Høsten 2014

FYS100 Fysikk
Eksamen/Exam

Here are the solutions to the exam. Also indicated is the approximate number of points for each problem; each sub-question is also given some specific number of points (not indicated).

Please note that even making some of these mistakes, partial credit (some points) are awarded for a given question. It is not all-or-nothing. And I have almost not subtracted for rounding off errors, except in extreme cases.

Finally, consider that to pass, you need 40 percent. And so even if you have done half completely correctly, you still only get E or D. Keep this in mind.

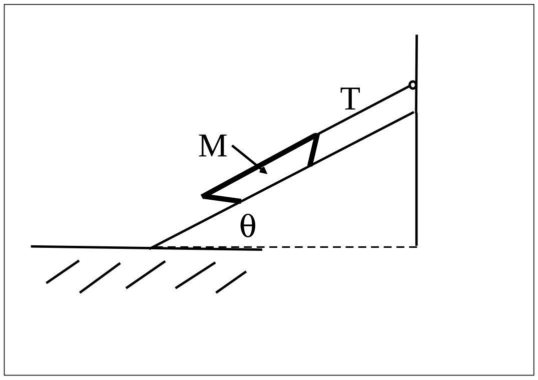
The general impression is that almost everybody has done good and thoughtful stuff on some of the problems, but very few got everything right. That's why there are so many C, D, and E. And most of the F's have 20-40 points, so that some things are correct. But alas, not enough to pass. The 10 percent that got A or B have done a really good job.

With each solution, I have stated examples of the most common mistakes made, and I would appreciate that you check those against your own results, before contacting about further clarification.

You are entitled to get a clarification, if you have further questions. These you can get by filling in a particular form, and hand it in in the appropriate place. I will reply by email. Again, I would appreciate if you would check your results against this document first.

About complaining formally: After the clarification, you can in principle file a complaint, if you really feel that an actual mistake has been made. But be aware, that the external censor is usually more severe that I am in terms of grading, and that his result overrules mine, even if it makes your grade go down! So this option should be not be used at random.

Problem 1 (10 points)



A boat of mass $M = 100$ kg is lying on an evenly sloping rock-face, completely out of the water. It is attached to a rope from the end of the boat, and the coefficient of static friction between boat and rock is $\mu_s = 0.300$. The rock-face makes an angle of $\theta = 20.0°$ with the horizontal (see figure). You do not have to take into account possible rotation of the boat in this problem. The gravitational constant is $g = 9.80$ m/s^2.

a) List important and unimportant information in the problem description, and draw a sketch of the relevant forces.

Solution: Important: Slope. θ, M, μ_s, g. Static. No rotation. Unimportant: Boat. Water. Rock.

b) What is the minimum string force T required to keep the boat in place? Provide both the algebraic expression, and the numerical result.

Solution: Write force equilibrium along slope and perpendicular to it, and solve for T:

$$T + F_s = Mg\sin\theta, \tag{1}$$

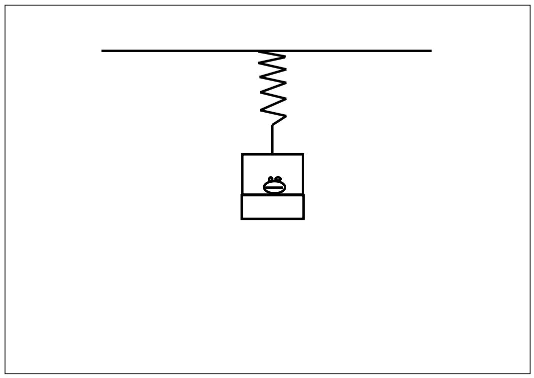
$$|n| = Mg\cos\theta, \tag{2}$$

$$T_{\min} \rightarrow F_s^{\max} = \mu_s |n|, \tag{3}$$

$$T = Mg(\sin\theta - \mu_s\cos\theta) = 58.9N. \tag{4}$$

Typical mistakes: Forgetting friction. Swapping sin and cos. Forgetting cos in the friction force. Thinking that friction pulls opposite to T (there is no movement in the direction of T!). Typing in numbers wrong. Rounding off errors.

Problem 2 (15)



A frog sits in a small basket, which is hanging at rest from the ceiling at the end of a spring (see figure). The basket has a mass of 0.500 kg and the frog has a mass of 0.100 kg. The spring constant is $k = 20.0$ N/m. The gravitational constant is $g = 9.80$ m/s^2.

a) List important and unimportant information in the problem description, and draw a sketch of the relevant forces.

Solution: Important: m_f, m_b, k, g. Vertical. Unimportant: Basket. Frog.

b) How far is the spring stretched, compared to the equilibrium position when the basket is empty?

Solution: Basket stretches some. With frog stretches more $= dx$. Spring force gives

$$m_f g = k dx \rightarrow dx = \frac{m_f g}{k} = 4.90 \text{ cm}. \qquad (5)$$

Typical mistakes: Not computing the *difference* between with frog and without frog. Gives $24 - 29$ cm. Rounding off errors.

c) The frog now falls out of the basket which starts oscillating up and down. Find the period of the oscillation and the total mechanical energy associated with this oscillation.

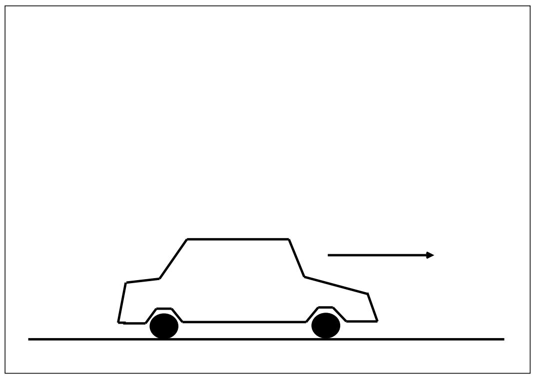
Solution: Harmonic oscillator motion from initial amplitude $A = dx$, from rest, with angular frequency $\omega = \sqrt{k/m_b}$. Then

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m_b}{k}} = 0.993 \text{ s}, \qquad (6)$$

$$K = \frac{1}{2} k dx^2 = 2.40 \times 10^{-2} \text{ J}. \qquad (7)$$

Typical mistakes: Wrong equation for T. Follow-mistake from using wrong dx from b). Wrong equation for energy, including some kinetic energy. Typing in the numbers wrong. Rounding off errors particularly common.

Problem 3 (20)



A 4-wheel drive car of mass $M = 2100$ kg accelerates from rest to 100 km/h in 3.40 seconds, and we will assume that the acceleration is constant. In the following, ignore resistance from the air and the rolling friction of the wheels against the surface. The wheels are rolling without sliding, and they have a diameter of $D = 50.0$ cm. Each wheel has a mass of $m = 18.0$ kg, and has a moment of inertia around their centre of mass corresponding to a disc, $I = 1/2\, mR^2$. The mass of the wheels is included in the total 2100 kg. The wheels are taken to be identical and carry the same amount of weight. The gravitational constant is $g = 9.80$ m/s^2.

a) List important and unimportant information in the problem description, draw a sketch of the forces on the car, and the forces and torques acting on each wheel.

Solution: Important: 4-wheel. M. $v_{\max}$. t_a. Constant a. No resistance. Rolling, no sliding. D. m. I. Same weight on all wheels. g. Unimportant: car. Remember that friction force is forwards! Hand-in problem!!

b) What is the required torque that the engine has to provide for each wheel, to have this acceleration? Provide both the algebraic expression, and the numerical result.

Solution: Write down torque equation for one wheel; force equation for whole car; and rolling without sliding condition; and solve for τ:

$$\tau - F_s D/2 = I\alpha = \frac{1}{2}m\left(\frac{D}{2}\right)^2, \tag{8}$$

$$4F_s = Ma, \tag{9}$$

$$a = \frac{D}{2}\alpha, \tag{10}$$

$$\tau = aR\left(\frac{M}{4} + \frac{m}{2}\right) = 1.09 \times 10^3 \text{ Nm (not Joule)}. \tag{11}$$

Typical mistakes: Friction the wrong way. Not including the friction (mistaking rolling friction for friction and ignoring it). Ignoring the rotation of the wheels. Algebraic mistakes with signs. Typing in the numbers wrong. Forgetting to divide by 4. Mistaking radius and diameter of wheel, getting a factor of 2 wrong. Including/excluding mass of wheels into the total mass. Mistaking a and α. Rounding off errors.

c) What is the required coefficient of static friction between road and wheel, to avoid spinning? Provide both the algebraic expression, and the numerical result.

Solution: Write down force equation and the relation of static friction to normal force, and solve for μ_s:

$$4F_s = Ma, \tag{12}$$

$$F_s = \mu_s |n|, \tag{13}$$

$$|n| = \frac{Mg}{4}, \tag{14}$$

$$\mu_s \geq \frac{a}{g} = 0.834. \tag{15}$$

Typical mistakes: Rounding off errors. Forgetting factor of 4. Just writing down the result (remembering the result from the hand-in...but not the reason why?).

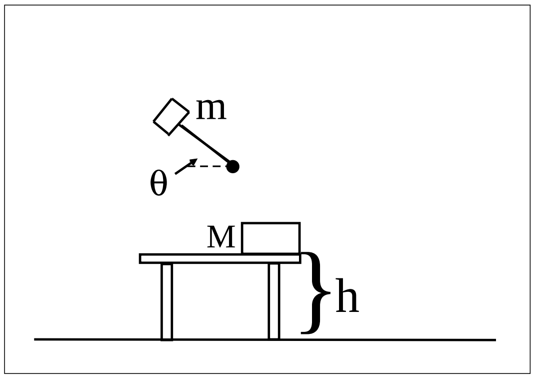
d) What is the total kinetic energy of the car as it reaches 100 km/h?

Solution: The total kinetic energy is the sum of the translation energy and the rotational energy of the 4 wheels:

$$K = \frac{1}{2}Mv^2 + 4\frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2 + 4\frac{1}{2}\frac{1}{2}m\left(\frac{D}{2}\right)^2\omega^2 = \left(\frac{1}{2}M + m\right)v^2 = 824 \text{ kJ}. \tag{16}$$

Typical mistakes: Forgetting the contribution from the wheels. Forgetting the contribution from the linear motion. Only including one wheel. Typing in numbers wrong. Rounding off errors. Having the wheels give negative contribution.

Problem 4 (35)



A block of mass M is standing on top of a horizontal, smooth table of height h, initially at rest. Above it, a hammer is attached to the wall, in such a way that it can rotate in the vertical plane (see figure). The hammer has a head of mass m, and an arm of length l and negligible mass. It rotates without friction, and for the purpose of this problem, you may take its moment of inertia around the point of attachment to the wall to be $I = ml^2$. The hammer is now released from rest from an angle of θ above the horizontal. It swings down and hits the block perfectly elastically. The collision takes place just as the hammer arm is in the vertical position. The gravitational constant is g.

a) List important and unimportant information in the problem description, and draw a sketch of the situation.

Solution: Important: M, m, h, l, g, I. Horizontal. Smooth. At rest. Rotate. No friction. From rest. Elastic. Collision at vertical position.

b) What is the angular acceleration for the hammer, just as it is release?

Solution: Compute the torque of gravity and use the torque-angular acceleration relation

$$\tau = mgl\cos\theta, \tag{17}$$

$$\alpha = \frac{\tau}{I} = \frac{mgl\cos\theta}{ml^2} = \frac{g}{l}\cos\theta. \tag{18}$$

Typical mistakes: Swapping sin and cos. Forgetting the cos. Forgetting the l in the torque.

c) What is the speed of the hammer immediately before it hits the block?

Solution: Energy conservation (axis does not perform work), gives

$$-\Delta U = \Delta K, \tag{19}$$
$$mgl(1+\sin\theta) = \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2, \tag{20}$$
$$v = \sqrt{2gl(1+\sin\theta)}. \tag{21}$$

Typical mistakes: Thinking that there is constant α. Linearizing as a pendulum, using θ and not sin. Factor of 2 missing.

d) What are the speeds of the block and hammer immediately after the collision?

Solution: Elastic collision, energy and momentum conservation. Solve for v_f and u:

$$mv_i = mv_f + Mu, \tag{22}$$
$$\frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}Mu^2, \tag{23}$$
$$v_f = \frac{m-M}{m+M}\sqrt{2gl(1+\sin\theta)}, \tag{24}$$
$$u = \frac{2m}{m+M}\sqrt{2gl(1+\sin\theta)}. \tag{25}$$

[Note that there is a possible issue of the force supplied by the axis at the moment of collision, which can potentially be non-zero! But because all mass is assumed point-like at the end of the stick, this force turns out to be 0, and there is momentum conservation. None of you made this point, but I did not subtract any points for this.]

Typical mistakes: Thinking that block and hammer stick together as for a perfectly *in*elastic collision. Not using energy conservation. Not deriving the equations correctly. Using constant α. Using forces. Mistakes in algebra. Follow-mistakes from having the initial speed wrong. Thinking that hammer stops when hitting the block (only for equal masses!).

e) To which height does the hammer swing up after the collision?

Solution: Again, energy conservation of the hammer after the collision.

$$\Delta h m g = \frac{1}{2}mv_f^2, \tag{26}$$
$$\Delta h = \frac{v_f^2}{2g} = \left(\frac{m-M}{m+M}\right)^2 l(1+\sin\theta). \tag{27}$$

Typical mistakes: Using constant α. Assuming that hammer stops. Algebraic mistakes. Follow-error from not doing c) and d) correctly.

f) The block slides along the table and over the edge. At what distance from the table does it hit the floor, and with what speed?

Solution: Projectile motion with u as the initial horizontal velocity, dropping a height h.

$$h - \frac{g}{2}t^2 = 0, \tag{28}$$

$$d = ut, \tag{29}$$

$$d = u\sqrt{\frac{2h}{g}} = \frac{4m}{m+M}\sqrt{hl(1+\sin\theta)}. \tag{30}$$

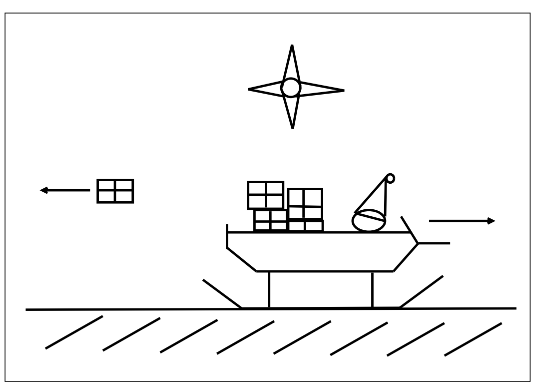
The speed again comes from energy conservation

$$\frac{1}{2}Mv^2 = \frac{1}{2}Mu^2 + Mgh, \tag{31}$$

$$v = \sqrt{u^2 + 2gh}. \tag{32}$$

Typical mistakes: Follow-errors. Using the range equations. Not realising that the initial velocity is horizontal and having an additional angle in the problem, in the usual projectile motion calculation.

Problem 5 (20)



Santa Claus is travelling in his sleigh over an icy lake on Christmas eve. Suddenly, all his reindeer decide to call it a day, and fly off to meet up with all the cute girl-reindeer down at the pub. Santa Claus is left behind in his sleigh, motionless on the lake. The gravitational constant is $g = 9.80\ \text{m/s}^2$.

What to do? He has a bunch of 50 presents, each with a mass of 1.00 kg. He and his sleigh, not counting the 50 presents, have a total mass of 300 kg. Santa decides to throw a present out the back of the sleigh, at a horizontal speed of 10.0 m/s.

a) List important and unimportant information in the problem description, and draw a sketch of the situation.

Solution: Important: Icy, no friction. Rest. 50, m, M. $v_e = 10$ m/s. Horizontal. Unimportant: Santa Claus. Reindeer. Pub. Christmas. Presents. g. Sleigh.

b) What is the resulting speed of the sleigh + Santa + rest of the presents?

Solution: Momentum conservation, initially 0.

$$mv_e = (M + (N-1)m)v_1, \tag{33}$$

$$v_1 = \frac{mv_e}{M}\frac{1}{1+(N-1)m/M} = 2.87 \times 10^{-2} \text{ m/s}. \tag{34}$$

Typical mistakes: Using energy conservation instead of momentum conservation. Rounding off errors. Using 300 instead of 350 kg. Doing something with gravity.

c) That's quite slow, and Christmas doesn't last for ever, so Santa decides to throw out another present, again at 10.0 m/s relative to himself. What is now the speed of the sleigh with the remaining content?

Solution: Same thing, but now with one less present in the sleigh, and relative to the moving frame of the sleigh:

$$v_2 = v_1 + \frac{mv_e}{M}\frac{1}{1+(N-2)m/M} = 5.74 \times 10^{-2} \text{ m/s}. \tag{35}$$

Typical mistakes: Thinking that the sleigh comes to rest between every throw. Not realising that the second package has speed v_e relative to the sleigh, not the lake. Rounding off errors. Notice that numerically, the result is very similar to the correct one, if you do this. But the equation is wrong.

d) After throwing the third present? Fourth?

Solution: Again the same, but with less and less present, and all the time relative to the previous moving frame.

$$v_3 = v_2 + \frac{mv_e}{M}\frac{1}{1+(N-3)m/M} = 8.62 \times 10^{-2} \text{ m/s}. \tag{36}$$

$$v_4 = v_3 + \frac{mv_e}{M}\frac{1}{1+(N-4)m/M} = 11.5 \times 10^{-2} \text{ m/s}. \tag{37}$$

Typical mistakes: Thinking that the sleigh comes to rest between every throw. Not realising that the subsequent packages have speed v_e relative to the sleigh, not the lake. Rounding off errors. Notice that numerically, the result is very similar to the correct one, if you do this. But the equation is wrong. If you got 11.6 after the fourth present, you did it wrong. But some got 11.5 anyway through a rounding off error.

e) **[Up to 10 extra points: Only attempt this when everything else is done!]** How fast is he going after having thrown out all 50 presents, one at a time? See if you can find a general algebraic expression when having thrown out N presents.

Solution (10 extra): The general expression is

$$v_j = \frac{mv_e}{M} \sum_{i=1}^{j} \frac{1}{1 + \frac{m}{M}(N - i)}. \tag{38}$$

which for $N = j = 50$ is $v_{50} = 1.54$ m/s.

Typical mistakes: Same as above. Some got the above right, but then it went wrong here. I gave some points for any effort, but only full points for a correct expression, with an explanation why, and the correct final number. (Throwing all the presents out at the same time would give $50/300 v_e = 1.67$ m/s; which is the wrong result here).