Høsten 2015

FYS100 Fysikk
Problems week 44

Have a go at these. And for each, make a little sketch to illustrate the solution.

Some problems from the book:

- 10.15, 10.18, 10.20, 10.27, 10.31, 10.40, 10.47, 10.53.

Have fun!

Solution, 10.15: a) If it has a speed of $v = 45$ m/s the angular speed is just $\omega = v/R = 45/250 = 0.18$ s^{-1}. b) Since the speed is constant, there is no tangential acceleration, and so the only acceleration is the centripetal acceleration of magnitude $a_c = v^2/R = R\omega^2 = 8.1$ m/s^2, with direction towards the centre.

Solution, 10.18: a) If the cyclist provides 76 revolutions/minutes, that's an angular speed of of $\omega_{sprocket} = 76/60 \times 2\pi = 7.96$/s. Since the speed of the chain is determined by it going over the larger sprocket of diameters 0.152 m, this gives a linear speed of a link of $v_{chain} = 0.152\omega_{sprocket}/2 = 0.605$m/s.
b) Now the chain pulls a sprocket of radius 0.07/2 m, so that the angular speed of that is $\omega_{wheel} = v_{chain}/0.035 = 17.3$/s. That is then also the angular speed of the wheel.
c) Finally, the linear speed of the bicycle is the same as the speed of the outer rim of the wheel, with radius 0.675/2m, and so $v_{bike} = R_{wheel}\omega_{wheel} = 5.82$ m/s (or 21 km/h).
d) The length of the pedal cranks is irrelevant given the other information in the problem. However, if we had been asked for (for instance) how much force the legs have to provide to give some specific torque, that additional information would be needed.

Solution, 10.20: a) If the car goes from rest to v_f in a time t, the constant acceleration is $v_f/t = 2.44$ m/s^2. Then the distance is $h = at^2/2 = v_f t/2 =$ 99m. To find the number of revolutions N, divide by the circumference of the tire, to find

$$N = \frac{h}{\pi D} = 54.3. \tag{1}$$

b) The final angular speed is $\omega = v_f/R = 22/0.29 = 75.9$/s, which is (by dividing by 2π) 12.1 rev/s.

Solution, 10.27: The three forces appear to all be orthogonal to radius vector (although the whole 30° is trying to confuse us!). The two at the outer rim pull clockwise, the third counterclockwise. hence, counting countercockwise as the positive direction

$$\tau = \tau_1 + \tau_2 + \tau_3 = -10b - 9b + 12a = -3.55 \text{ Nm}. \tag{2}$$

Solution, 10.31: Being a disc, the merry-go-round has a moment of inertia (around the CM axis) $I = MR^2/2$, and we want an angular acceleration of

$$\alpha = \frac{\omega}{t} = \frac{0.5(2\pi)}{2} = 1.57 \text{ s}^{-2}. \tag{3}$$

Then we need a torque around the CM of $I\alpha = 265$ Nm. If this is provided by a pulling at the edge, this requires a force of $\tau/R = 177$ N.

Solution, 10.40: The moment of inertia around an axis at distance x from M is (note that the x-axis has origin where M is)

$$I = Mx^2 + m(L - x)^2. \tag{4}$$

The minimum of that is when $dI/dx = 0$, so

$$\frac{dI}{dx} = 2Mx - 2m(L - x) = 0 \to x = \frac{mL}{m + M}. \tag{5}$$

This is the CM in the coordinate system centred on M as defined $((mL + M0)/(m + M))$.

Solution, 10.47: Several ways to do it. One using energy conservation: The maximum speed is when as much potential energy as possible is converted to kinetic energy. That's when the big stone is at the bottom and the small stone is at the top, with the stick vertical. Then

$$\Delta E_{kin} = -m_1(L - x)g + m_2 x g = \frac{m_1}{2}v_1^2 + \frac{m_2}{2}v_1^2 = \frac{I}{2}\omega^2. \tag{6}$$

Then solving for ω, one finds

$$\omega = \sqrt{2g\frac{-m_1(L - x) + m_2 x}{I}} = 8.56 \text{ rad/s}. \tag{7}$$

where we have used that $I = m_1(L-x)^2 + m_2 x^2$. Then the speed of the small mass is $v_1 = (L-x)\omega = 24.5$ m/s. The answers to b), c), d), e) and f) are no, no, no, no and yes. Because the force of gravity is not always perpendicular to the radius vector and so torque is not constant. Momentum is not conserved when accelerating. And gravity is a conservative force.

Solution, 10.53: Energy considerations are easiest to use. When sliding h along the table, friction takes $f_k h$ energy out of the system. $f_k = \mu_k |\vec{\mathbf{n}}| = \mu_k m_2 g$. At the same time, m_1 drops also h, and so potential energy $m_1 gh$ becomes available. This goes into kinetic energy of m_1, m_2 and the pulley; as well as the energy lost to friction. So we write

$$m_1 gh = \frac{m_1}{2}(v_f^2 - v_i^2) + \frac{m_2}{2}(v_f^2 - v_i^2) + \frac{I}{2}(\omega_f^2 - \omega_i^2) + \mu_k m_1 gh. \tag{8}$$

Using the moment of inertia of a hollow cylinder (see table) $I = \frac{M}{2}(R_2^2 + R_1^2)$, and the rolling condition $v = R_2 \omega$, we find

$$v_f = \sqrt{v_i^2 + \frac{2gh(m_1 - \mu_k m_2)}{m_1 + m_2 + \frac{M}{2}\left(1 + \frac{R_1^2}{R_2^2}\right)}}. \tag{9}$$

Inserting the numbers, we get $v_f = 1.59$ m/s. b) then $\omega = v_f / R_2 = 53.1$ rad/s.