Diffusivitetslikningen:

$$(*) \quad \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) = \frac{1}{\eta}\frac{\partial p}{\partial t}, \quad \eta = \frac{k}{\mu \varphi c_t}$$

Initial betingelser: $p(r,0) = p_i$

$$\lim_{r\to\infty} p(r,t) = p_i$$

$$\lim_{r\to 0}\left(r\frac{\partial p}{\partial r}\right) = \frac{Q B_o \mu}{2\pi k h}$$

assumptions:
- homogeneous, isotropic formation
- fully penetrated well
- constant viscosity
- small constant compressibility

(e.g. flow of undersaturated oil)

surface rate

Darcy lov: $q = \frac{k 2\pi r h}{\mu}\frac{\partial p}{\partial r} = Q B_o$

change of variables (Boltzman transformation)

$$\xi = \frac{r^2}{4\eta t} \Rightarrow (*) \quad \underbrace{\frac{dp}{d\xi}}_{=y} + \xi\frac{d}{d\xi}\left(\frac{dp}{d\xi}\right) = -\cancel{\xi}\,\frac{dp}{d\xi}$$

$$\Rightarrow y + \xi\frac{d}{d\xi}y = -\xi y$$

$$\frac{dy}{y} = -\frac{(1+\xi)\,d\xi}{\xi}$$

$$\ln y = -\ln\xi - \xi + c$$

$$y = C_2\frac{e^{-\xi}}{\xi}$$

$$p = \int C_1 \frac{e^{-\xi}}{\xi}$$

$$p_i - p_w(t) = \frac{Q B_o \mu}{4\pi k h}\left[-E_i(-\xi)\right]$$

$$E_i(\xi) = -\int_{-\xi}^{\infty}\frac{e^{-t}}{t}\,dt$$

if $\xi < 0.01$ $\quad E_i(\xi) \approx -\ln\xi \underbrace{-0.5772}_{\text{Eulers const.}}$

$$\Rightarrow p_i - p_w(t) = \frac{Q B_o \mu}{4\pi k h}\left[\ln\frac{k t}{(\gamma)\varphi\mu c_t r_w^2} \underbrace{+0.80907}_{-0.5772+\ln 4} + 2S\right]$$

Darcy

if skin is a time-independent variable

$$\gamma = e^{0.5772} = 1{,}781$$

OFU :

$$p_i - p_w(t) = \frac{162{,}6\, QB\mu}{hk}\left[\log\frac{kt}{\varphi\mu c_t r_w^2} - 3{,}2275 + 0{,}875\right]$$

is $\xi < 0.01$ a good approximation?

$k = 50\ mD$

$h = 30\ ft$

$r_w = 6$ inches

$\varphi = 0.3$

$\mu = 3\ cp$

$c = 10^{-5}\ 1/psi$

$B_o = 1.25\ rb/stb$

$$\xi = \frac{r^2}{4\eta t} = \frac{r^2 \cdot \mu \varphi C_t}{4 \cdot k \cdot t}$$

$$\xi = \frac{r^2 \mu \varphi C_t}{4 k t}$$

$\Rightarrow$ Darcy units

$k = 0.05\ D$

$h = 30 \cdot 30.48\ cm = 914.4\ cm$

$r_w = 6 \cdot 2.54\ cm = 15.24\ cm$

$c = 10^{-5}\ 1/psi = 10^{-5} \cdot 14.7\ 1/atm$

$c = 14.7 \cdot 10^{-5}\ 1/atm$

$q = 400\ stb/d = 400 \cdot 1.25 \cdot \frac{159 \cdot 10^3\ cm^3}{24 \cdot 60 \cdot 60} = 920\ \frac{Rcm^3}{s}$

$$\Rightarrow \frac{\varphi \mu r_w^2 C_t}{4kt} < 0.01 \Rightarrow t > \frac{\varphi \mu r_w^2 C_t}{0.04 k}$$

$$= \frac{0.3 \cdot 3 \cdot (15.24)^2 \cdot 14.7 \cdot 10^{-5}}{0.04 \cdot 0.05} = 15.4\ sec$$

$$\underline{\underline{t > 15.4\ sec}}$$

b) Forklar kort med ord hvordan en generelt går fram for å sette opp en materialbalanseligning.

c) Angi hvilke 3 hovedtyper av data som er nødvendige for å kunne utføre en materialbalanseberegning og forklar hvordan de kan bli framskaffet.

d) Materialbalanseligningen for et oljereservoar med gasskappe er

$$N_p(B_o + (R_p - R_s)B_g) = NB_{oi}\left(\frac{(B_o - B_{oi}) + (R_{si} - R_s)B_g}{B_{oi}} + m(\frac{B_g}{B_{gi}} - 1)\right).$$

Forklar i prinsipp hvordan denne ligningen kan brukes til å bestemme opprinnelig oljevolum og størrelsen på gasskappen dersom begge disse verdiene er usikre fra volumetriske overslag.

Oppgave 4 sep 2003

Linjekildeløsningen ("line source solution") i praktiske enheter er gitt ved

$$p_{wf} = p_i - \frac{162.6\,Q\mu B}{kh}\left[\log\left(\frac{kt}{\phi\mu c r_w^2}\right) - 3.23 + 0.87S\right]. \qquad \ldots\ldots\ldots (1)$$

Praktiske enheter	
p : psi	h : ft
Q : stb/d	t : timer
q : rb/d	c : psi^{-1}
B : rb/stb	r : ft
k : mD	μ : cp

$y = -226 \cdot \underbrace{x}_{\log t} + 2082$

$\Rightarrow \log\frac{kt}{\varphi\mu c r_w^2} = \log\frac{k}{\varphi\mu c r_w^2} + \log t$

a) Forklar hva som menes med halvstasjonær periode ("pseudo-steady state" eller "semi-steady state") og vis ved en enkel materialbalansebetraktning at p_{wf} i denne perioden kan uttrykkes ved

$$\frac{dp_{wf}}{dt} = \frac{QB}{cV_p},$$

hvor V_p er porevolumet.

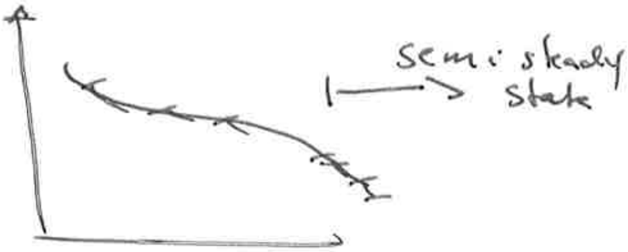
$\log a \cdot b = \log a + \log b$

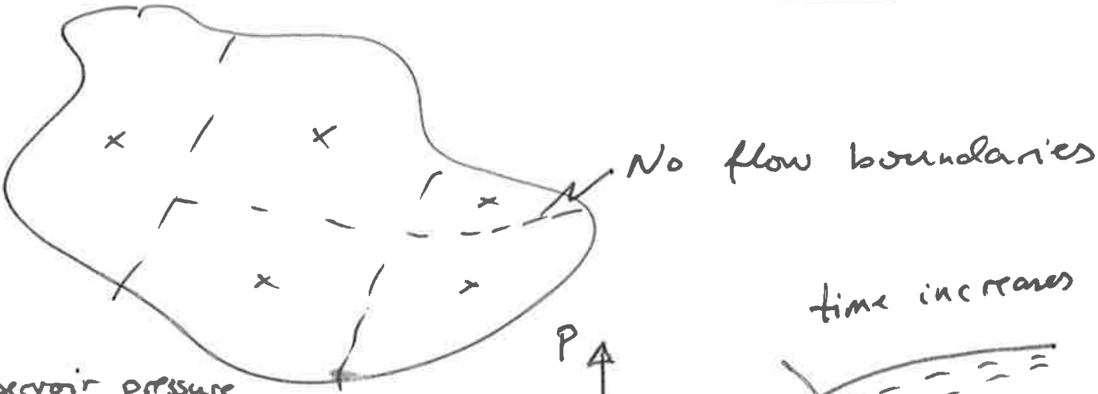
$\Rightarrow -\frac{162.6\,Q\mu B}{kh} = \underbrace{-226}_{\beta \cdot t}$

$$k = \frac{162.6 \cdot q \mu B}{226 \cdot h}$$

$$k = \frac{162.6}{226} \cdot \frac{800 \cdot 1 \cdot 1.25}{8} = \underline{\underline{90 \text{ mD}}}$$

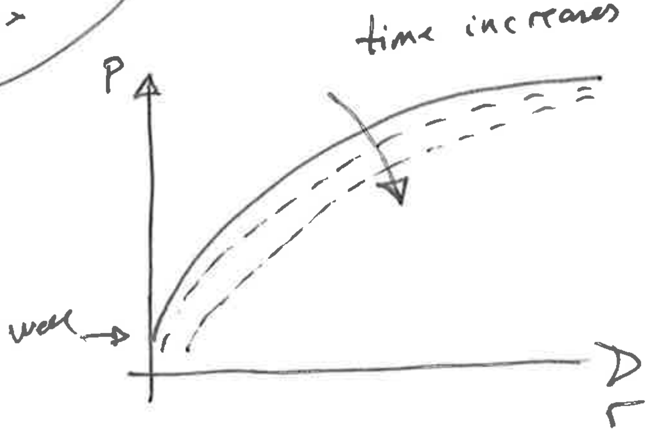
Semi steady state





reservoir pressure

$$\frac{dp}{dt} = \text{const}$$



We can always use material balance

$$C = -\frac{1}{V_p}\frac{dV_p}{dp} \simeq -\frac{1}{V_p}\frac{\Delta V_p}{\underbrace{\Delta P}_{P-P_i}}$$

$\cancel{\int c\,dp =}$

average reservoir pressure

$$\underbrace{\Delta V_p}_{q\cdot\Delta t} = -V_p C\,(p-p_i) = V_p C\,(p_i - P)$$

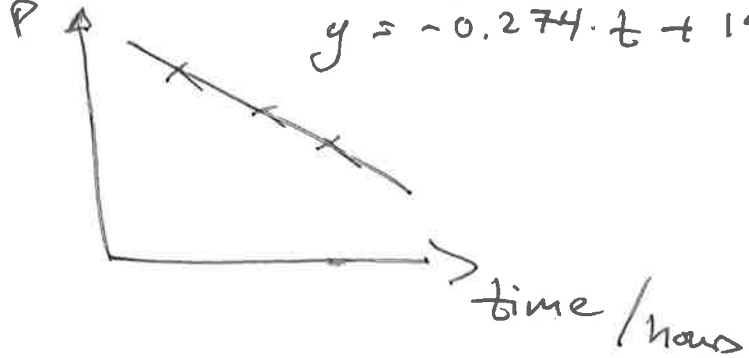
$$\underbrace{q}_{QB_o}\cdot\Delta t = V_p\,C\,(p_i-P) \Longrightarrow Q B_o \Delta t = V_p C \underbrace{(p_i-P)}_{\equiv \Delta P}$$

$$\boxed{\frac{\Delta P}{\Delta t} = -\frac{QB_o}{V_p\cdot C}}$$

$\Rightarrow$ In Semi-Steady state

$$\frac{dp}{dt} = \text{const} = -\frac{QB_o}{V_p C}$$

Ex 2003 sep

a) 

$$y = \overbrace{-0.274}^{\frac{dp}{dt}} \cdot t + 1539$$

$$\frac{dp}{dt} = -0.274 \ psi/hour = -6.576 \ psi/day$$

$$V_p = -\frac{QB}{c_o \, dp/dt} = \frac{800 \cdot 1{,}25}{17.7 \cdot 10^{-6} \cdot 6{,}576} = \underline{\underline{8.6 \cdot 10^6 \ bbl}}$$

$$(p_i - \bar{p}) = \frac{\overbrace{QB_o}^{6.576}}{cV_p} \Delta t$$

$t = 2000$ hrs $\qquad p_w = 986$ psi

$$\bar{p} = \left(1900 - 6.576 \cdot \frac{2000}{24}\right) psi = \underline{\underline{1352}}$$