone of the opposing team (white stones) is currently in the middle of the target circle, with several black stones around.

a) What happens, if the black team choose to hit the central white stone exactly in the middle? Why?

Solution Exactly central collision with momentum and energy conservation in one dimension. Then the only option is for the black stone to stop and the white one to go off with the same speed as the black had. The white will then hit the black one ahead of it, stop, while that black goes off with the same speed as the original black had (ignoring friction).

b) The team choose to hit the central white stone slightly to the left of middle, so that it leaves the target circle at an angle θ, exactly halfway between two other black stones (1 and 2, as shown in the figure). Assume that the speed of the incoming black stone immediately before impact is $v = 1.00$ m/s. What is the speed of the white stone, immediately after impact? Does the black stone itself (the one they throw) leave the circle? What is the speed of that stone immediately after impact?

Solution We need to write down the equations for momentum and energy conservation in two dimensions, but remembering that all masses are the same (and cancel out)

$$\text{x -direction:} \quad v_w^x + v_b^x = 0, \tag{10}$$

$$\text{y -direction:} \quad v_w^y + v_b^y = v, \tag{11}$$

$$\text{Energy:} \quad (v_w^x)^2 + (v_w^y)^2 + (v_b^x)^2 + (v_b^y)^2 = v^2, \tag{12}$$

Hitting exactly between the two black stones, means and angle of 45°, so that $v_w^x = v_w^y$. Also, equal mass stones with one at rest scatter at a relative angle of 90°, so that the black stone goes at 45° to the left. Hence $v_b^x = -v_b^y$. As a consequence

$$v_w^y = v_b^y = v_w^x = -v_b^x = \frac{v}{2} = \mathbf{0.500} \text{ m/s}, \tag{13}$$

$$|v_w| = |v_b| = \sqrt{\frac{1}{2}} = \mathbf{0.707} \text{ m/s}. \tag{14}$$

Both stones leave the circle, not hitting any of the other black stones.

c) Now assume that the ice and stone have a small friction between them, parametrized by a coefficient of kinetic friction of $\mu_k = 0.02$. How far does the black stone slide after the collision before coming to rest? Does it in fact leave the circle? How far does the white stone slide before coming to rest? Does it leave the circle?

Solution Simplest is to use energy considerations. The work done by friction on the stone is $W = Mg\mu_k d$, when sliding a distance d. And so it comes to a stop after

$$Mg\mu_k d = \frac{1}{2}M|v_b|^2 \rightarrow d = \frac{|v_b|^2}{2\mu_k g} = \mathbf{1.28} \text{ m}. \tag{15}$$

Since radius of the circle is 2.00 m, the black stays inside. Same thing with the white one.

d) What possible angles θ could the black team choose to push the white stone instead, if they want the white stone to leave the circle (including friction), but have the black stone stay in the circle (including friction)? Is that possible without hitting any of the other black stones, given the configuration of stones shown in the figure? Why/why not?

Solution If instead of 45°, the angle of scattering would have been θ (measured from forward, the y-axis), we would have the equations

$$\text{x -direction:} \quad v_w^x + v_b^x = 0, \tag{16}$$

$$\text{y -direction:} \quad v_w^y + v_b^y = v, \tag{17}$$

$$\text{Energy:} \quad (v_w^x)^2 + (v_w^y)^2 + (v_b^x)^2 + (v_b^y)^2 = v^2, \tag{18}$$

$$\rightarrow \quad |v_b| = v \sin\theta, \qquad |v_w| = v\cos\theta. \tag{19}$$

In order for the white stone to leave the circle, we want

$$\frac{|v_w|^2}{2\mu_k g} = \frac{v^2\cos^2\theta}{2\mu_k g} > R \rightarrow \theta < \cos^{-1}\sqrt{\frac{2R\mu_k g}{v^2}} \simeq \mathbf{27.7}°. \tag{20}$$

In order to miss stone 1, the centres of that and the white stone must be at least 30.0 cm from each other, corresponding to (see figure)

$$\frac{0.3}{R} > \sqrt{\sin^2\theta + (1-\cos\theta)^2} = \sqrt{2(1-\cos\theta)} \rightarrow \theta > \cos^{-1}\left(\frac{2-(0.3/R)^2}{2}\right) \simeq \mathbf{8.60}°. \tag{21}$$

So the white stone will be kicked all the way out of the circle for an angle between 8.60° and 27.7°.