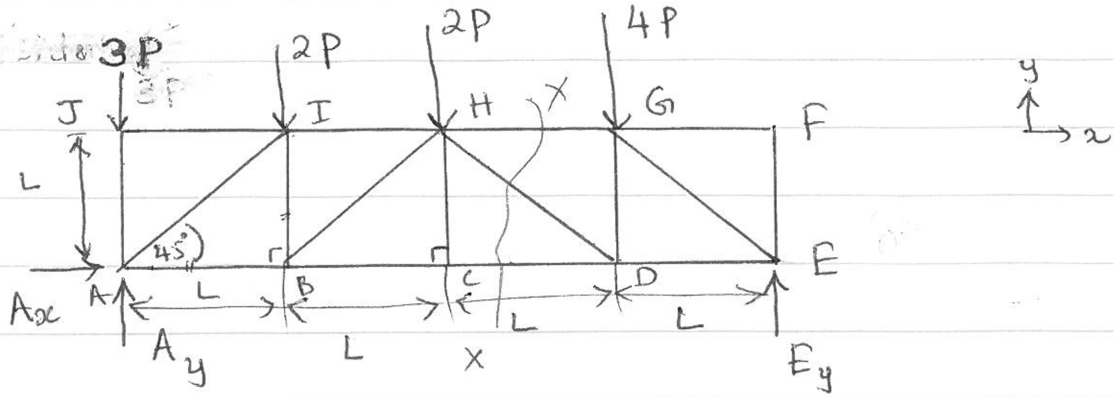
# Answers, 2012 May

## Question 1

(I)




Free-body diagram of the truss

Considering the equilibrium of the truss,

$y \uparrow (+)$ $\quad A_y + E_y - 3P - 2P - 2P - 4P = 0$

$$A_y + E_y = 11P \quad — ①$$

$\xrightarrow[x]{(+)}$ $\quad \sum F_x = 0; \quad A_x = 0$ //

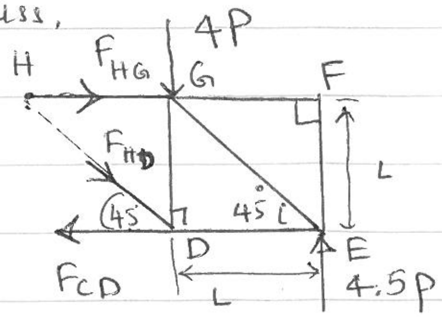
$(+\circlearrowleft) \sum M_A = 0; \quad -L \times 2P - 2P \times (2L) - 3L \times 4P + E_y \times (4L) = 0$

$$E_y = 4.5P \text{ //}$$

①, $\quad 4.5P + E_y = 11P$

$$A_y = 6.5P \text{ //}$$

(II) Considering an imaginary cut x – x, equilibrium of right part of the truss,




$(+\circlearrowleft) \sum M_H = 0; \quad -(L) \times F_{CD} - (L) \times 4P + (2L) \times 4.5P = 0$

$$F_{CD} = 5P \text{ kN (Tension)}$$

$(+\circlearrowleft)\ \Sigma M_D = 0;\ -L \times F_{HG} + 4.5P \times L = 0$

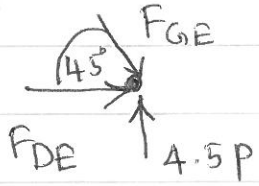
$F_{HG} = \underline{\underline{4.5P\ kN}}$ (Compression)

$y \uparrow\ \Sigma F_y = 0;\ -F_{HD}\cos 45^\circ - 4P + 4.5P = 0$

$F_{HD} \simeq \underline{\underline{0.707P}}$ (Compression)

(III) Zero-force members - GF, FE, CH, JI

(IV) Consider the equilibrium of joint 'E'




$F_{FE} = 0,$

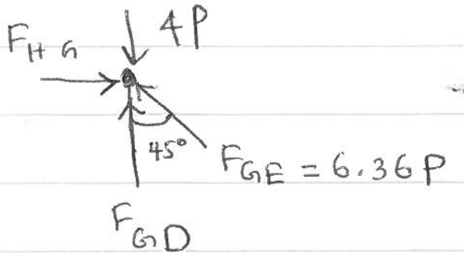
$y \uparrow\ \Sigma F_y = 0;\ F_{GE}\sin 45^\circ - 4.5P = 0$

$F_{GE} \simeq 6.36P$ kN (Compression)

$\overrightarrow{x}\ \Sigma F_x = 0;\ F_{DE} + F_{GE}\cos 45^\circ = 0$

$F_{DE} = \underline{\underline{4.5P\ kN}}$ (Tension)

Joint 'G'




$F_{GF} = 0,$

$y \uparrow\ \Sigma F_y = 0;$

$-4P + F_{GD} + 6.36P \times \cos 45^\circ = 0$

$F_{GD} = \underline{\underline{0.5P\ kN}}$ (Tension)

$F_{CD} = F_{BC} = \underline{\underline{5P\ kN}}$ (Tension)

V Cross sectional area, for GE

Cross-section, Area $= \dfrac{F_{GE}}{\sigma_{allow}} = \dfrac{6.36 \times 10 \times 10^3\ N}{140 \times 10^6\ Pa} \simeq 454\ mm^2$

For, CD,

Area $= \dfrac{F_{CD}}{\sigma_{allow}} = \dfrac{5 \times 10 \times 10^3\ N}{140 \times 10^6\ Pa} \simeq 357\ mm^2$

VI GE, A·cross-sectional area $= 454\ mm^2$, $P = 6.36 \times 10^4\ N$

$\delta_{GE} = \dfrac{PL}{AE}$ $L = \sqrt{4^2+4^2} = 5.66\ m$

From Table 1 → $E = 200\ GPa$

$\delta_{GE} = \dfrac{6.36 \times 10^4\ N \times 5.66\ m}{454 \times 10^{-6}\ m^2 \times 200 \times 10^9\ Pa} \simeq \underline{\underline{3.96\ mm}}$ (Contracted)

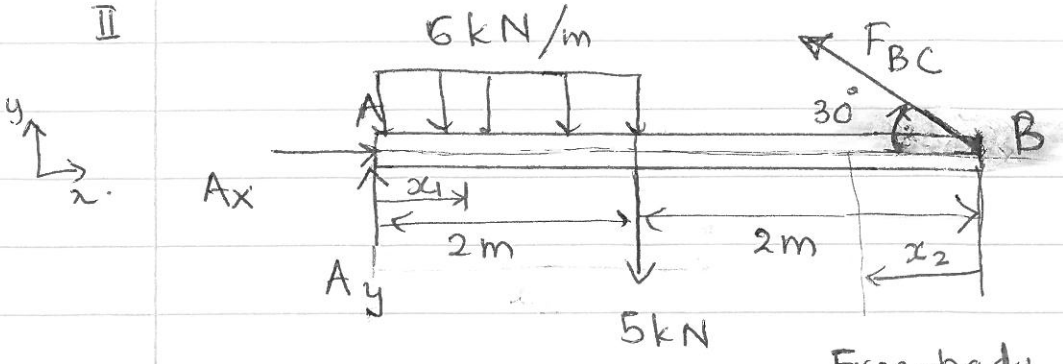
CD, $A = 357\ mm^2$, $P = 5 \times 10\ kN$ $L = 4\ m$, $E = 200\ GPa$.

$\delta_{CD} = \dfrac{5 \times 10 \times 10^3\ N \times 4\ m}{351 \times 10^{-6}\ m^2 \times 200 \times 10^9\ Pa} \simeq \underline{\underline{2.80\ mm}}$ (elongated)

Question 2

I BC ②

II




Free-body diagram for 'AB'

Considering equilibrium of beam 'AB',

$(+\circlearrowleft)\ \Sigma M_B = 0;\ +5\,kN \times (2m) + (6\times 2\,kN)\times(3m) - A_y \times 4m = 0$

$A_y = 11.5\,kN$ //

$y\uparrow\ \Sigma F_y = 0;\ A_y - 5kN - (6\times 2kN) + F_{BC}\sin 30^\circ = 0$

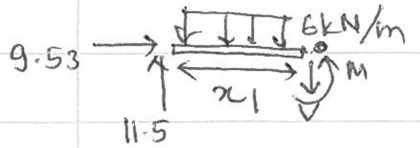
$F_{BC} = 11\,kN$ //

$\xrightarrow{+}_x\ \Sigma F_x = 0;\ A_x - F_{BC}\cos 30^\circ = 0$

$A_x = 9.53\,kN$ //

III Moment & shear equations interms of 'x'

$0 \le x_1 \le 2m$



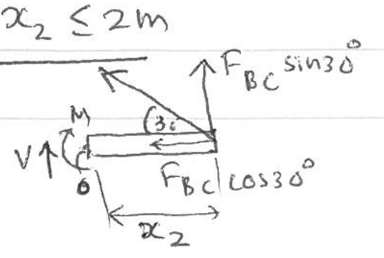

$(+\circlearrowleft)\ \Sigma M_O = 0;\quad M - 11.5x_1 + 6x_1 \times x_1/2 = 0$

$M = -3x_1^2 + 11.5x_1$

$\uparrow\quad -V - 6x_1 + 11.5 = 0$

$V = +11.5 - 6x_1$

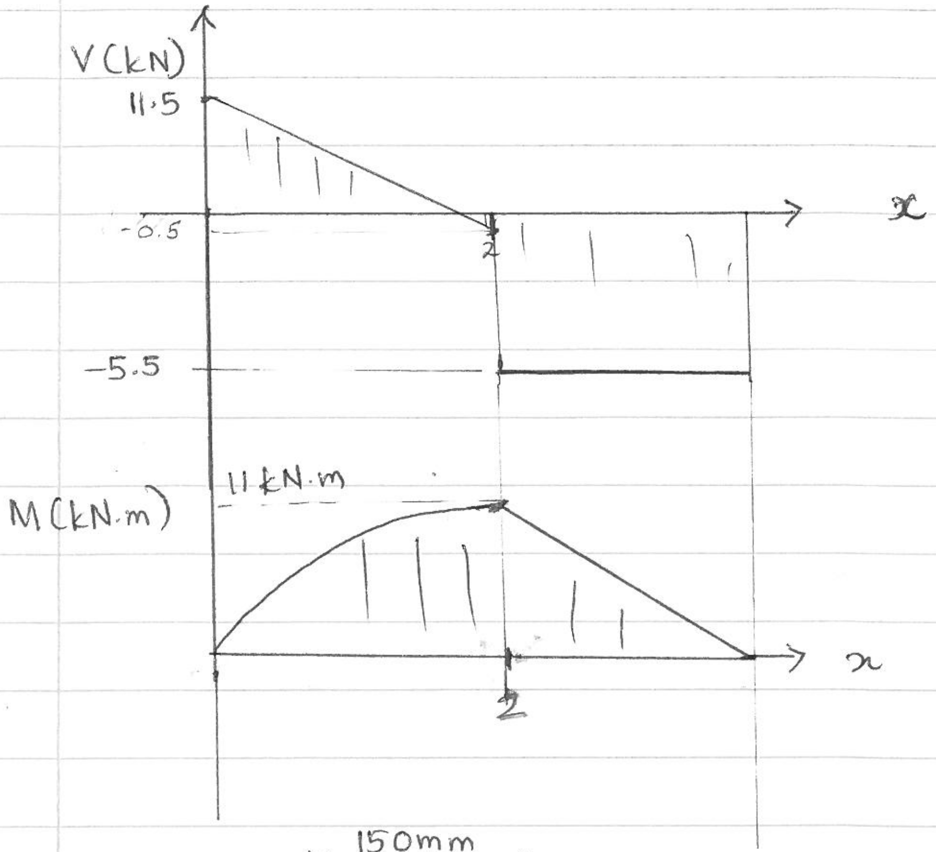
$0 \le x_2 \le 2m$



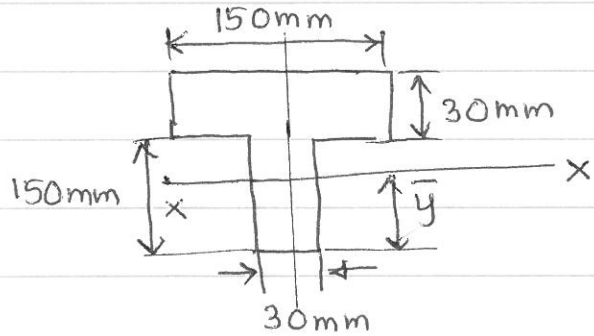

$(+\circlearrowleft)\ \Sigma M_O = 0;\ -M + 11 \times \sin 30\, x_2 = 0$

$M = 5.5x_2$

$\uparrow\ V + F_{BC}\sin 30^\circ = 0 \rightarrow V = -5.5$ //




IV



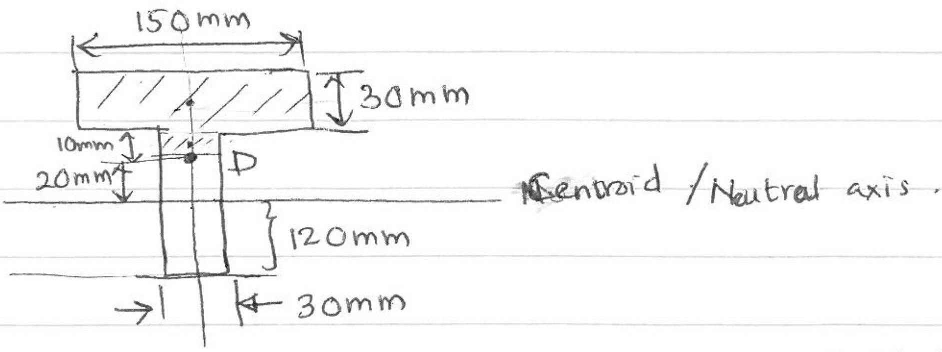

$$\bar{y} = \frac{(150\times30)\times(15+150) + (150\times30\times75)\ \text{mm}^3}{(150\times30 + 150\times30)\ \text{mm}^2}$$

$$= 120\text{mm}$$

$$I_{xx} = \tfrac{1}{12}\times(0.030)\times(0.150)^3 + \tfrac{1}{12}\times(0.030)^3\times0.150 + (0.150\times0.030)\times(0.120-0.075)^2 + 0.030\times(0.150)\times(0.165-0.120)^2$$

$$= 27\times10^{-6}\ \text{m}^4$$

V




Bending moment from 1m distance from B = 5.5 kN.m,

Shear force = 5.5 kN.

$$\text{Bending stress} = \frac{M}{I} \times y = \frac{5.5 \times 10^{3}\ \text{N.m} \times (0.020\ \text{m})}{27 \times 10^{-6}\ \text{m}^4}$$

$$= 4.07\ \text{MPa}$$

Axial stress (compressive)

$$= \frac{P}{A} = \frac{9.53 \times 10^{3}\ \text{N}}{(0.150 \times 0.030) \times 2\ \text{m}^2} = 1.06\ \text{MPa}$$

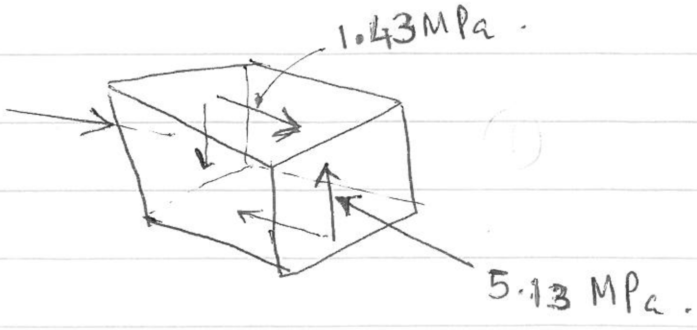
Ax (P)

Normal stress (compressive) $= 4.07\ \text{MPa} + 1.06\ \text{MPa} = 5.13\ \text{MPa}$

$$\text{Shear stress} = \frac{V_D Q_D}{I_x t_D} = \frac{5.5 \times 10^{3}\ \text{N} \times 2.1 \times 10^{-4}}{27 \times 10^{-6}\ \text{m}^4 \times 0.03\ \text{m}} = 1.43\ \text{MPa}$$

$$Q_D = \Sigma \bar{y}' A' = (0.010 \times 0.030) \times (0.020 + 0.005) + (0.150 \times 0.030) \times (0.030 + 0.015)$$

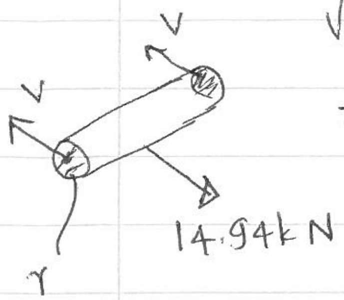
$$= 2.1 \times 10^{-4}\ \text{m}^3$$




Volume element. (State of stress at D)

VI Resultant force at Support A

$$\sqrt{A_x^2 + A_y^2} = \sqrt{11.5^2 + 9.53^2} = 14.94\ kN$$

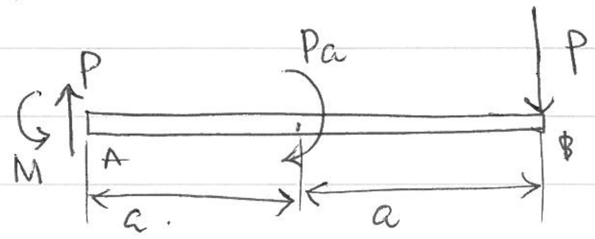



$$\tau_{allow} = \frac{V}{area} \ ; \quad \pi r^2 = \frac{14.94/2 \times 10^3}{60\times10^6\ Pa}$$

$r = 6.23mm \rightarrow d \simeq 13mm$

Round off to nearest 5mm } $d \simeq 15mm$

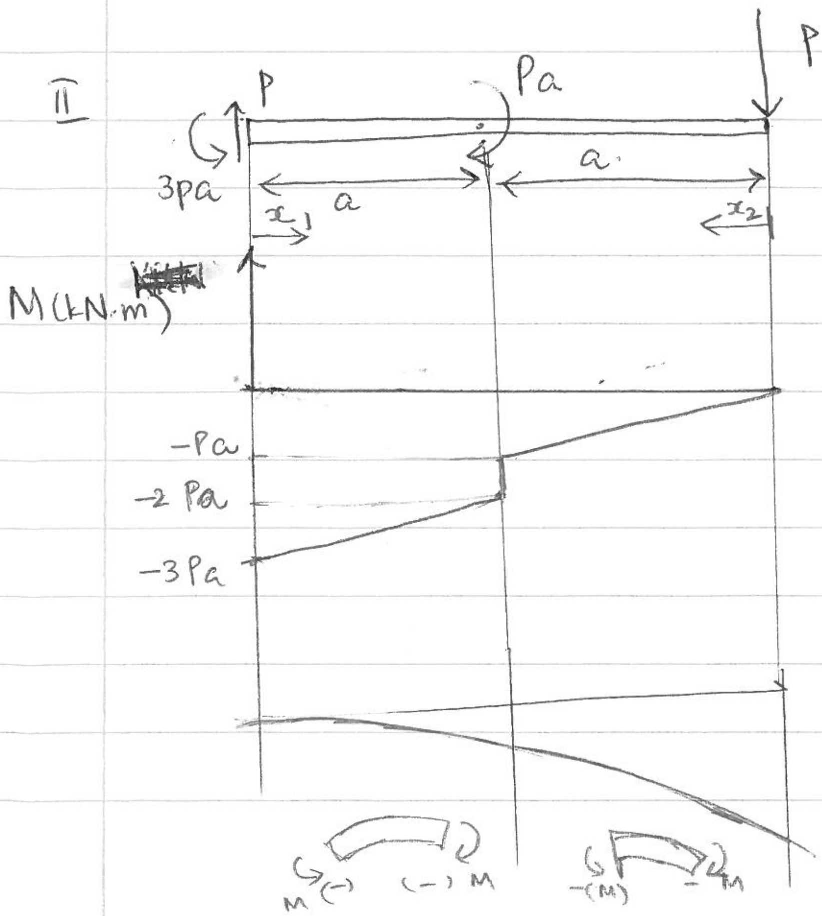
Q3/ I



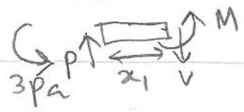

Free-body diagram of the beam.

$\circlearrowleft_A \ \Sigma M_A = 0; \quad M - Pa - 2Pa = 0, \qquad M = 3Pa$

II



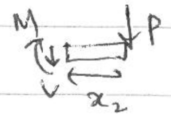

$0 \le x_1 \le a$




$M - Px_1 + 3Pa = 0$

$M = -3Pa + Px_1$

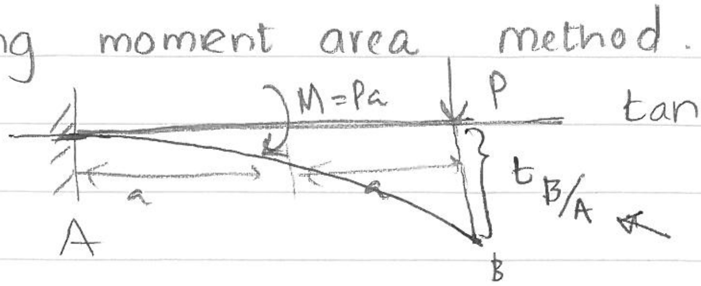
$0 \le x_2 \le a$



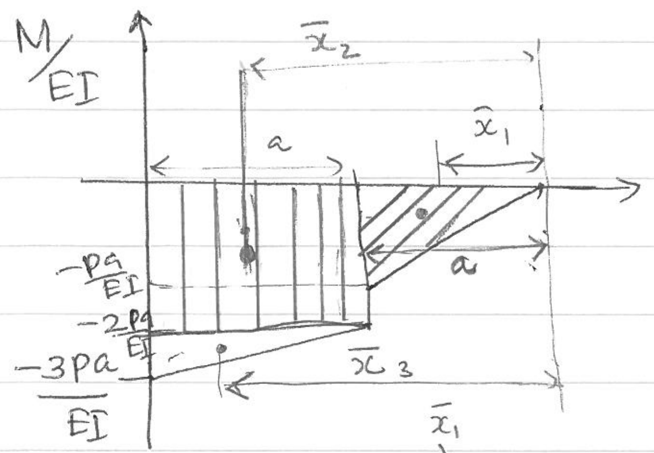

$M = -Px_2$

Deflected shape.

III using moment area method.




$$t_{B/A} = \bar{x}\int_A^B \frac{M}{EI}\,dx$$




$$t_{B/A} = \left(-\underbrace{\frac{2a}{3}}_{\bar{x}_1} \times \frac{Pa}{EI} \times a \times \frac{1}{2} - \frac{Pa}{2EI} \times \underbrace{(a + 2a/3)}_{\bar{x}_3} \times a - \underbrace{(a + a/2)}_{\bar{x}_2} \times \frac{2Pa}{EI} \times a\right)$$

$$= -\frac{25\,Pa^3}{6EI}$$

IV W 250X22,

$M_{max}$ (Maximum moment) $= 3\times12\times1 = 36$ kN·m

$a=1$, $P = 12$ kN,

$V_{max} = P = 12$ kN,

$$\frac{M_{max}}{\sigma_{allow}} = S_{required} = \frac{36\ \text{kN·m} \times \frac{10^3\ \text{N}}{1\ \text{kN}}}{170\times10^6\ \text{N·m}} = 212\times10^3\ \text{mm}^3$$

From Table 2, $S = 227\times10^3\ \text{mm}^3 > 212\times10^3\ \text{mm}^3$

$$\tau_{avg} = \frac{V_{max}}{A_{web}} = \frac{12000N}{0.254 \times 0.00584} = 8.08\,MPa$$

$$\tau_{avg} < \tau_{allowable}$$

Beam can safely carry the load.

V Beam is statically indeterminate.
4 unknows & 3 equations.

Question 4

I Stress at point A.

Axial force, $\rightarrow \sigma_x = P/A = -\frac{12.5 \times 10^3}{\pi \times (0.020)^2} = \underline{-9.94\,MPa}$

Bending stress, $\sigma_b = -\frac{My}{I} = -\frac{(2.25 \times 10^3)N \times 0.020}{1/4\,\pi \times (0.020)^4}$

$= -357.95\,MPa$

Normal stress at A, $\sigma_n = -9.94 - 357.95$

$= -368\,MPa$

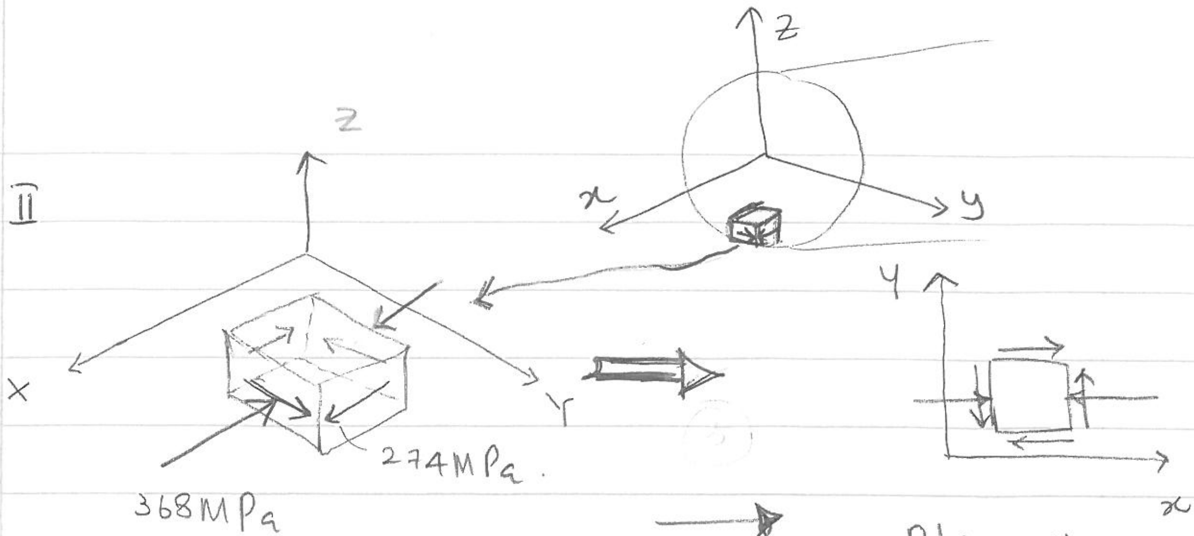
Shear stress, at B $\tau = \frac{T\rho}{J} = \frac{3.45 \times 10^3 \times (0.02)}{\pi/2 \times (0.02)^4}$

$= 274\,MPa$

Stress, at B, $\sigma_n = -9.94\,MPa$, (Bending stress = 0)

Shear stress, $\tau = 274\,MPa$

II




Volume element

3

plane stress

Can be analysed as a plane stress problem

III principal stresses at point 'A'

$$(\sigma_1)_A = \frac{(\sigma_x + \sigma_y)}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$\sigma_x = -368$ MPa $\quad \sigma_y = 0, \quad \tau_{xy} = 274$ MPa.

$$(\sigma_1)_A = \frac{(-368 + 0)}{2} \pm \sqrt{\left(\frac{-368-0}{2}\right)^2 + 274^2}$$

$\sigma_1 = -514$ MPa,

$\sigma_2 = +146$ MPa

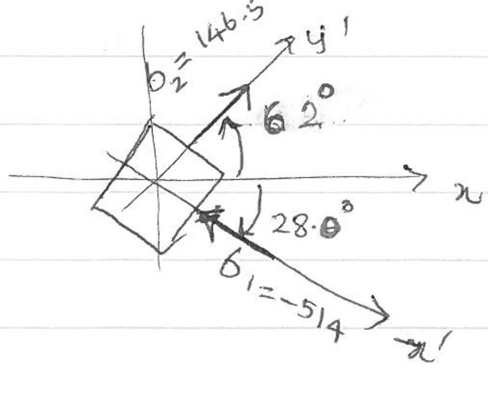
Orientation, $\tan 2\theta_p = \tau_{xy} / (\sigma_x - \sigma_y)/2$

$$\tan 2\theta_p = \frac{274}{(-368/2)}, \quad 2\theta_p \simeq -56^\circ \rightarrow \theta_{p_1} \simeq -28.0^\circ$$

$$\theta_{p_2} \simeq 62.0^\circ,$$

When, $\theta = 62.0^\circ$, $\rightarrow \sigma_x = (-368/2) + -368/2 \times \cos(2\times 62.0) + 274 \sin(2\times 62.0)$

$\sigma_x = \sigma_2 = +146$ MPa



IV $\tau_{In\text{-}plane} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{368}{2}\right)^2 + 274^2}$

$= 330 \text{ MPa}$ //

$45^\circ$ degree's apart from principal planes.
$\theta_s$ & $\theta_p$ are $45^\circ$ apart.

V i) To avoid the failure of ductile material the absolute maximum shear stress should be less than or equal to $\frac{\text{yielding stress}}{2}$ $(\sigma_y/2)$.

$|\sigma_1| \le \sigma_y$
$|\sigma_2| \le \sigma_y$ } $\sigma_1, \sigma_2$ same sign.

$|\sigma_1 - \sigma_2| \le \sigma_y \rightarrow \sigma_1, \sigma_2$ opposite sign.

ii) $\sigma_1, \sigma_2$ opposite,

$|\sigma_1 - \sigma_2| = |-514 - 146| = 660 \text{ MPa} \le 703$

no failure according to the theory.

VI ~~Ex~~ Table 1, $E = 200 \text{ GPa}$, $\nu = 0.32$
$G = 75 \text{ MPa}$

$\epsilon_x = \frac{1}{200\times10^9}(-368\times10^6 - 0.32(0+0)) \simeq -1.84\times10^{-3}$ //

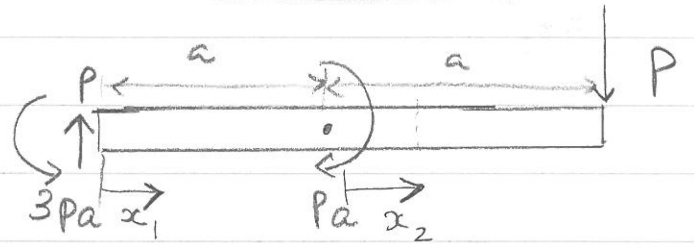
$\epsilon_y = \frac{1}{200\times10^9}(0 - 0.32(-368\times10^6 + 0)) \simeq 5.89\times10^{-4}$ //

$\epsilon_z = \frac{1}{200\times10^9}(0 - 0.32(-368\times10^{-6} + 0)) \simeq 5.89\times10^{-4}$ //

$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{274\times10^6}{75\times10^9} \simeq 3.65\cdot10^{-3} \text{ rad}$ /

Q 3/ <u>Method of Integration</u>

III




<u>$0 \le x_1 \le a$;</u>

$$EI \frac{d^2 v_1}{dx_1^2} = M(x) = Px - 3Pa$$

$$\frac{EI}{P} \frac{dv_1}{dx_1} = \frac{x_1^2}{2} - 3x_1 a + C_1$$

$$\frac{EI}{P} v_1 = \frac{x_1^3}{6} - \frac{3x_1^2 a}{2} + C_1 x_1 + C_2$$

Boundary conditions; $x_1 = 0$; $dv_1/dx_1 = 0$, $v_1 = 0$

then, $C_1 = 0$, $C_2 = 0$.

$$EI/P \; v_1 = x_1^3/6 - \frac{3x_1^2 a}{2} \quad — (1)$$

<u>$a \le x_2 \le 2a$</u>

$$EI \frac{d^2 v_2}{dx_2} = M(x) = Px_2 - 2Pa$$

$$\frac{EI}{P} dv_2/dx_2 = \frac{x_2^2}{2} - 2ax_2 + C_3$$

$$\frac{EI}{P} v_2 = \frac{x_2^3}{6} - \frac{2ax_2^2}{2} + C_3 x_2 + C_4$$

Continuity: $\frac{d(v_1)}{dx_1} = \frac{d(v_2)}{dx_2}$, $v_1 = v_2$

$\left.\begin{array}{l} x_1 = a \\ x_2 = a \end{array}\right\}$

$\left.\begin{array}{l} x_1 = a \\ x_2 = a \end{array}\right\}$, $v_2 = v_1 = \frac{P}{EI}\left(a^3/6 - 3a^3/2\right) = \frac{P}{EI}\left(\frac{a^3}{6} - a^3 + C_3 a + C_4\right)$ — (2)

$\left.\begin{array}{l} x_1 = a \\ x_2 = a \end{array}\right\}$, $\frac{dv_1}{dx_1} = \frac{dv_2}{dx_2} = \cancel{P}/EI\left(a^2/2 - 3a^2\right) = \cancel{P}/EI\left(\frac{a^2}{2} - 2a^2 + C_3\right)$ — (3)

(3), $C_3 = -a^2$

② $$-\frac{8a^3}{6} = -\frac{5a^3}{6} + ax - a^2 + C_4$$

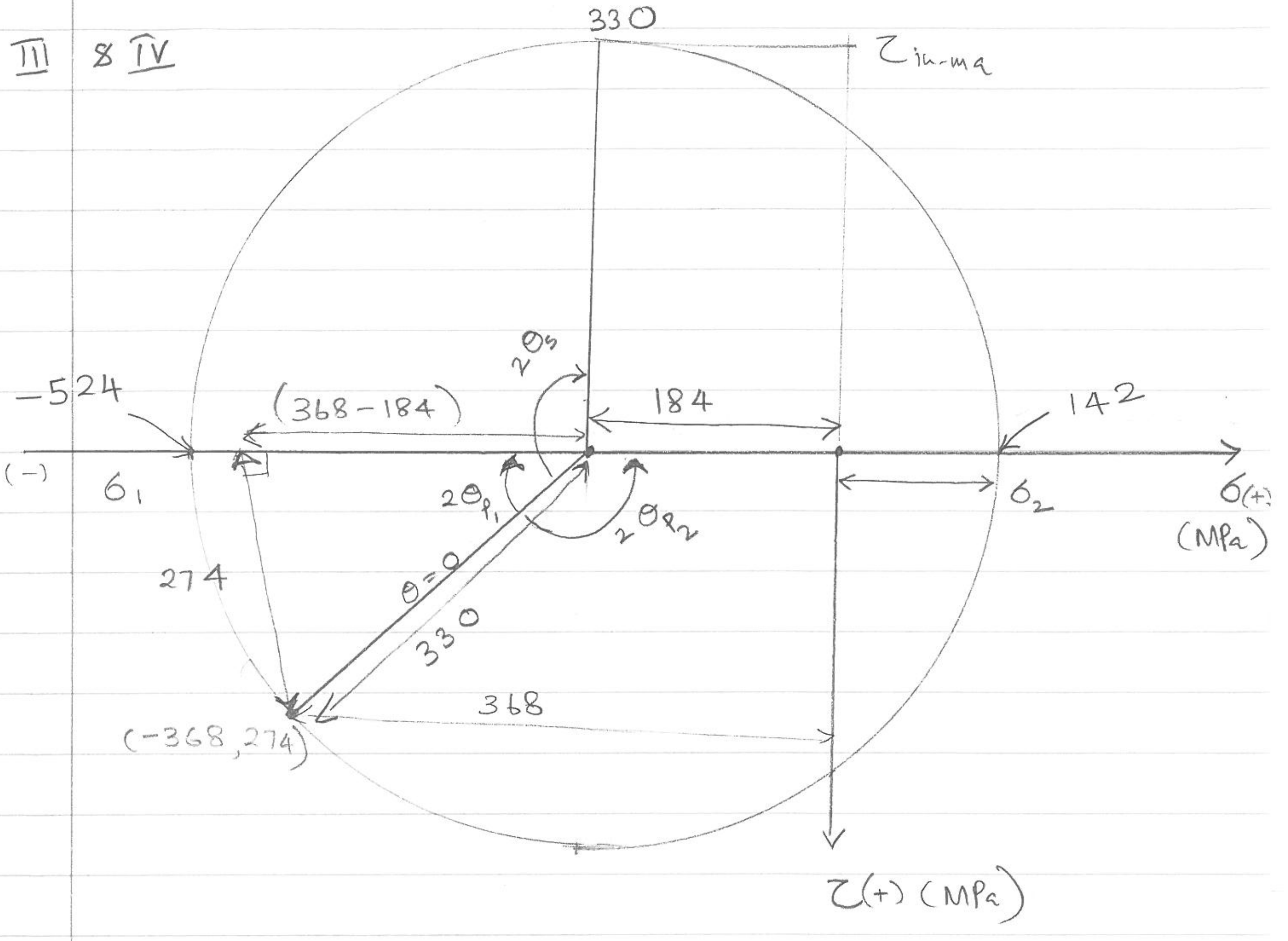
$$C_4 = \tfrac{1}{2}a^3,$$

$$\frac{EI}{P}v_2 = \frac{x_2^3}{6} - ax_2^2 - a^2x_2 + \tfrac{1}{2}a^3$$

$x = 2a,$ $$V_{max} = \frac{P}{EI}\left(\frac{8a^3}{6} - 4a^3 - 2a^3 + \tfrac{1}{2}a^3\right) = -\frac{25a^3P}{6EI}$$ //

# Mohr's circle Q4 (Method 2)

III & IV




$\sigma_{ave} = (\sigma_x + \sigma_y)/2 = (-368+0)/2 = -184 \text{ MPa}$

Co-ordinate : $(\sigma_x, \tau_{xy}) = (-368, 274)$

$$R = \sqrt{(-368/2)^2 + 274^2} = 330 \text{ MPa}$$

$\tau_{In\text{-}max} = 330 \text{ MPa}$ at 45° apart from principal planes

$\sigma_1 = -184 - 330 = -524 \text{ MPa}$ at $\tan 2\theta_{p_1} = \dfrac{274}{(368-184)}$

$\theta_{p_1} \simeq 28$ clockwise

$\sigma_2 = -184 + 330 = 146 \text{ MPa}$

at $180° - 2\theta_{p_1} = 2\theta_{p_2}$

$\theta_{p_2} \simeq 62°$ (counterclockwise)