$A + B + C$ [Rm^3] = produced volumes [Rm^3]

A:

i) expansion of oil

N = original oil in place [Sm^3]

P_i $\qquad\qquad$ P ($< P_i$)

$N B_o(P_i) \equiv N B_{o,i}$ $\qquad$ $N B_o(P) \equiv N B_o$

$$N B_o - N B_{o,i}$$

ii) dissolved gas in oil

$$R_s = \frac{V_{g,o}^{s}}{\underbrace{V_{o,o}^{s}}_{N}}$$

@ p_i $\quad V_{g,o}^{s} = R_{s,i} N$ [Sm^3]

@ p $\quad V_{g,o}^{s} = R_s N$ [Sm^3]

liberated gas $\quad R_{s,i} N - R_s N$ [Sm^3]

expansion dissolved gas in oil

$[R_{s,i} N - R_s N] \times B_g$ [Rm^3]

B: expansion of gas cap:

original gas in place G [Sm^3]

volume of gas @ p_i $\quad G\,B_{g,i}$ [Rm^3]

———— @ p $\quad G\,B_g$ [Rm^3]

$$G\,B_g - G\,B_{g,i}$$

m = ratio between gas & oil in the reservoir

$$m = \frac{V_g^R}{V_o^R} = \frac{G\,B_{g,i}}{N\,B_{o,i}} \Rightarrow G = \frac{m\,N\,B_{o,i}}{B_{g,i}}$$

$$\rightarrow G\,[B_g - B_{g,i}] = \frac{m\,N\,B_{o,i}}{B_{g,i}}\,[B_g - B_{g,i}]$$

$$= m\,N\,B_{o,i}\left[\frac{B_g}{B_{g,i}} - 1\right]$$

C: change i $\underbrace{\text{hydrocarbon pore volume}}_{\text{HCPV}}$

$$V_p = V_o + V_w + V_g = \underbrace{V_{HCPV}}_{V_o + V_g} + V_w$$

$$dV_p = dV_{HCPV} + dV_w$$

$$dV_{HCPV} = dV_p - dV_w$$

From thermodynamics (ch 8)

compressibility : tells how fluid volumes, gass, solid change when P, T changes

$$C_w \equiv -\frac{1}{V_w}\left(\frac{dV_w}{dP_f}\right)_T$$

keep T constant

fluid pressure

$$C_p \equiv -\frac{1}{V_p}\left(\frac{dV_p}{dP_g}\right)_T$$

grain pressure

$$dV_w = -C_w V_w \, dP_f$$

$$dP_f = -dP_g$$

$$dV_p = -C_p V_p \, dP_g$$
$$= +C_p V_p \, dP_f$$

$$dV_{HCPV} = c_p V_p \, dp_f + c_w V_w \, dp_f$$

$$V_p = V_{HCPV} + V_w = V_{HCPV} + S_w V_p$$

$$(1 - S_w) V_p = V_{HCPV}$$

$$V_p = \frac{V_{HCPV}}{1 - S_w}$$

$$V_w = S_w V_p = \frac{S_w V_{HCPV}}{1 - S_w}$$

$$dV_{HCPV} = \frac{c_p + c_w S_w}{1 - S_w} V_{HCPV} \, dp_f$$

$$\int_{p_i}^{p} dp = p - p_i = -(p_i - p) = -\overset{>0}{\Delta P}$$

$$\Delta V_{HCPV} = -\frac{c_p + c_w S_w}{1 - S_w} V_{HCPV} \, \Delta P$$

$$C = -\Delta V_{HCPV} = \frac{c_p + c_w S_w}{1 - S_w} V_{HCPV} \, \Delta P + W_e B_w$$

+ influx of water

volume of water into the reservoir [Sm^3]

Produced oil : N_p

producing gas oil ratio : $R_p = \dfrac{V_{g,g}^{s} + \overbrace{V_{g,o}^{s}}^{R_s}}{\underbrace{V_{o,o}^{s}}_{N_p} + \cancel{(V_{o,g}^{s})}}$

$$V_{g,g}^{s} = N_p R_p - V_{g,o}^{s}$$

$$R_s = \frac{V_{g,o}^{s}}{\underbrace{V_{o,o}^{s}}_{N_p}} \Longrightarrow V_{g,o}^{s} = N_p R_s$$

$$V_{g,g}^{s} = N_p R_p - N_p R_s = N_p [R_p - R_s] \quad [Sm^3]$$

$$V_{g,g}^{s} B_g = N_p [R_p - R_s] B_g$$

$$V_{o,o}^{s} B_o = N_p B_o$$

produced volumes $= N_p B_o + N_p [R_p - R_s] B_g + W_p B_w$

produced water vol

$$A = N [B_o - B_{o,i}] + N [R_{s,i} - R_s] B_g$$

$$B = m N B_{o,i} \left[\frac{B_g}{B_{g,i}} - 1 \right]$$

$$\boxed{\frac{V_g^R}{V_o^R}} \quad m = \frac{V_g^R}{\underbrace{V_o^R}_{N [Sm^3]}}$$

$$C = \frac{C_p + S_w C_w}{1 - S_w} \underbrace{V_{HCPV}} \Delta p + W_e B_w$$

$$V_{HCPV} = V_g^R + V_o^R = m V_o^R + V_o^R = (1+m) V_o^R$$

$$= (1+m) N B_{o,i}$$

$$C = \frac{C_p + S_w C_w}{1 - S_w} (1+m) N B_{o,i} \Delta p + W_e B_w$$

Material balance :

$[Rm^3]$ produced volumes $= A + B + C$

Definitions:

$$F \equiv N_p \left[B_o + (R_p - R_s) B_g \right] + W_p B_w$$

(A) $$E_o \equiv (B_o - B_{o,i}) + (R_{s,i} - R_s) B_g = A/N$$

(B) $$E_g \equiv B_{o,i} \left[\frac{B_g}{B_{g,i}} - 1 \right] = B/mN$$

(C) $$E_{f,w} \equiv B_{o,i}(1+m) \left(\frac{c_w S_w + c_p}{1 - S_w} \right) \Delta p = {\sim}C/N$$

$$\underbrace{F}_{\text{known}} = N \left[\underbrace{E_o}_{\text{known}} + m \underbrace{E_g}_{\text{known}} + \underbrace{E_{f,w}}_{\text{(almost) known}} \right] + B_w \underbrace{W_e}_{\text{unknown}}$$

schematic examples

Ex 1 i) oil reservoir without gas cap

$m = 0$

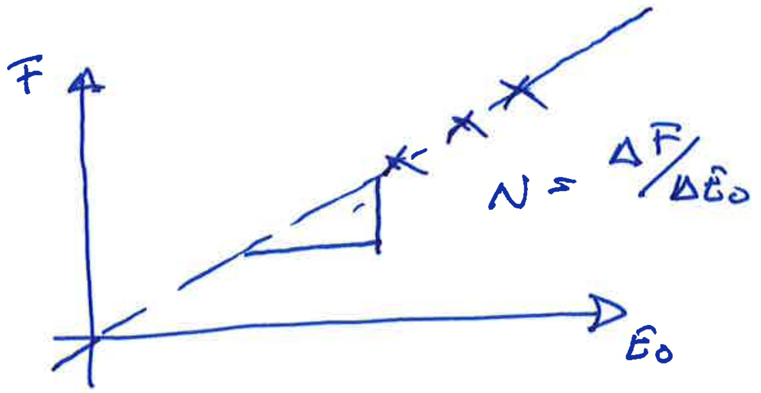
ii) no influx of water

$\Rightarrow W_e = 0$

iii) no compaction & water is incompressible

$\Rightarrow E_{f,w} = 0$

$$F = N E_o \Rightarrow N = F / E_o$$



Assume that W_e cannot be ignored