Derive Darcy law:

- assuming a-bundle of tubes
- Poiseuille law $\boxed{q = \frac{\pi R^4}{8\mu} \frac{\Delta P}{L}}$

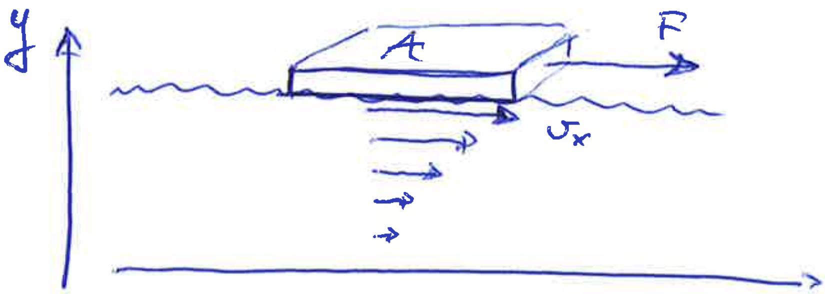
$$\Longrightarrow \boxed{k = \frac{R^2 \cdot \varphi}{8}}$$

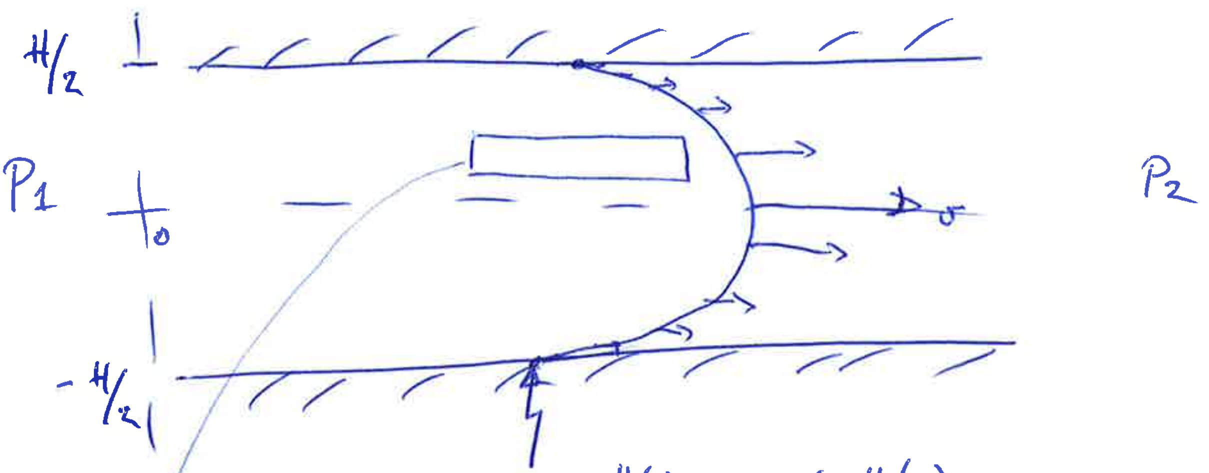
viscous forces: shear rate

$$\tau = \mu \overbrace{\frac{\Delta v_x}{\Delta y}}$$ Newtonian fluid

shear stress

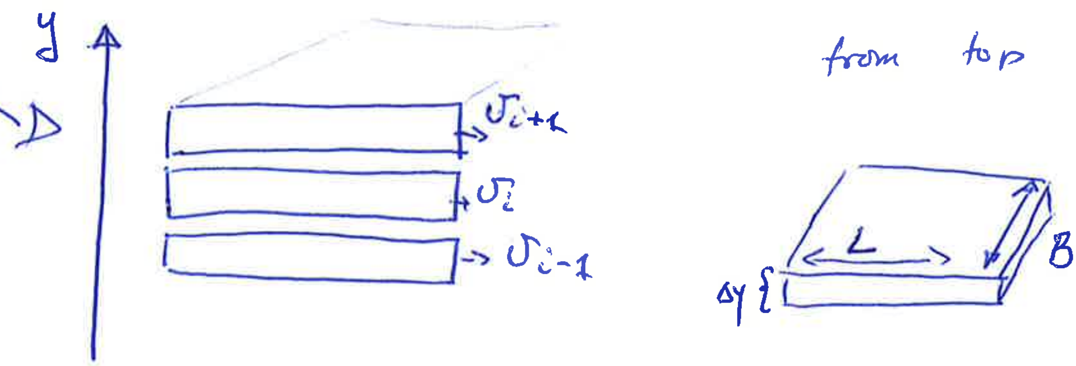
$$F = A \cdot \mu \frac{\Delta v_x}{\Delta y}$$





$v(H/2) = v(-H/2) = 0$

usually reffered to as no-slip condition



Assumption:
- no slip
- laminar flow
- Newtonian fluid

N2: $\sum F = \underbrace{ma}$

inertia effects, assume they can be neglected compared to viscous forces (Reynolds number < 1)

$$(1)\ \sum F = P_1 B\cdot\Delta y - P_2 B\cdot\Delta y + B\cdot L\mu\left(\frac{v_{i+1}-v_i}{\Delta y} + \frac{v_{i-1}-v_i}{\Delta y}\right) = 0$$

$$\frac{v_{i+1}-v_i}{\Delta y} + \frac{v_{i-1}-v_i}{\Delta y} = \frac{v_{i+1}-2\overbrace{v_i}^{v_i}+v_{i-1}}{\Delta y} = \Delta y\frac{d^2v}{dy^2}$$

$$v_{i+1} = v(y+\Delta y) = \overbrace{v(y)}^{v_i} + \frac{dv}{dy}\Delta y + \frac{1}{2}\frac{d^2v}{dy^2}\Delta y^2 + \cdots$$

$$v_{i-1} = v(y-\Delta y) = v(y) - \frac{dv}{dy}\Delta y + \frac{1}{2}\frac{d^2v}{dy^2}\Delta y^2 + \cdots$$

$$(1) \Rightarrow \quad P_1\cdot B\cancel{\Delta y} - P_2 B\cancel{\Delta y} + B\cdot L\cdot\mu\cdot\cancel{\Delta y}\frac{d^2v}{dy^2} = 0$$

$$\boxed{\mu\frac{d^2v}{dy^2} + \frac{\Delta P}{L} = 0}$$

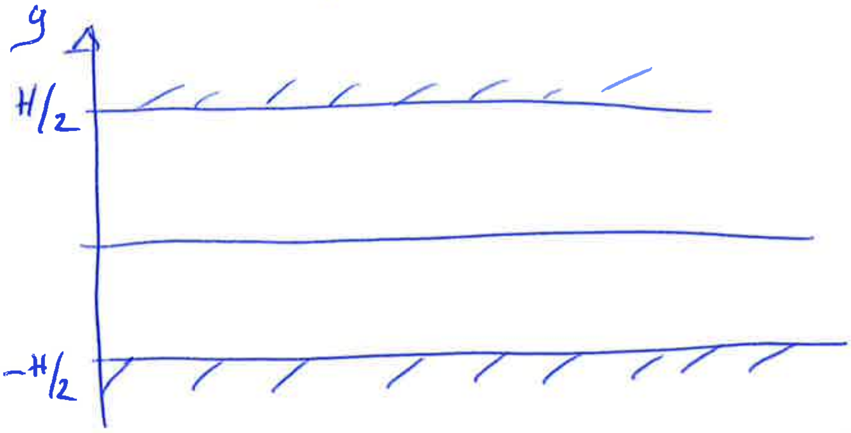
general $\quad \mu\nabla^2 v - \vec{\nabla}\cdot P = 0$

if we donot neglect inertia forces

$$\rightarrow \underbrace{\rho}_{\sim m}\underbrace{\frac{D\vec{v}}{Dt}}_{a} + \vec{\nabla}p - \mu\nabla^2\vec{v} = 0 \quad \text{(Navier-Stokes)}$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{v}\cdot\vec{\nabla}$$

$$\mu \frac{d^2 v}{dy^2} + \frac{\Delta p}{L} = 0 \qquad (2)$$



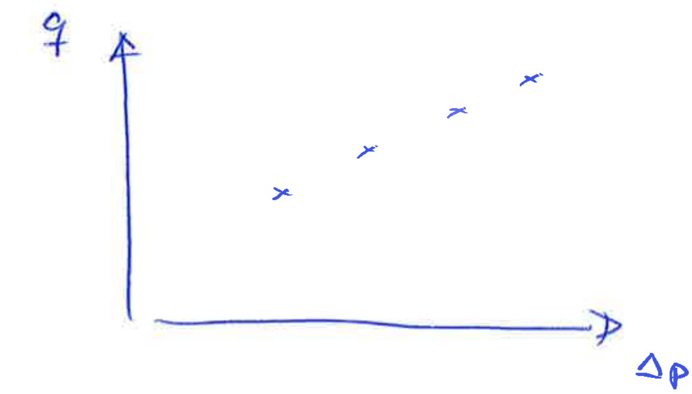
(2) integrate twice & $v(H/2) = v(-H/2) = 0$

$$\Longrightarrow v(x) = \frac{1}{2\mu}\left(\left(\frac{H}{2}\right)^2 - x^2\right)\frac{\Delta p}{L} \quad \text{plates}$$

$$v(r) = \frac{1}{4\mu}\left(R^2 - r^2\right)\frac{\Delta p}{L} \quad \text{cylinder}$$

$$\hookrightarrow q = \int_0^R v(r)\, 2\pi r\, dr = \underline{\frac{\pi R^4}{8\mu} \frac{\Delta p}{L}}$$

gas is used to measure permeability



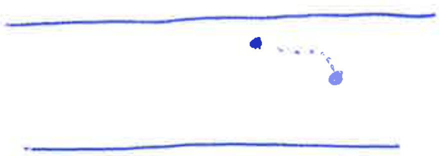
if gas has a low density then

$v_g(R) \neq 0$

pore wall

if the mean free path ~~of~~ between collision of gas molecules $\sim R$ Darcy law breaks down

an $v(R) \neq 0$



Klinkenberg effect: $q = -\frac{k^{eff}}{\mu} A \frac{\partial p}{\partial x}$

$$k_{eff} = k\left[1 + \frac{b}{\bar{p}}\right]$$

b = Klinkenberg parameter

inertia forces $<$ viscous forces

$$\rho \frac{Dv}{Dt} < \mu \nabla^2 v$$

$Dv \sim v$, $Dt \sim d/v$, $\nabla^2 v = v/d^2$

$$Re = \left[\frac{\rho \frac{Dv}{Dt}}{\mu \nabla^2 \vec{v}}\right] = \frac{\rho \cdot \frac{v}{d/v}}{\mu \; v/d^2} = \frac{\rho v \cdot d}{\mu}$$

grain diameter

$$Re = \frac{\rho v \cdot d}{\mu}$$

North sea $v = 1 m/day$, $\mu = 10^{-3} Pa\,s$

$\rho = 1000 kg/m^3$, $d = 10^{-6} m$

$$\Rightarrow \quad \underline{Re = 10^{-5} \ll 1}$$