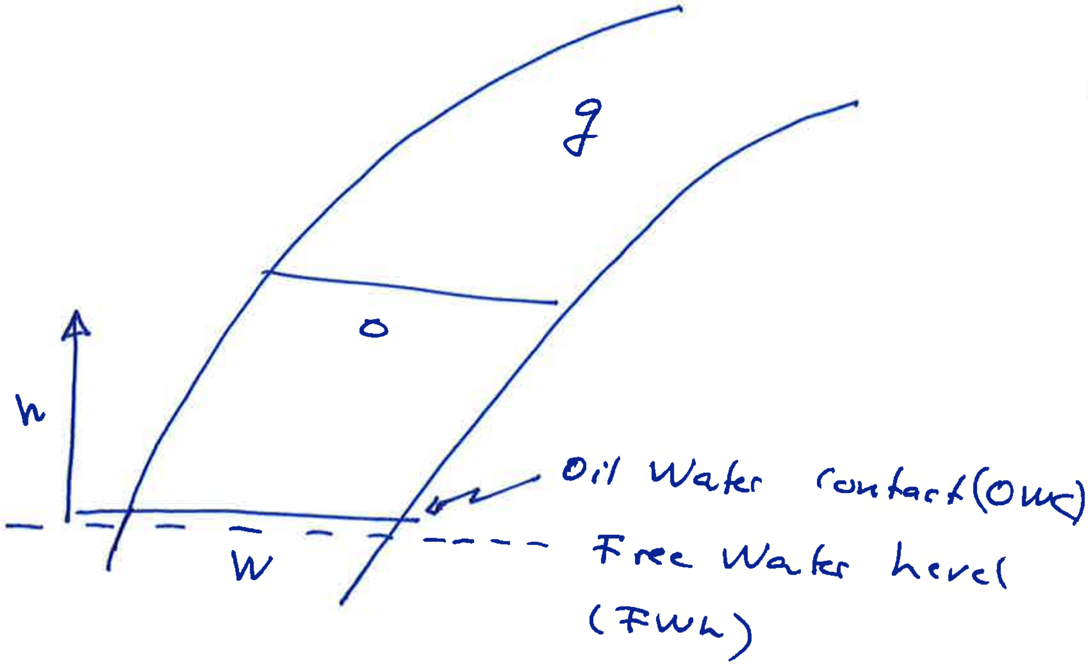
chap 6 wettability & capillary pressure

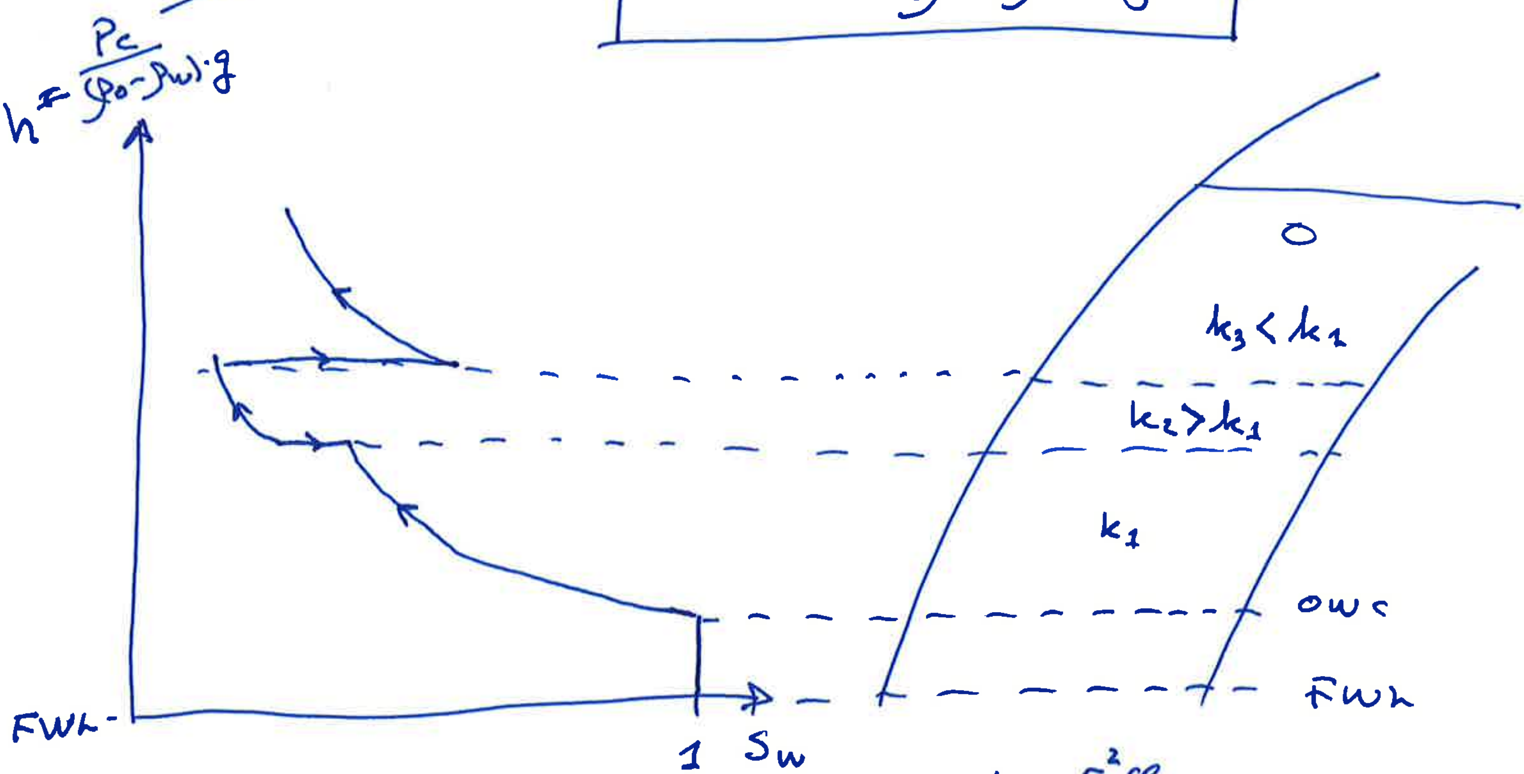


FWL $P_o = P_w = P_{ref}$

$$\left.\begin{aligned} P_w &= P_{ref} - \rho_w g h \\ P_o &= P_{ref} - \rho_o g h \end{aligned}\right\} \quad P_c \equiv P_o - P_w$$

$$= (P_{ref} - \rho_o g h) - (P_{ref} - \rho_w g h)$$

$$\boxed{P_c = (\rho_w - \rho_o) g h}$$



$k = \frac{r^2 \cdot \varphi}{8}$

explanation :

- surface forces

A surface usually has a preference for one phase

θ = contact angle (measured through the densest phase)

θ < 90° wetting

θ = 90° neutral wetting

θ > 90° non-wetting

surface tension: $\sigma = \left(\frac{\partial W}{\partial A}\right)_{P,T,\mu}$

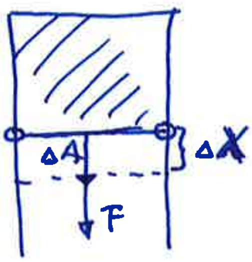
∂W is the energy needed to increase the surface area with the amount ∂A at constant pressure (P), temperature (T), chemical potential μ.

$$\left[\frac{\partial W}{\partial A}\right] = \left[\frac{\text{Energy}}{\text{area}}\right] = \left[\frac{F\cdot \text{length}}{(\text{length})^2}\right] = \left[\frac{F}{\text{length}}\right]$$

An interface could be:

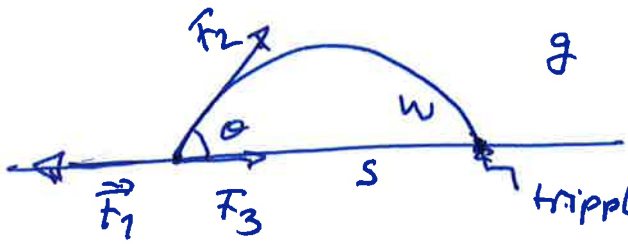
- solid / fluid interface
- fluid / fluid interface

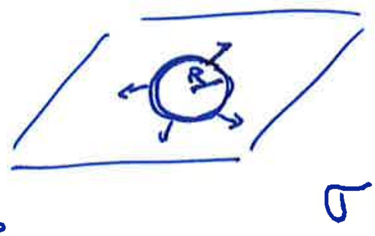
How can we measure surface forces?



$$\sigma = \frac{\Delta W}{\Delta A} = \frac{F\cdot\Delta x}{\Delta A}$$

water	air	$\sim 70\ mN/m$
oil	air	$\sim 50\ mN/m$
water	oil	$\sim 25\ mN/m$





F_1 force between gas / rock : $\sigma_{gs}\cdot 2\pi R$

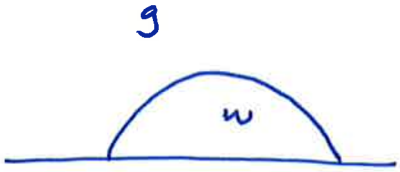
F_2 —— " —— gas / water : $\sigma_{gw}\cdot 2\pi R$

F_3 —— " —— water / rock : $\sigma_{ws}\cdot 2\pi R$

$$\sum \vec{F} = 0 \Rightarrow F_1 - F_3 - F_2\cos\theta = 0$$

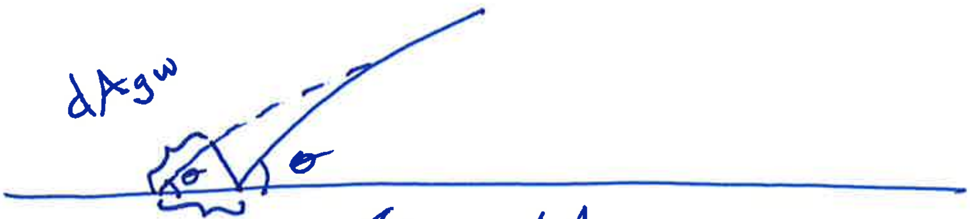
$$\underline{\sigma_{gs} - \sigma_{ws} = \sigma_{gw}\cos\theta}$$ Young-Dupre equation

Energy:



~~Energy~~ $E = \sigma_{gw} A_{gw} + \sigma_{ws} A_{ws} + \sigma_{gs} A_{gs}$

$$dE = 0 = \sigma_{gw}\, dA_{gw} + \sigma_{ws}\, dA_{ws} + \sigma_{gs}\, dA_{gs}$$



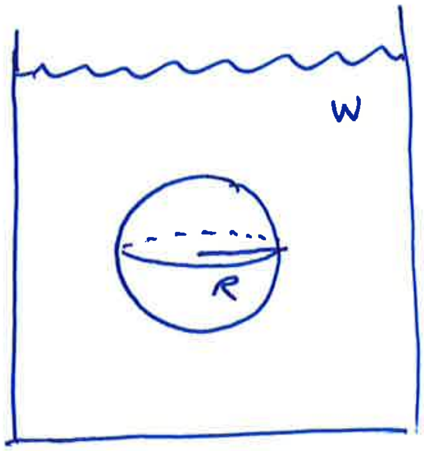
$$dA_{ws} = -dA_{gs}$$

$$dA_{gw} = \cos\theta \, dA_{ws}$$

$$\boxed{\sigma_{gw} \cos\theta = \sigma_{gs} - \sigma_{ws}}$$

Capillary pressure

$$P_c = P_o - P_w$$



$$dE = p_o\, dV_o - p_w\, dV_o - \sigma_{ow}\, dA = 0 \quad (*)$$

$$V_o = \frac{4}{3}\pi R^3$$

$$dV_o = 4\pi R^2\, dR$$

$$A = 4\pi R^2$$

$$dA = 8\pi R\, dR$$

$$(*) \Rightarrow p_o 4\pi R^2 \cancel{dR} - p_w 4\pi R^2 \cancel{dR} - \sigma_{ow}\, 8\pi R \cancel{dR} = 0$$

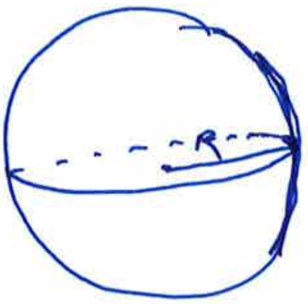
$$\boxed{p_o - p_w = \frac{2\sigma_{ow}}{R}}$$

Young-Laplace equation : $P_c = \sigma\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$

R_1, R_2 are the main curvature in ~~of~~ ~~an~~ ~~in~~ ~~it~~ a point.
(Euler 1760)

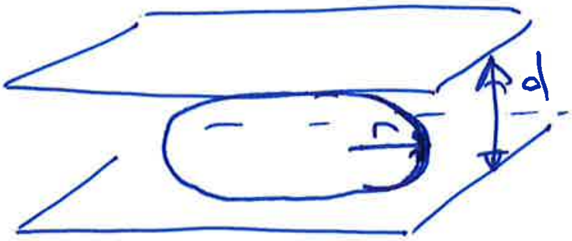
The main curvature is the radii of the largest and smallest circles in a point. These circles are orthogonal

Eks



$R_1 = R_2 = R$

$\Rightarrow P_c = \sigma\left[\frac{1}{R} + \frac{1}{R}\right] = \frac{2\sigma}{R}$



$r = d/2$

$R_1 = R, \quad R_2 = d/2$

$$P_c = \sigma\left[\frac{1}{R} + \frac{1}{d/2}\right]$$

- If the interface is curved, there will be a pressure drop between the interfaces