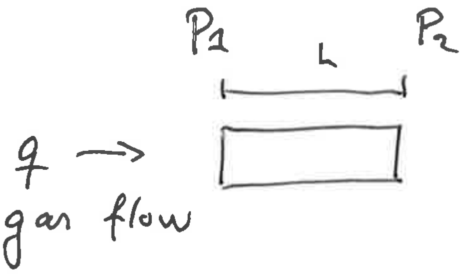
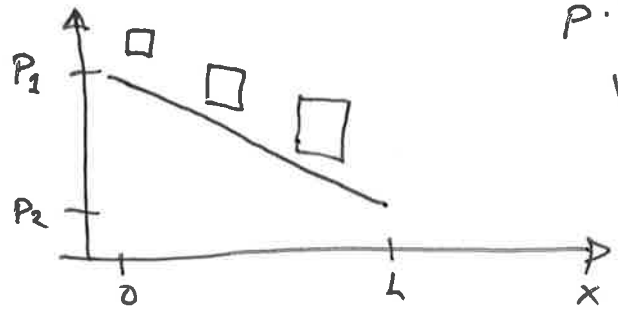
Darcy law : $q = -\frac{k \cdot A}{\mu} \frac{\partial \Psi}{\partial x}$ ℓ w gravity

$$q = -\frac{k \cdot A}{\mu} \frac{\partial p}{\partial x}$$

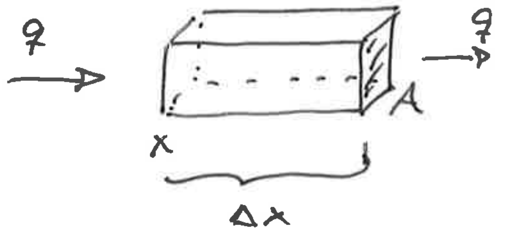




$p \cdot V = konst$

$V = \frac{konst}{p}$

conservation of mass:



$\Delta V = A \cdot \Delta x$

mass flow in per time − mass flow out per time

= change of mass in ΔV

mass flow in $q \cdot \rho$ $\left[\frac{vol}{time} \frac{mass}{vol} = \frac{mass}{time} \right]$

$$(q \cdot \rho)_x - (q \cdot \rho)_{x+\Delta x} = \frac{dm}{dt}$$

Taylor expansion $f(x+\Delta x) = f(x) + \frac{1}{1!} f'(x) \cdot \Delta x + \frac{1}{2!} f''(x) \Delta x^2 + \dots$

$$(q \cdot \rho)\big|_{x+\Delta x} = q \cdot \rho\big|_x + \frac{1}{1!} \frac{\partial}{\partial x}(q \cdot \rho)\Delta x + \dots$$

$$\to -\frac{\partial}{\partial x}(q \cdot \rho)\Delta x = \frac{dm}{dt}, \quad m = \rho \cdot \Delta V = \rho \cdot A \cdot \Delta x$$

$q = A \cdot v$

$$-\frac{\partial}{\partial x}(\cancel{q}A \cdot v \cdot \rho)\,\Delta x = \frac{d}{dt}(\rho \cdot A \cdot \Delta x)$$

$$\boxed{\frac{d\rho}{dt} + \frac{\partial(\cancel{u}\cdot\rho)}{\partial x} = 0}$$

3 D $\quad \frac{\partial}{\partial t}\rho + \vec{\nabla}\cdot(\vec{v}\rho) = 0 \quad , \vec{\nabla} = \left[\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right]$

in our case $\quad \frac{\partial}{\partial x}(v \cdot \rho) = 0$

or $\quad v \cdot \rho = \text{const} \Rightarrow \boxed{q \cdot \rho = \text{const}}$

ideal gas : $\rho = \frac{m}{V}$, $\quad p \cdot V = \text{konst}$

$\Rightarrow V = \frac{\text{konst}}{p}$

$\rho = \text{konst} \cdot p$

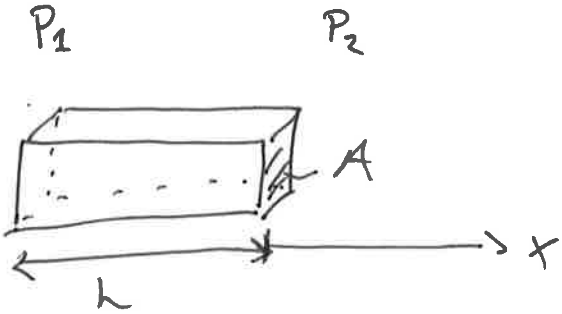
$\Longrightarrow$ ideal gas $\quad q \cdot p = \text{const} = K \; (*)$

$q = -\frac{k \cdot A}{\mu}\frac{dp}{dx}$, from $(*)$ $\; q = \frac{K}{p}$

$$\frac{K}{p} = -\frac{k \cdot A}{\mu}\frac{dp}{dx} \iff \int_0^L K\,dx = -\frac{k \cdot A}{\mu}\int_{P_1}^{P_2} p\,dp$$

$$K \cdot L = -\frac{k \cdot A}{\mu}\frac{1}{2}p^2\Big|_{P_1}^{P_2} = -\frac{k \cdot A}{\mu}\frac{1}{2}(P_2^2 - P_1^2)$$

$\cancel{q \cdot A}$



$$q \cdot p = K = q_a p_a$$

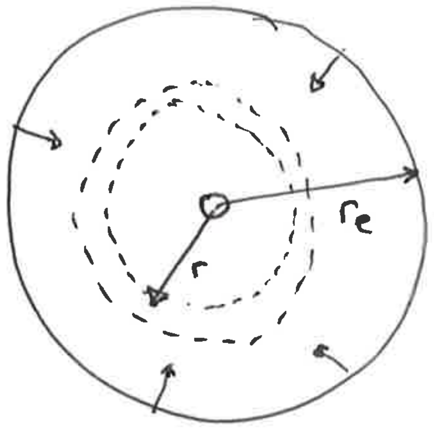
$$q_a \cdot p_a \cdot L = -\frac{k \cdot A}{\mu} \frac{1}{2} \left(p_2^2 - p_1^2\right)$$

$$\boxed{q_a = \frac{k \cdot A}{\mu} \frac{1}{2 p_a} \frac{\left(p_1^2 - p_2^2\right)}{L}}$$

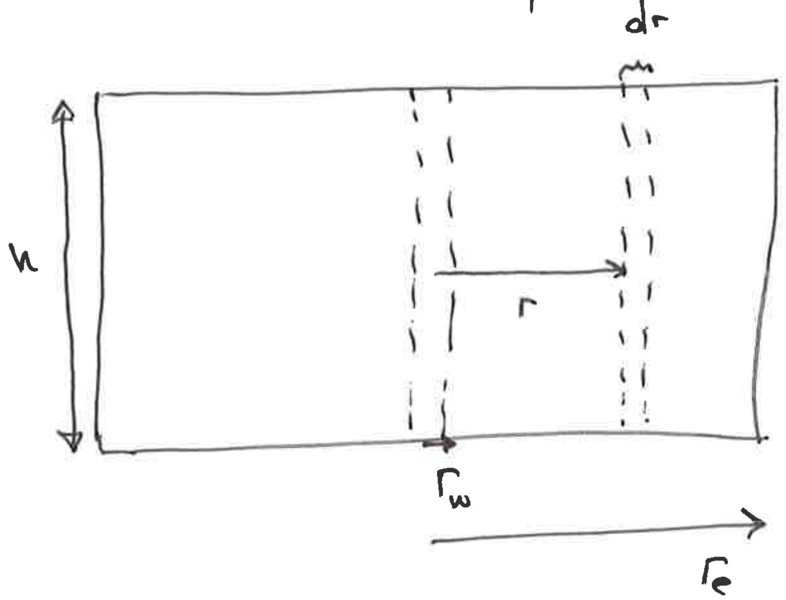
Radial flow

$$q = -\frac{k \cdot A}{\mu}\frac{\partial p}{\partial x}$$

From top



horizontally



$$A = 2\pi r h \quad , \quad dx \to dr \quad , \quad q \to -q$$

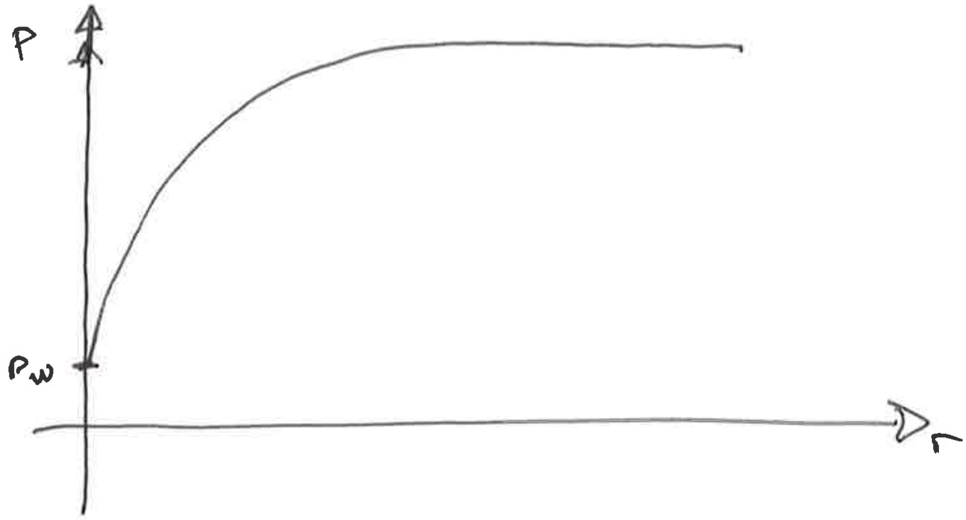
$$\boxed{q = \frac{k\, 2\pi r h}{\mu}\frac{\partial p}{\partial r}}$$

- homogeneous reservoir
- well perforated in the whole section
- well is producing at constant rate

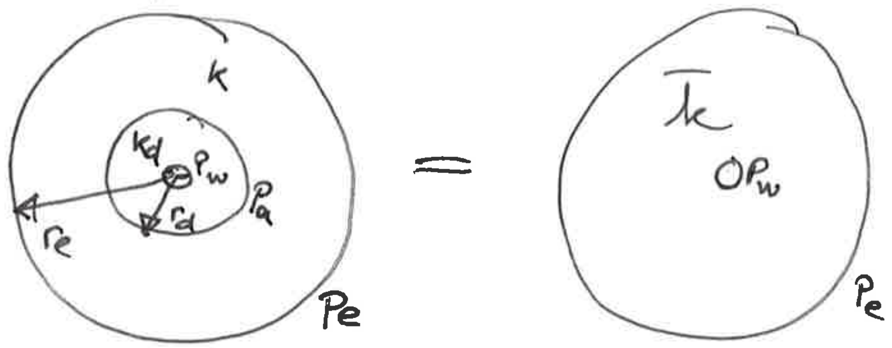
$$\int_{r_w}^{r} \frac{q}{r}\, dr = \frac{k\, 2\pi h}{\mu}\int_{p_w}^{p} dp$$

$$q \ln\frac{r}{r_w} = \frac{2\pi h k}{\mu}(p - p_w)$$

$$p - p_w = \frac{q \cdot \mu}{2\pi h k}\ln r/r_w$$



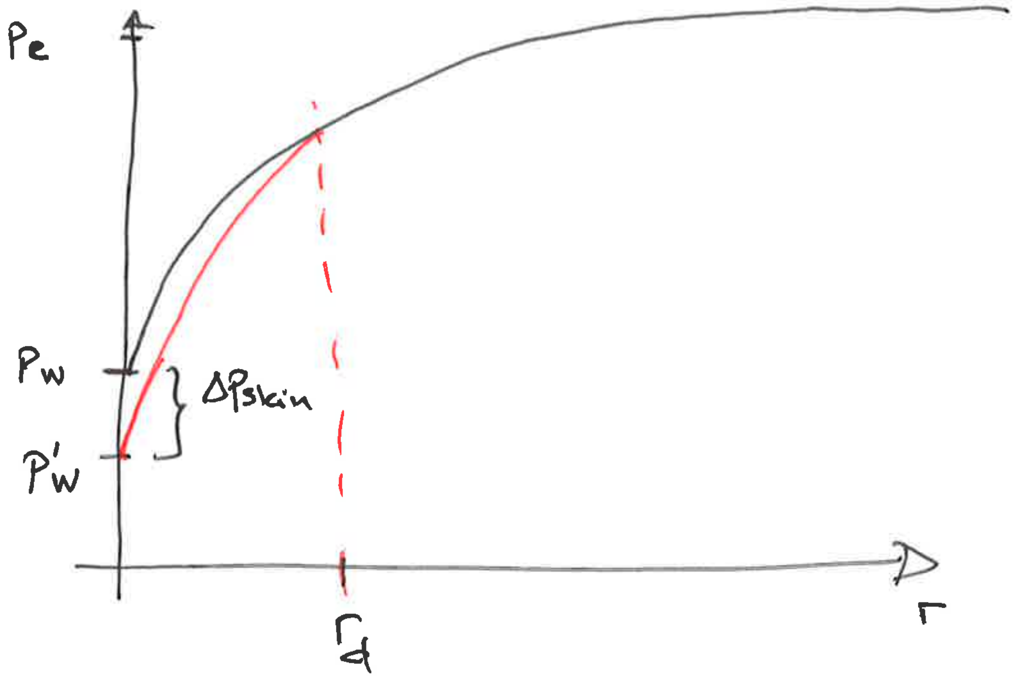
consider a reservoir with a damaged zone



$$P_e - P_w = P_e - P_a + P_a - P_w$$

$$\frac{q \cdot \mu}{2\pi h \bar{k}} \ln r_e/r_w = \frac{q \cdot \mu}{2\pi h k} \ln r_e/r_d + \frac{q \cdot \mu}{2\pi k_d} \ln r_d/r_w$$

$$\boxed{\frac{1}{\bar{k}} \ln r_e/r_w = \frac{1}{k} \ln r_e/r_d + \frac{1}{k_d} \ln r_d/r_w}$$



(1) $P_e - P_w = \frac{q \cdot \mu}{2\pi h k} \ln r_e/r_w$

(2) $P_e - P_w' = \frac{q \cdot \mu}{2\pi h \bar{k}} \ln r_e/r_w$, $\frac{\ln r_e/r_w}{\bar{k}} = \frac{\ln r_e/r_d}{k} + \frac{\ln r_d/r_w}{k_d}$

production engineer : $PI = \frac{q}{P_e - P_w}$ as high as possible

Skin factor $S = \frac{2\pi h k}{q \cdot \mu} \Delta P_{skin}$

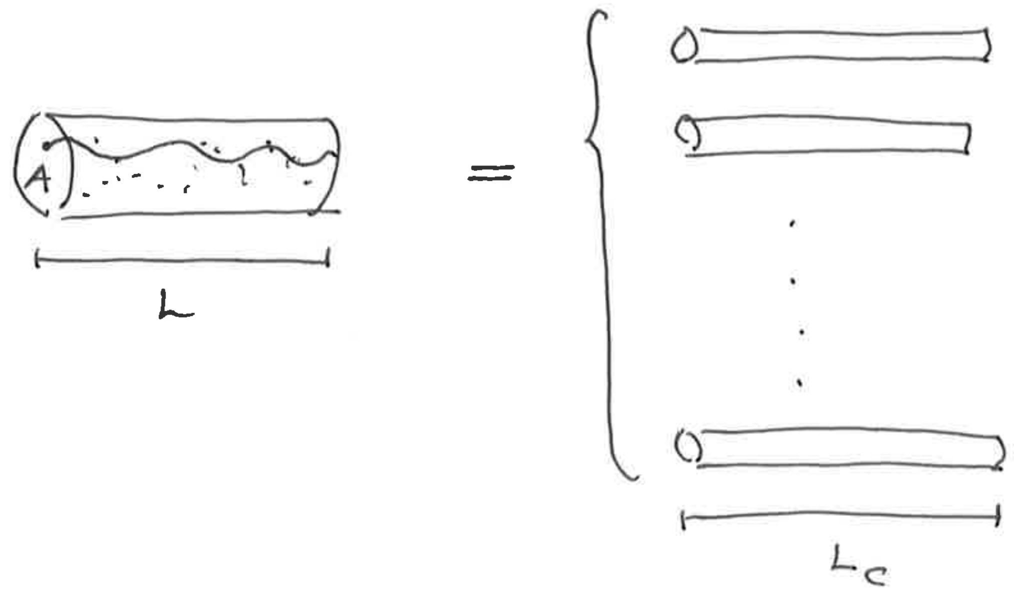
$\Delta P_{skin} = P_w - P_w' = \underbrace{(P_e - P_w')}_{(2)} - \underbrace{(P_e - P_w)}_{(1)}$

$\Longrightarrow \boxed{S = \ln r_d/r_w \; \frac{k - k_d}{k_d}}$

$k = 200\,mD$, $k_d = 50\,mD$, $r_d = 10\,ft$, $r_w = 0.5\,ft$

$\underline{\underline{S \simeq 9}}$

Bundle of tubes model



N-tubes with radius R

Poiseuille law: $$q_i = \frac{\pi R^4}{8\mu} \frac{\Delta P}{L_c}$$

$$q = \sum_{i=1}^{N} q_i = N \cdot q_i = \frac{N \pi R^4}{8\mu} \frac{\Delta P}{L_c} \qquad (1)$$

porosity $$\varphi = \frac{V_p}{V_b} = \frac{A_p \cdot L_c}{A \cdot L} = \frac{N \pi R^2 \cdot L_c}{A \cdot L}$$

$$N = \frac{\varphi \cdot A \cdot L}{\pi R^2 \cdot L_c}$$

$$(1) \Rightarrow \quad q = \frac{\varphi \cdot A \cdot L R^2}{8\mu L_c} \frac{\Delta P}{L_c} = \frac{k \cdot A}{\mu} \frac{\Delta P}{L}$$

$$\boxed{k = \frac{\varphi \cdot L^2 R^2}{8 L_c^2} = \frac{\varphi R^2}{8 \tau^2}} , \quad \tau = \frac{L_c}{L}$$

chalk $k \sim 1\,mD = 10^{-3}\,D \simeq 10^{-3}\,\mu m^2$

$\varphi \sim 0.5$ (ignore $\tau \sim 1$)

$$R = \sqrt{\frac{8k}{\varphi}} = \sqrt{\frac{8 \cdot 10^{-3}\,\mu m^2}{0.5}} \simeq \underline{\underline{0.12\,\mu m}}$$