Darcy law with gravity

$$q = \frac{k \cdot A}{\mu} \frac{\Delta \Psi}{L}$$

Ψ = fluid potential ($\Phi = \frac{\Psi}{\rho}$)

$$\Psi = p + G \cdot \rho g Z$$

correction for Darcy units

Darcy units $[\rho] = \mathrm{gram}/\mathrm{cm}^3$, $[g] = \mathrm{cm}/\mathrm{s}^2$

$[Z] = \mathrm{cm}$, $[p] = \mathrm{atm}$

$$[G \cdot \rho \cdot g Z] = \mathrm{atm}$$

$$1\,\mathrm{atm} = 1.01 \cdot 10^5\,\mathrm{Pa} = 1.01 \cdot 10^5\,\mathrm{N}/\mathrm{m}^2 = 1.01 \cdot 10^5 \frac{\mathrm{kg} \cdot \mathrm{m}/\mathrm{s}^2}{\mathrm{m}^2}$$

$$= 1.01 \cdot 10^5 \frac{\mathrm{kg}}{\mathrm{m} \cdot \mathrm{s}^2} = 1.01 \cdot 10^5 \cdot 10^3 \frac{\mathrm{gram}}{10^2\,\mathrm{cm} \cdot \mathrm{s}^2}$$

$$1\,\mathrm{atm} = 1.01 \cdot 10^6 \frac{\mathrm{gram}}{\mathrm{cm} \cdot \mathrm{s}^2}$$

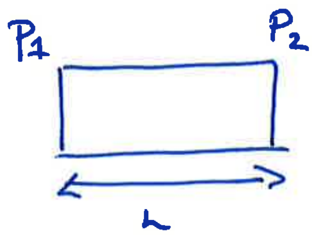
$$[\rho g Z] = \frac{\mathrm{gram}}{\mathrm{cm}^3} \cdot \mathrm{cm}/\mathrm{s}^2 \cdot \mathrm{cm} = \frac{\mathrm{gram}}{\mathrm{cm} \cdot \mathrm{s}^2}$$

$$\Psi = p + G \rho \cdot g Z, \quad G = \frac{1}{1.01 \cdot 10^6}$$

ignore gravity

$$q = \frac{k.A}{\mu}\,\frac{\overbrace{\Delta P}^{P_1 - P_2}}{L}$$

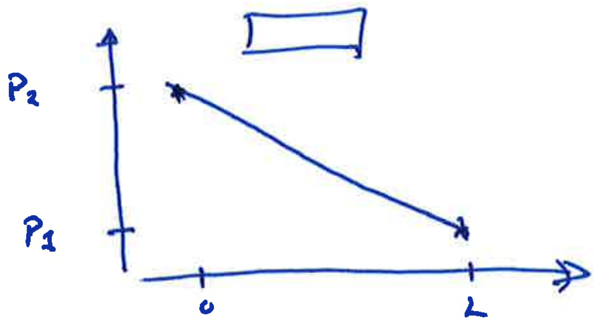
$L \rightarrow dx$



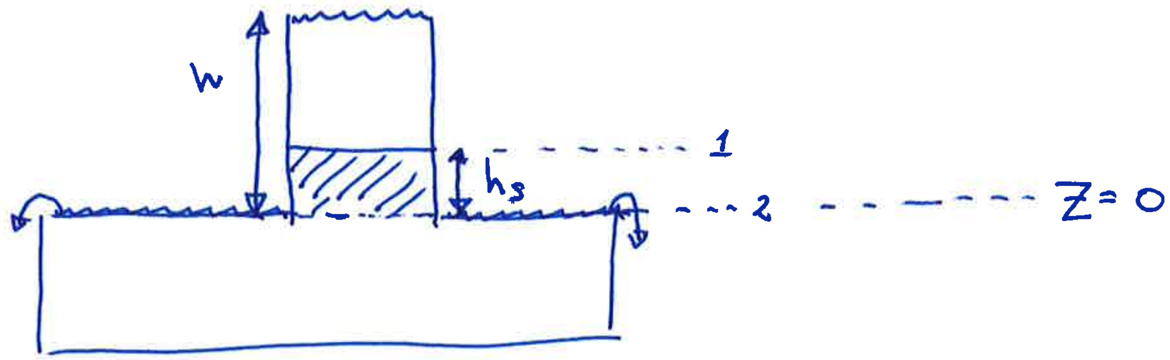
$L \rightarrow dx$

$P_2 = P_1 + dP$

$$q = \frac{k.A}{\mu}\,\frac{(P_1 - (P_1 + dp))}{dx} = \underline{-\frac{k.A}{\mu}\,\frac{dp}{dx}}$$



Exam 97 1



$$q = \frac{k \cdot A}{\mu} \frac{\Delta \Psi}{h}$$

$$\Psi_1 = P_1 + G \rho_w g Z = P_1 + G \rho_w g h_s$$

$$\Psi_2 = P_2 = \text{atmospheric pressure} = p^\circ$$

$$P_1 = p^\circ + G \rho_w g (h - h_s)$$

$$\Rightarrow \cancel{\Delta\Psi =} \; \Psi_2 - \Psi_1 = p^\circ - (\underbrace{P_1}_{p^\circ + G\rho_w g (h - h_s)} + G \rho_w g h_s)$$

$$= p^\circ - (\cancel{p^\circ} + G \rho_w g (h - \cancel{h_s}) + G \rho_w g \cancel{h_s})$$

$$\cancel{\Delta \Psi =} \; - G \rho_w g h \quad , \quad \Delta \Psi = \Psi_1 - \Psi_2 = G \rho_w g h$$

$$q = \frac{k \cdot A}{\mu} \frac{G \rho_w g h}{h_s}$$

stop to add fluid and

h 100 cm – 80cm in 400 seconds

$h_s = 5\,cm$, $\mu_w = 1\,cP$

Determine the permeability of the sandpack

The fluid speed $u = -\dfrac{dh}{dt}$

$$u = \frac{q}{A} = \frac{k}{\mu} G \frac{\rho_w g h}{h_s}$$

$$-\frac{dh}{dt} = \frac{k}{\mu} \frac{G \rho g h}{h_s}$$

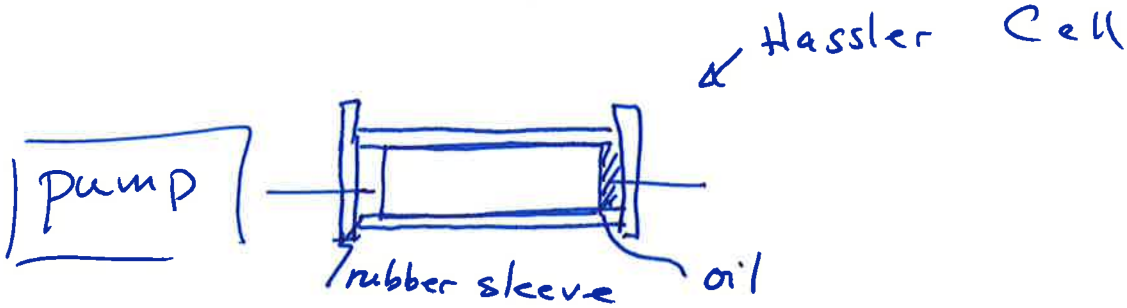
$$-\int_{h_1}^{h_2} \frac{dh}{h} = \frac{k}{\mu} \frac{G \rho g}{h_s} \int_{t_1}^{t_2} dt$$

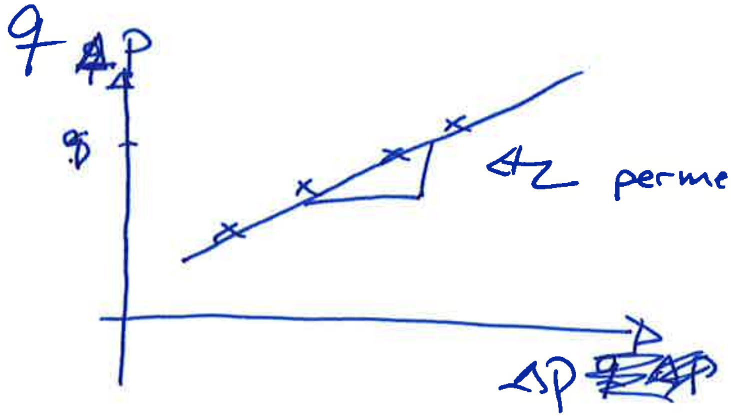
$$-\ln h \Big|_{h_1}^{h_2} = \frac{k}{\mu} \frac{G \rho g}{h_s} t \Big|_{t_1}^{t_2}$$

$$-\underbrace{[\ln h_2 - \ln h_1]}_{-\ln h_2/h_1} = \frac{k}{\mu} \frac{G \rho g}{h_s} (t_2 - t_1)$$

$$k = -\frac{\mu h_s \ln h_2/h_1}{G \rho g (t_2 - t_1)}$$

$$k = -\frac{1\,cP \cdot 5cm \ln 100/80}{\frac{1}{1.01 \cdot 10^6} \cdot 1 cm^3/s \cdot 980 cm/s^2 \, 400 s} = \underline{2.88\ D}$$





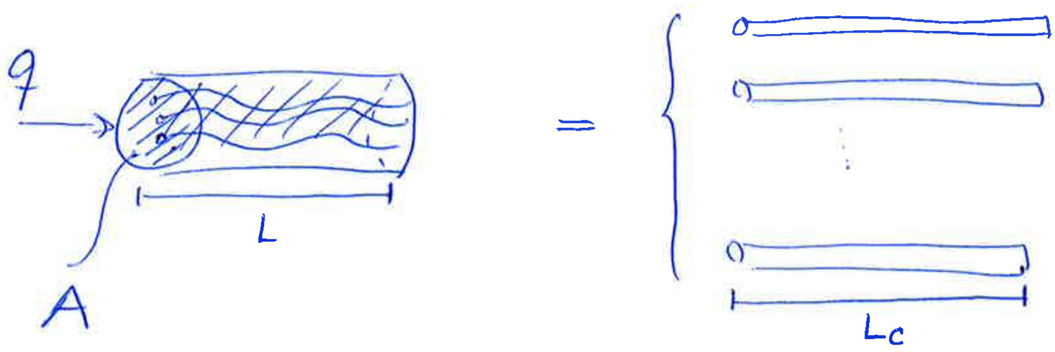
Δz permeability from the slope

$$q = \frac{k \cdot A}{\mu} \frac{\Delta P}{h}$$

$$\cancel{k = \frac{\mu \cdot h}{A} \frac{\Delta P}{q}}$$

$$q = \frac{k \cdot A}{\mu \cdot h} \Delta P$$

— slope



N-tubes with the same radii r

$L_c \geqslant L$

$$q = u \cdot A = v \cdot A_p$$

$$v = u \cdot \frac{A}{A_p} = u \cdot \frac{A \cdot L \cdot L_c}{A_p \cdot L \cdot L_c} = \frac{u \cdot V_b\, L_c}{V_p \cdot L}$$

$$\tau = \frac{L_c}{L}$$ = ~~tourtuosity~~ tortuosity

$$\varphi = V_p / V_b$$

Darcy velocity

$$\boxed{v = u \cdot \frac{\tau}{\varphi}}$$

fluid speed

Dimmension of k $\qquad k = \frac{q \cdot \mu}{A} \frac{L}{\Delta P}$

$$1\,D = \frac{1\,\mathrm{cm}^{\cancel{3}\,2}/\mathrm{s} \cdot 1\,\mathrm{cP} \cdot 1\,\cancel{\mathrm{cm}}}{1\,\cancel{\mathrm{cm}^2} \cdot 1\,\mathrm{atm}} \overset{SI?}{=}$$

$$= \frac{\mathrm{cm}^2\,\mathrm{cP}}{\mathrm{atm}\cdot\mathrm{s}} = \frac{(10^{-2}\mathrm{m})^2\, 10^{-3}\,\mathrm{Pa}\cdot\mathrm{s}}{1.01325\cdot 10^{5}\,\mathrm{Pa}\cdot\mathrm{s}}$$

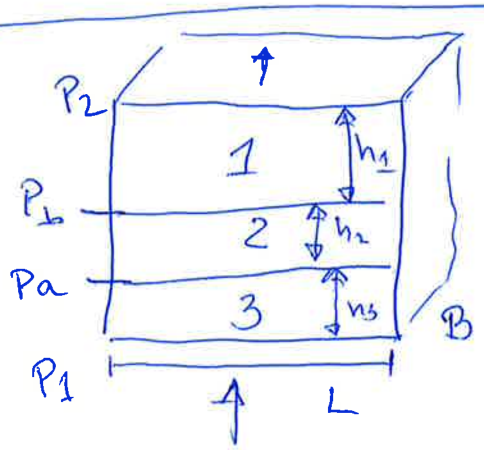
$$1\,D = \frac{1}{1.01325}\, 10^{-12}\,\mathrm{m}^2 = (0.993\cdot 10^{-6}\,\mathrm{m})^2 = \underline{(0.99\,\mu\mathrm{m})^2}$$

$$k_1 \cdot h_1 + k_2 \cdot h_2 + k_3 \cdot h_3 = \bar{k}\,(h_1+h_2+h_3)$$

$$\bar{k} = \frac{1}{h_1+h_2+h_3} \left\{ k_1 \cdot h_1 + k_2 \cdot h_2 + k_3 \cdot h_3 \right\}$$

N-layes $\quad \bar{k} = \frac{1}{\sum_{i=1}^{N} h_i} \sum_{i=1}^{N} k_i h_i$

Flow across layers :



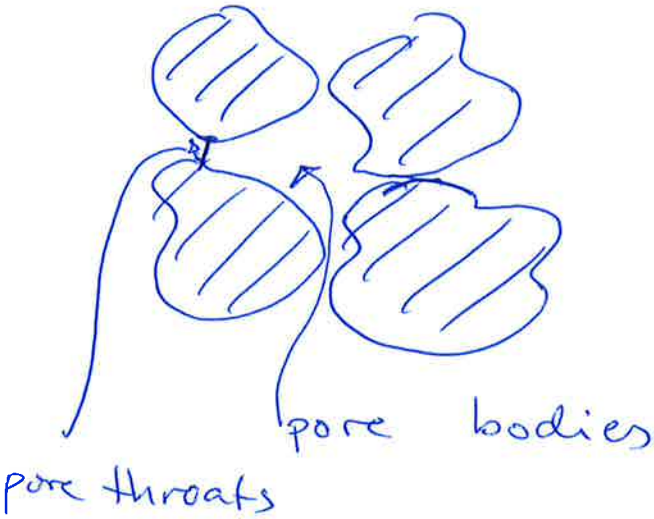
$$\Delta P = \frac{q \cdot \mu}{k} \cdot L$$

$$\underbrace{P_1 - P_2}_{} = P_1 - P_a + P_a - P_b + P_b - P_2$$

$$\frac{q \cdot \mu}{\bar{k}}(h_1+h_2+h_3) = \frac{\cancel{q} \cdot \mu\, h_3}{k_3} + \frac{\cancel{q} \cdot \mu\, h_2}{k_2} + \frac{\cancel{q} \cdot \mu\, h_1}{k_1}$$

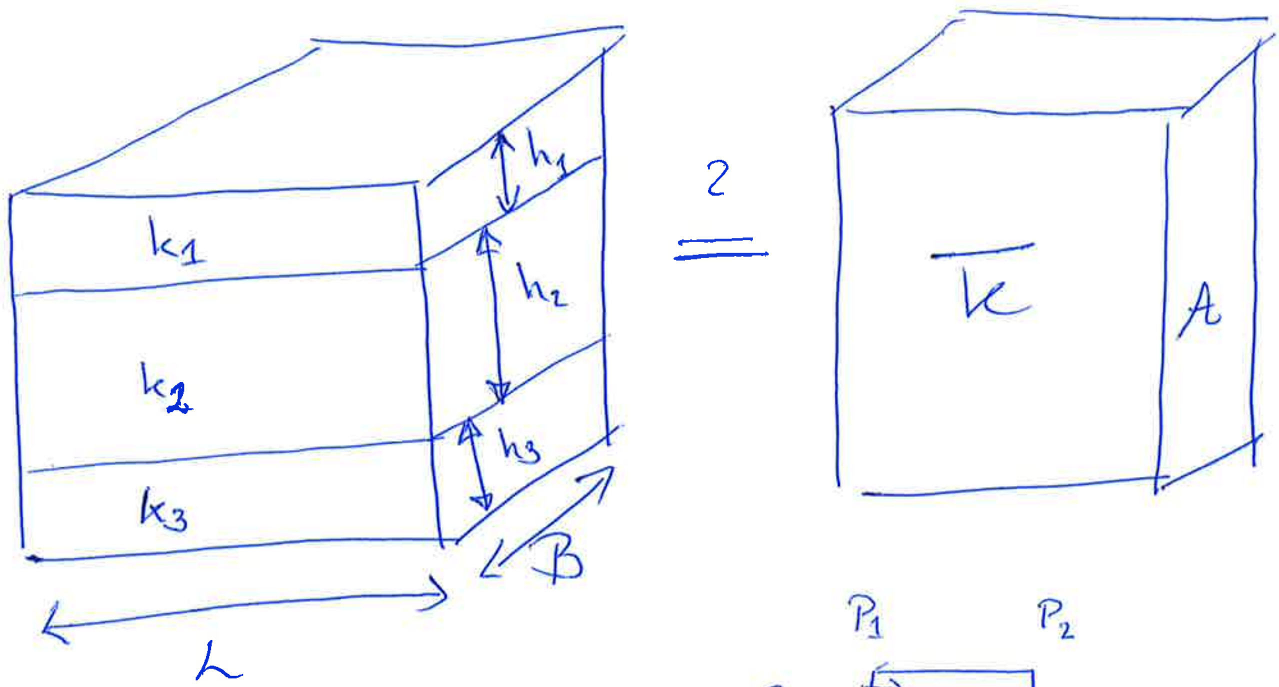
$$\frac{h_1+h_2+h_3}{\bar{k}} = \frac{h_3}{k_3} + \frac{h_2}{k_2} + \frac{h_1}{k_1}$$

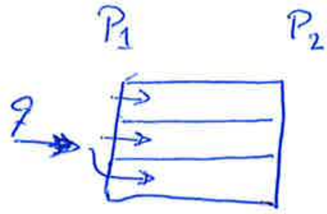
in general $\quad \dfrac{\sum_{i=1}^{N} h_i}{\bar{k}} = \sum_{i=1}^{N} \dfrac{h_i}{k_i}$



in a 1D rock
cross section (pore throats)
$\sim (1\mu m)^2$

Layered system





① Flow along layers

$$q = \underbrace{q_1}_{\frac{k_1 \cdot h_1 \cdot B}{\mu} \frac{\Delta P}{L}} + q_2 + q_3 = \frac{\bar{k} \cdot A}{\mu} \frac{\Delta P}{L}$$

$$\frac{k_1\, h_1 \cdot B}{\mu} \frac{\Delta P}{L} + \frac{k_2 \cdot h_2 \cdot B}{\mu} \frac{\Delta P}{L} + \frac{k_3 \cdot h_3 \cdot B}{\mu} \frac{\Delta P}{L} = \frac{\bar{k} \cdot B \cdot (h_1 + h_2 + h_3)}{\mu} \times \frac{\Delta P}{L}$$

Example :

$$h_1 = h_2 = h_3 = 1\,m$$

$$k_1 = k_2 = 1\,D$$

$$k_3 = 0.1\,D$$

flow along layers: $\bar{k} = \frac{1}{1+1+1}\left(1\cdot 1 + 1\cdot 1 + 0.1\cdot 1\right) D$

$$\bar{k} = \frac{2.1}{3} = \underline{0.7\,D}$$

flow across layers: $\frac{3m}{\bar{k}} = \frac{1m}{1D} + \frac{1m}{1D} + \frac{1m}{0.1D}$

$$= 12\,\frac{m}{D}$$

$$\bar{k} = \cancel{12}\,\frac{3}{12}\,D \doteq \underline{0.25\,D}$$

permeability is dependent on direction & usually lower in the vertical direction

$$\underline{k_v \sim 0.1\,k_h}$$