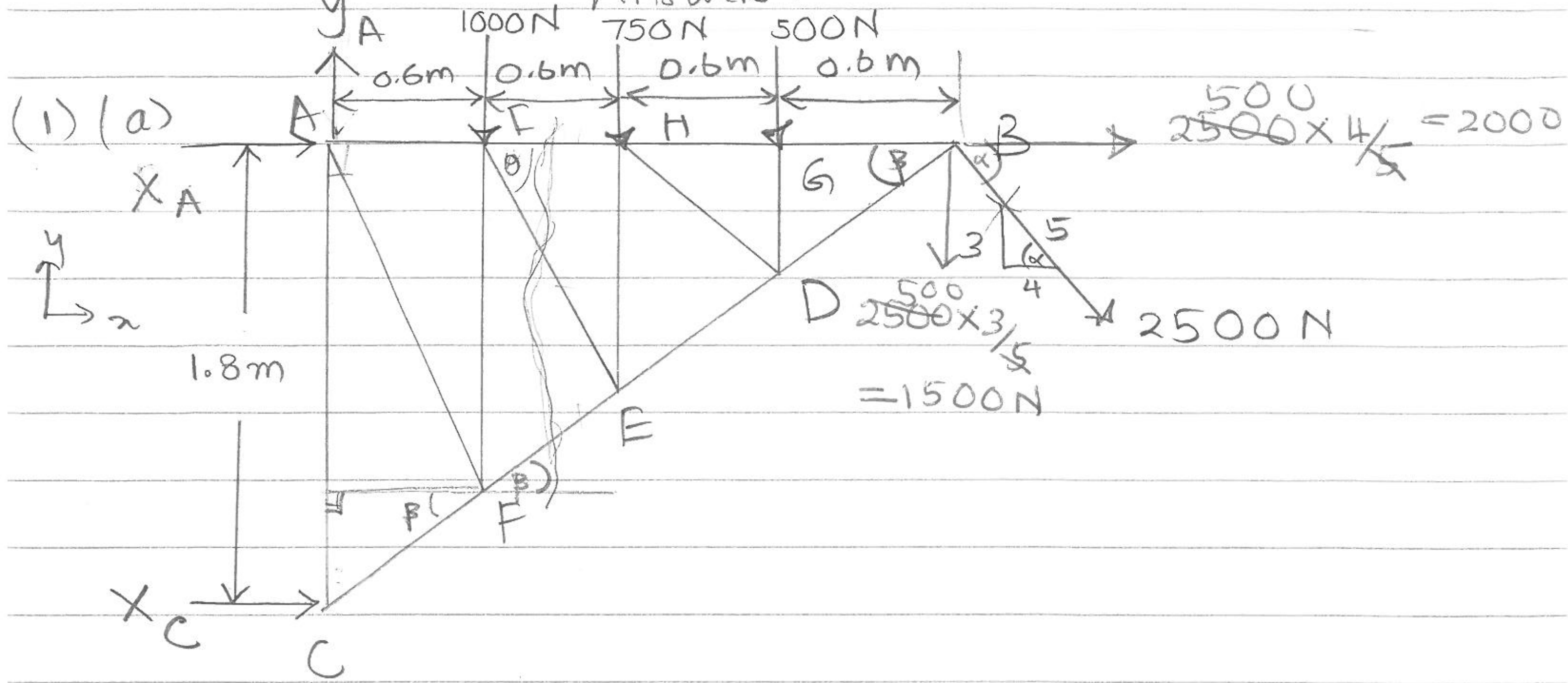
Exam – BIB 120 – September

Answers.

(1) (a)



Considering equilibrium of the truss,

$$\Sigma M_A = 0;\ -(1000N)\times 0.6m - (750N)\times 1.2m - 500N\times 1.8m - (1500N)\times 2.4m + X_C \times 1.8m = 0$$

$$X_C = 3333.3N = 3.33\,kN$$

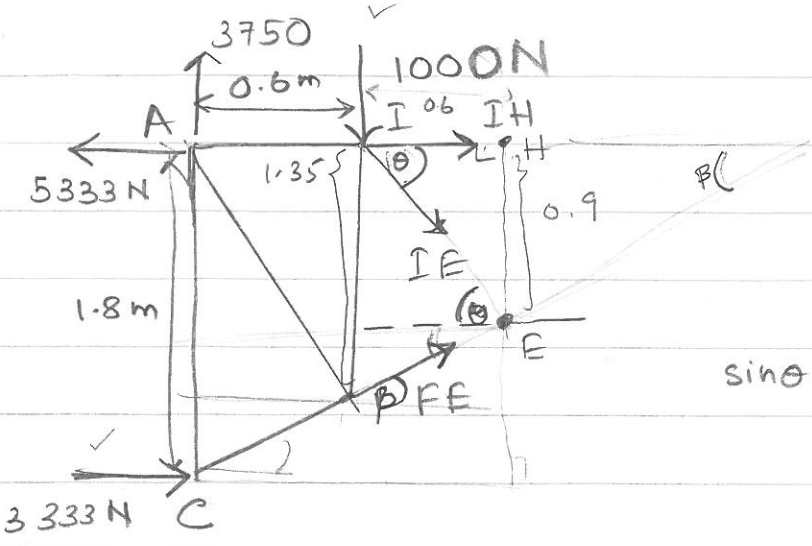
$$\Sigma Forces_x = 0;\ X_A + X_C + 2000 = 0$$

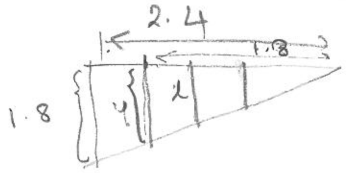
$$X_A = -2000 - 333.3 = -5333N = -5.33\,kN$$

$$\Sigma Forces_y = 0;\ Y_A - 1000 - 750 - 500 - 1500 = 0$$

$$Y_A = 3750N = 3.75\,kN$$

(b) Making imaginaring cut through HI, IE & EF considering the equilibrium of left.





$\frac{2.4}{1.8} = \frac{1.8}{y}$ $y = 1.35$

$\frac{2.4}{1.8} = \frac{1.2}{l}$ $l = 0.9$

$\sin\theta = \frac{0.9}{1.08}$

$\tan\beta = \frac{1.8}{2.4}$

$\cos\beta = \frac{2.4}{3}$ $\sin\beta = \frac{1.8}{3}$

$\Sigma M_E = 0;\ -F_{IH} \times 0.9m + 0.6m \times 1000 - 1.2m \times 3750N$
$5333 \times 0.9m + 3333 \times 0.9m = 0$

$F_{IH} = 4333\ N = 4.33\ kN$ (Ten)

$\Sigma M_I = 0;\ +1.35m \times F_{FE} \times \cos\beta + 1.8 \times 3333 - 3750 \times 0.6m = 0$

$F_{FE} = -3471\ N$ (comp)

$= -3.47\ kN$

$\uparrow \Sigma F_y = 0;\ -1000 + 3750 - F_{IE} \times \sin\theta + F_{FE} \sin\beta = 0$

$2750 - F_{IE} \times \frac{0.9}{1.08} - 3471 \times \frac{1.8}{3} = 0$

$F_{IE} = 801 N$
(tension)

(c) Yield stress of alluminium alloy = 352 MPa

Allowablle stress = $\frac{352}{1.275}$ MPa

= 276 MPa //

Required area,

FE, Area = $\frac{3471\ N}{276\ N/mm^2}$ $\frac{\text{Force}}{\text{area}}$ = stress

= 12.57 mm^2 // ✓

IH, Area = $\frac{4333\ N}{276\ N/mm^2}$ ≃ 16 mm^2

d) using stress-strain curve,

$E = \frac{\sigma}{\epsilon} = \frac{300 MPa}{0.004 mm/mm}$ = 75 GPa //

$\sigma = E\epsilon \leftarrow \delta/L$

$\sigma_{all} = \frac{\text{Force}}{\text{area}}$

$\delta/L = \frac{\sigma_{all}}{E}$

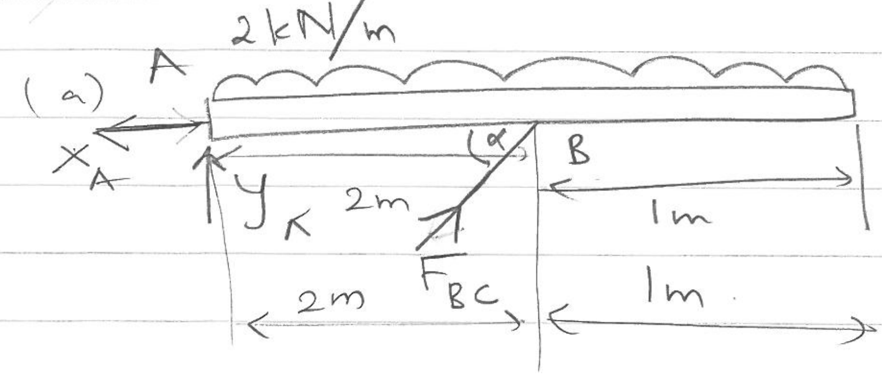
$L_{FE} = 0.6/\cos\beta = \cancel{0.6}^{0.1} \times 3/\cancel{2.4}_{0.4} = 3/4$ m

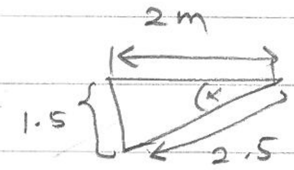
$\delta_{FE} = \frac{276 \times 10^6 \times L_{FE}}{75 \times 10^9}$ = 2.76×10^{-3} m // (shortned)

L_{HI} = 0.6m, $\delta_{HI} = \frac{276 \times 10^6 \times 0.6m}{75 \times 10^9}$ = 2.208×10^{-3} m (elongated)

Question 2

(a)





$\tan\alpha = \frac{1.5}{2}$

$\sin\alpha = 1.5/2.5$

$\cos\alpha = 2/2.5$

Considering the equilibrium of beam AB,

$\circlearrowleft \Sigma M_A = 0$; $-2\,kN/m \times 3m \times 1.5m + F_{BC} \times \sin\alpha \times 2 = 0$

$F_{BC} = 7.5\,kN$ //

$\overset{\rightarrow}{+}\ \Sigma F_x = 0$; $-X_A + F_{BC}\cos\alpha = 0$

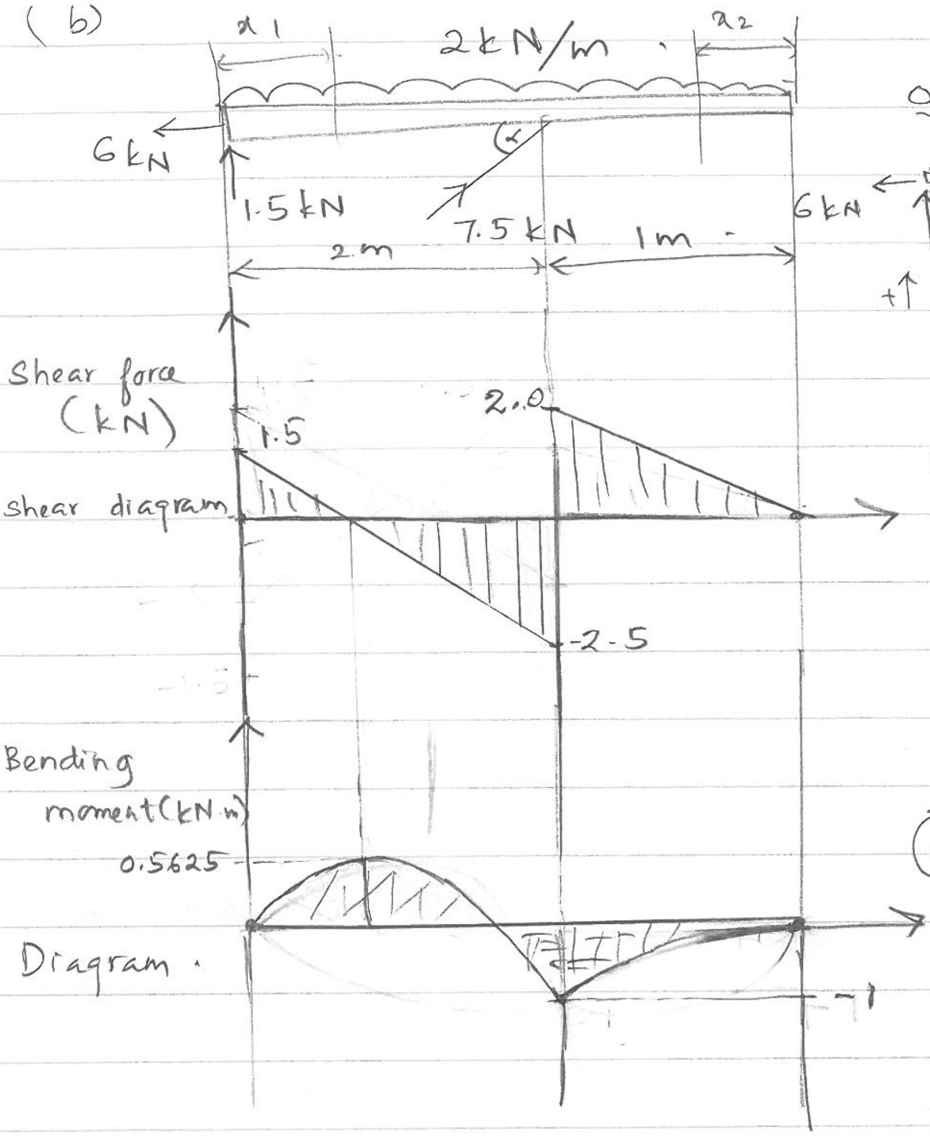
$\therefore X_A = 7.5 \times 2/2.5 = 6\,kN$ //

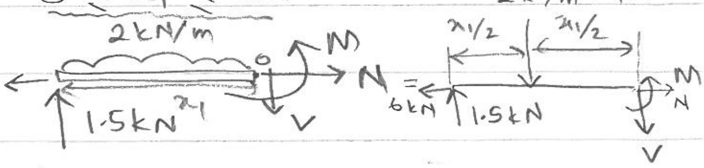
$+\uparrow\ \Sigma F_y = 0$; $Y_A - 2kN/m \times 3m + F_{BC} \times \sin\alpha = 0$

$Y_A = 6 - 7.5 \times \frac{1.5}{2.5} = 1.5\,kN$ //

(b)

(b)





$0 \leq x_1 \leq 2m$

$+\uparrow \sum F_y = 0; \quad 1.5 - 2x_1 - V = 0$

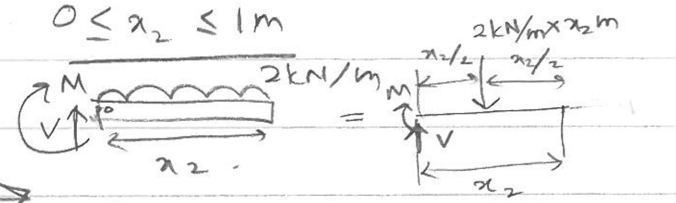
$$\boxed{V = -2x_1 + 1.5}$$

$\sum M_O = 0;$

$$+M + 2x_1 \times x_1/2 - 1.5 \times x_1 = 0$$

$$\boxed{M = 1.5x_1 - x_1^2}$$

$0 \leq x_2 \leq 1m$



$+\uparrow \sum F_y = 0;$

$$V - 2x_2 = 0$$

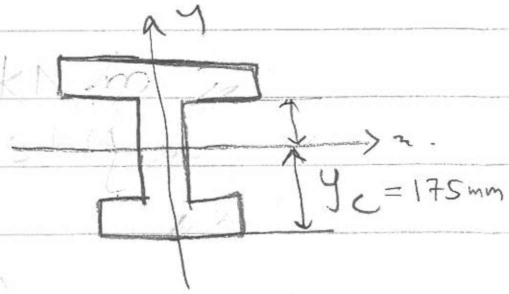
$$\boxed{V = 2x_2}$$

$\sum M_O = 0$

$$-2 \times x_2 \times x_2/2 - M = 0$$

$$\boxed{M = -x_2^2}$$

(C) $y_c = \dfrac{(125 \times 30 \times 15 + 25 \times 250 \times 155 + 200 \times 30 \times 295)}{(125 \times 30 + 25 \times 250 + 200 \times 30)}$



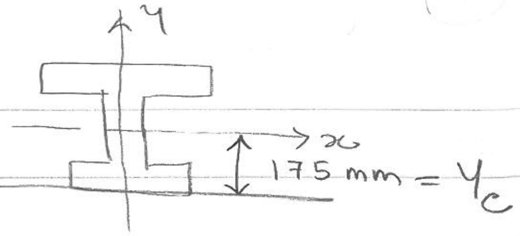
$= 174.69$ mm

$$I_x = \frac{1}{12} \times 250^3 \times 25 + 250 \times 25 \times (125 + 30 - 175)^2 +$$

$$\tfrac{1}{12} \times 30^3 \times 200 + 200 \times 30\,((250+30+30) - 0.5 \times 30 - 175)^2$$

$$+ \tfrac{1}{12} \times 30^3 \times 125 + 125 \times 30\,(0.5 \times 30 - 175)^2$$

(c) $I_x = 218.18 \times 10^{-6}\ m^4$ //

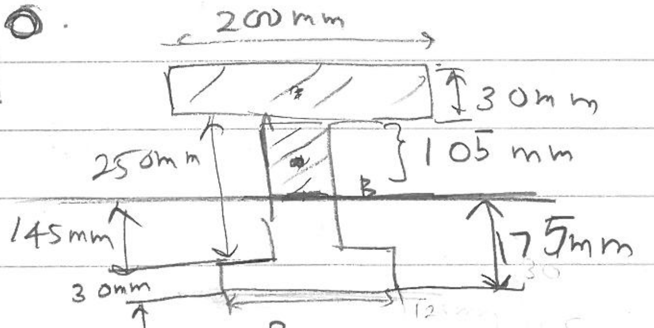


$y_c = 174.69\ mm$ //

Area = $200 \times 30 + 25 \times 250 + 30 \times 125 = 16000\ mm^2$

(d) (1) Axial stress = $\frac{P}{A} = \frac{6 \times 10^3\ N}{160\,000\ mm^2} = 375\ kPa$ or $0.375\ MPa$ //

(2) Bending stress at the point $= 0$.



Shear force = 0.5 kN

(3) Shear stress $(\tau_B) = \frac{VQ}{It} = \frac{0.5 \times 10^3 \times (857812.5 \times 10^{-9})}{218.18 \times 10^{-6} \times 25 \times 10^{-3}}$

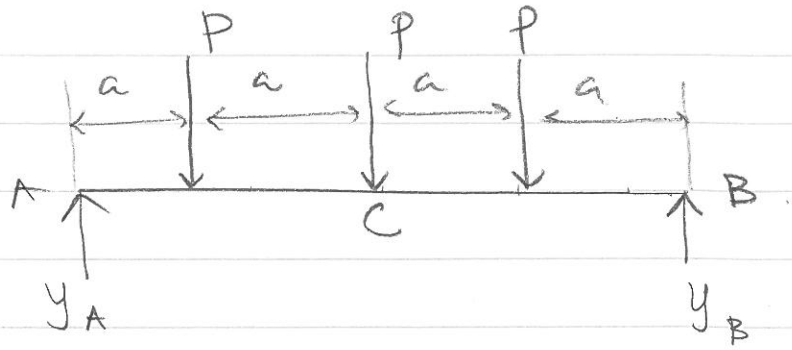
$Q = \bar{y}A = 200 \times 30 \times (105 + 15) + 105 \times 25 \times 105/2$

$= 857.812 \times 10^{-6}\ m^3$ //

$\tau_B = 78.6\ kPa$ or $0.079\ MPa$ //

Question ③

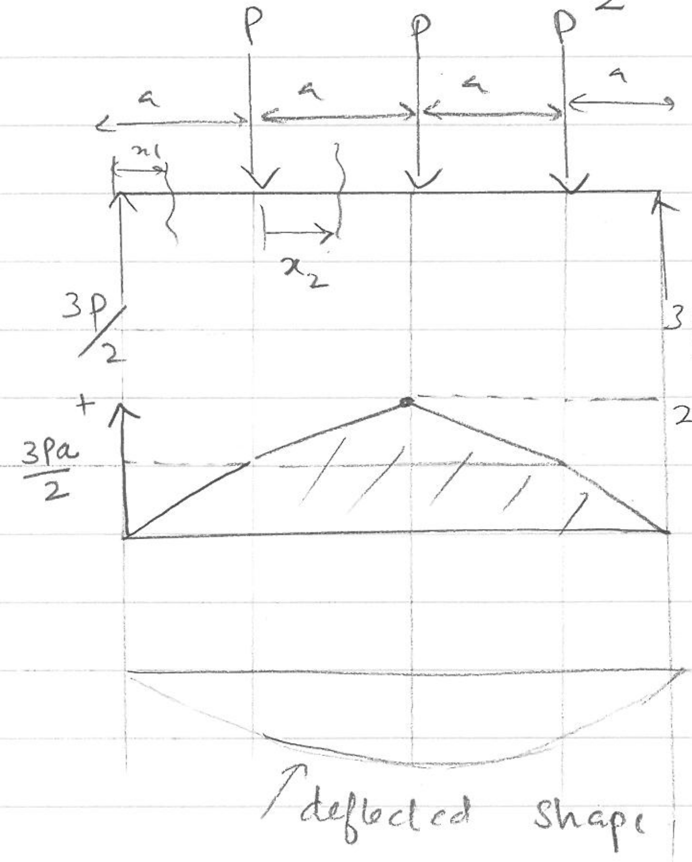
(a)



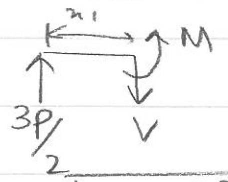
Considering the equilibrium of the structure,

$+\uparrow \Sigma F = 0;$ $\quad Y_A = Y_B = \frac{3P}{2}$ $\quad$ (symmetric)

(b)

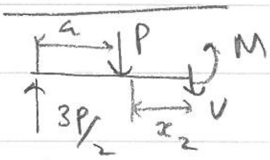


$0 \le x_1 \le a$



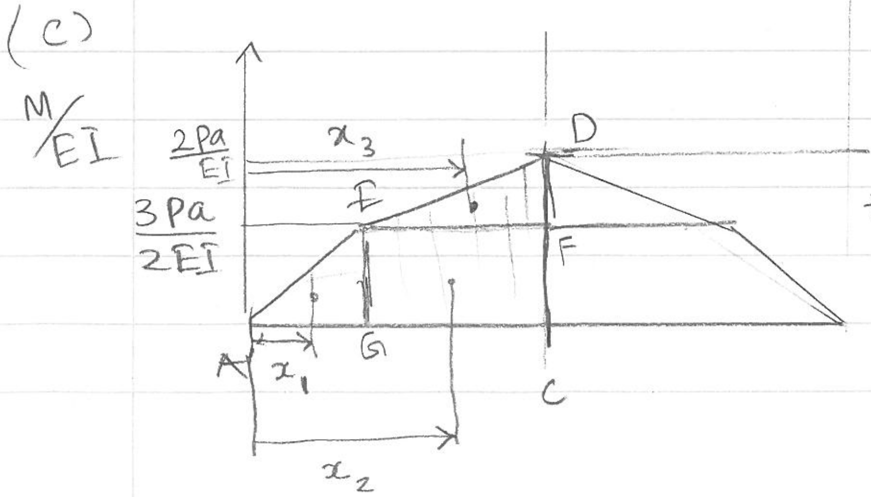
$$\boxed{M = \frac{3P}{2} x_1}$$

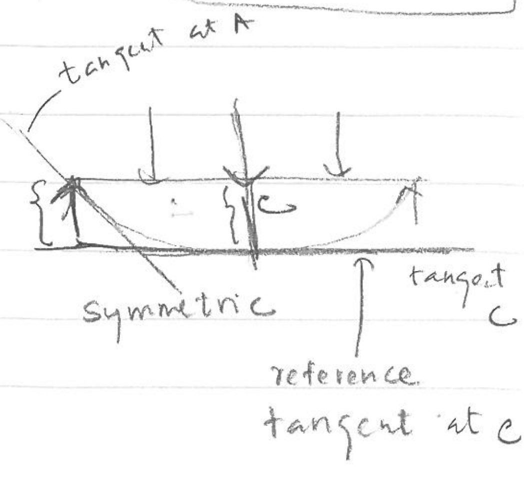
$a \le x_2 \le 2a$



$$\boxed{M = \frac{3P}{2}(a + x_2) - Px_2}$$

(c)





using moment area method, considering half of the beam (symmetric).

$$t_{A/C} = \bar{x} \int_A^C \frac{M}{EI}\, dx .$$

centroid

area of the M/EI diagram

$$t_{A/B} = x_1 \times (\text{area of AGE } \Delta) + x_2 \times (\text{area of EGCF rectangle}) + x_3 \times (\text{area of EFD trangle})$$

$$= \left(\frac{2a}{3}\right) \times \left\{\frac{1}{2} \times \frac{3Pa}{2EI} \times a\right\} + \left(a + \frac{a}{2}\right)\left\{\frac{3Pa}{2EI} \times a\right\}$$

$$+ \left(a + \frac{2a}{3}\right) \times \left\{\frac{1}{2} \times a \times \frac{Pa}{2EI}\right\}$$

$$= \frac{19}{6} \frac{Pa^3}{EI}$$

(d) $P = 10$ kN, $a = 2$ m,

$M_{max} = 2Pa = 2 \times 10 \times 2 = 40$ kN.m

$S_{required} = \dfrac{40 \times 10^3}{170 \times 10^6} = 235 \times 10^3 \text{ mm}^3$

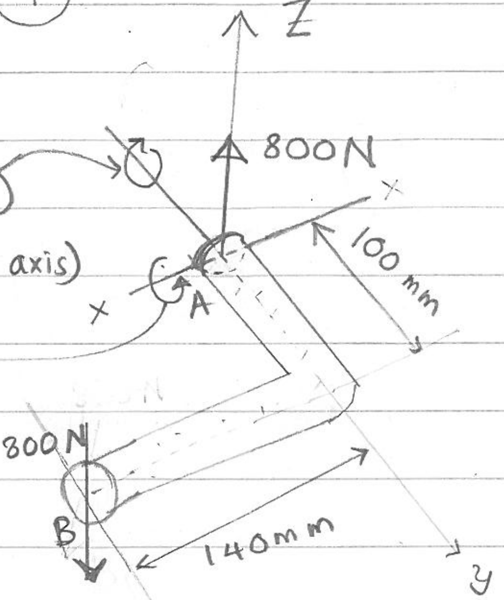
W 200×22 $\longrightarrow$ $S = 194 \times 10^3$ mm^3.

The beam can't widthstand the load.

Question 4

(a)

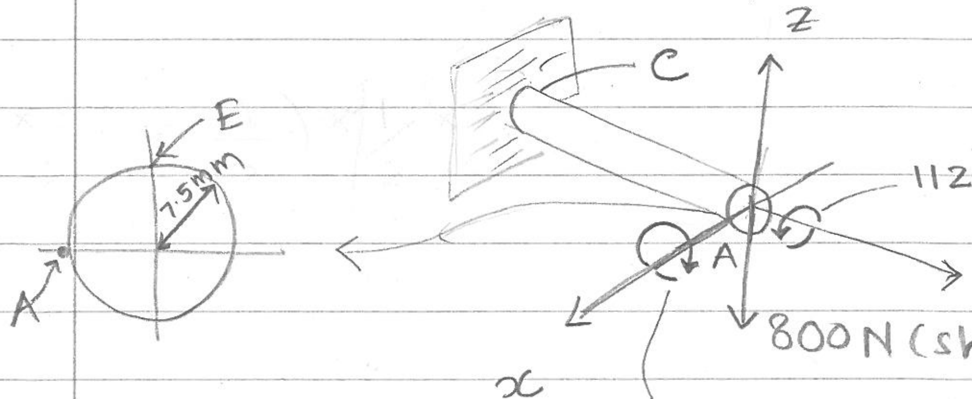
800N x (140mm) = 112 000 N.mm (Torque about y axis)



800N x (100mm) = 80 000 N.mm (Cbending moment about x axis)

Consider the equilibrium of segment 'AB'

Considering "BC" segment,



112 000 N.mm (torque)

800 N (shear force)

80 000 N.mm (bending moment)

(b) Stress at point A

Shear, $\rightarrow \tau_{y,z} = \dfrac{VQ}{It}$

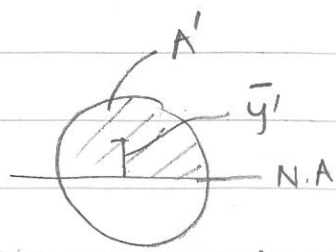
$V = 800N$, $Q = \bar{y}' A' = 4 \times \dfrac{(7.5mm)}{3\pi} \left[\dfrac{\pi \times 7.5mm^2}{2} \right]$

$= 281.25\ mm^3$

$I = \frac{1}{4}\pi (7.5mm)^4 \qquad t = 2 \times 7.5\ mm$

$\tau_{y,z} = \dfrac{(800N)(281.25\ mm^3)}{\left[\frac{1}{4}\pi (7.5mm)^4\right] 2 \times 7.5mm}$

$= 6.04\ MPa$



- Use attached Equations of statics.

$\bar{y}' = \dfrac{4r}{3\pi}$

$A' = \pi r^2/2$

Shear due to torque

$$\tau_{yz} = \frac{T\rho}{J}$$

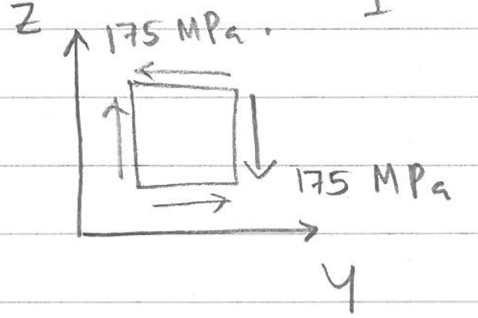
$T = 112000 N.mm$, $\rho = 7.5 mm$ $J = (\frac{1}{2}\pi(7.5mm)^4)$

$$\tau_{yz} = \frac{112000 N.mm (7.5mm)}{[\frac{1}{2}\pi (7.5mm)^4]}$$

$$= \underline{169 MPa}$$

Bending

$$\sigma_A = \frac{My}{I}$$ $y = 0$, $\sigma_A = 0$.

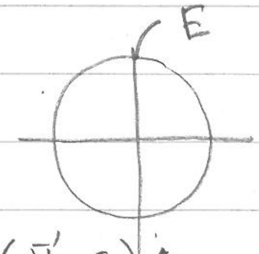


Total shear stress at A. $= 169 + 6.04$

$= \underline{175 MPa}$

Normal stress = 0 //

Stress at point E



Shear stress due to shear force (800N)

shear stress due to shear force = 0 // $(\bar{y}' = 0)$

Shear stress due torque $= \tau_{xy} = \frac{T\rho}{J} = 169 MPa$ //

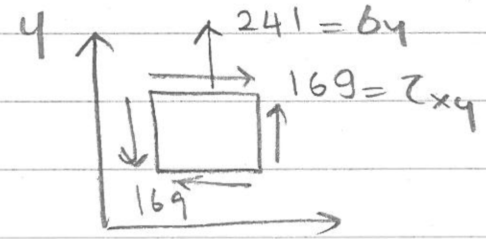
because, $T = 112000 N.mm$, $\rho = 7.5 mm$, $J = \frac{1}{2}\pi (7.5mm)^4$.

Bending stress, $\sigma_E = \frac{M}{I} y$

$M = 80\,000\,N.mm$, $\quad I = \frac{1}{4}\pi \times (7.5\,mm)^4$. $\quad y = 7.5\,mm$.

$$\sigma_E = \frac{80\,000\,N.mm \times 7.5\,mm}{\frac{1}{4}\pi \times (7.5\,mm)^4}$$

$$= \underline{241\ MPa}$$



State of stress at E

(c) Principal stresses at A $\quad$, $\sigma_x = 0$, $\sigma_y = 0$, $\tau_{xy} = -175$

$$(\sigma_1)_A = \frac{(\cancel{\sigma_x} + \cancel{\sigma_y})}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}.$$

$$= \pm 175\ MPa$$

Orientation, $\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} \Rightarrow \infty$.

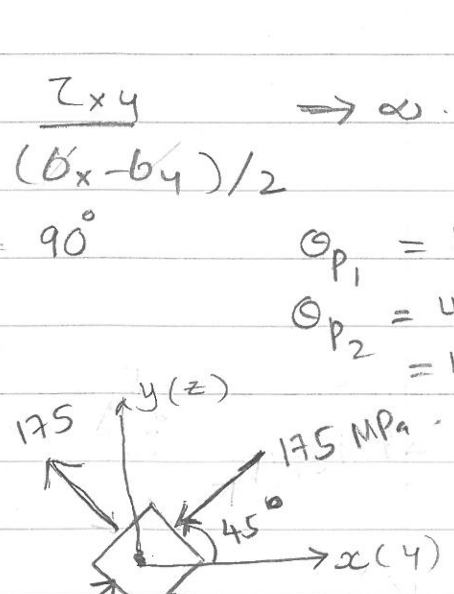
$2\theta_p = 90^\circ$ $\quad$ $\theta_{p_1} = 45^\circ$

$\theta_{p_2} = 45^\circ + 90^\circ = 135^\circ$

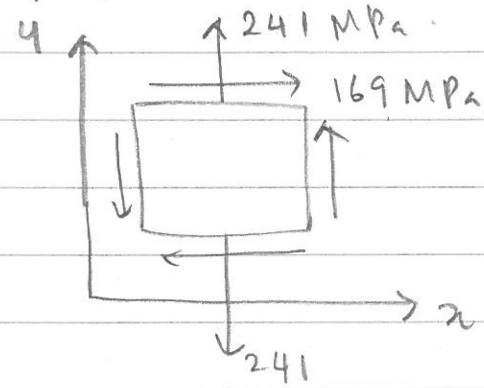
$\theta = 45^\circ$,

$\sigma_x = -175 \sin(2 \times 45^\circ)$

$= -175\ MPa$



Principal stresses at point E



$$(\sigma_1)_E = \frac{(\sigma_x + \sigma_y)}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$\sigma_x = 0$, $\sigma_y = 241$ MPa, $\tau_{xy} = 169$ MPa.

$$(\sigma_1)_E = \frac{(241 + 0)}{2} \pm \sqrt{\left(\frac{241}{2}\right)^2 + 169^2}$$

$$(\sigma_1)_E \text{ or } (\sigma_2)_E = 328 \text{ MPa} \quad \text{or} \quad -87 \text{ MPa}.$$

Orientation,

$$\tan 2\theta_p = \frac{169}{(0-241)/2} = -1.402$$

$$2\theta_p = -54 \rightarrow \theta_{p_1} = -27^\circ \ //$$

$$\theta_{p_2} = 62^\circ \ //$$

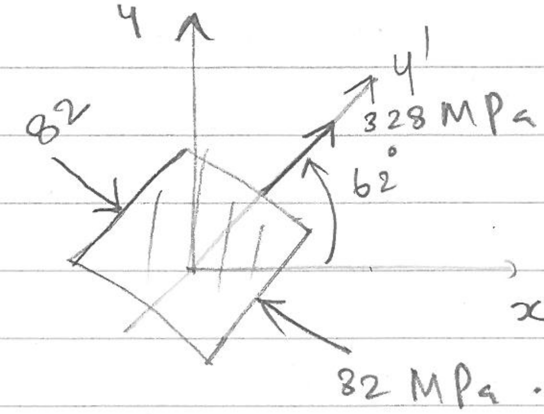
$\theta_{p_1} = 62^\circ$,

$$\sigma_x = \frac{(\sigma_x + \sigma_y)}{2} + \frac{(\sigma_x - \sigma_y)}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$\sigma_x = 0$, $\sigma_y = 241$ MPa $\tau_{xy} = 169$ MPa.

$$\sigma_x = \frac{(0 + 241)}{2} + \frac{(0 - 241)}{2} \cos(2 \times 62^\circ) + 169 \times \sin(2 \times 62^\circ)$$

$$= 328 \text{ MPa} \ //$$



Orientation

(d) $\tau_{in,plane} = \sqrt{\frac{(\sigma_x-\sigma_y)^2}{2} + \tau_{xy}^2}$

$= \pm 175$ MPa for point A.

45° degrees apart from principal planes.

$\tau_{in,plane} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{\left(\frac{241}{2}\right)^2 + 169^2}$

$= 207.5$ MPa for point E

45° degrees apart from principal planes.