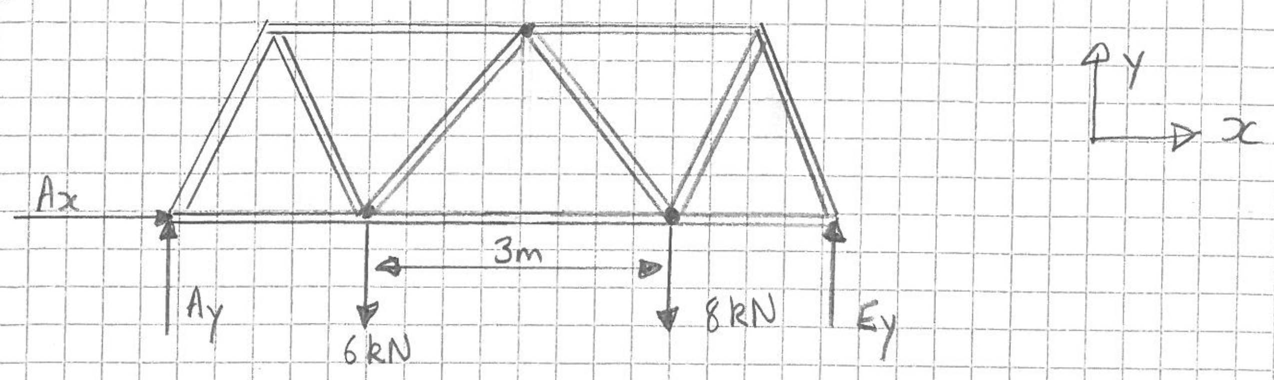
Q1



I Support Reactions

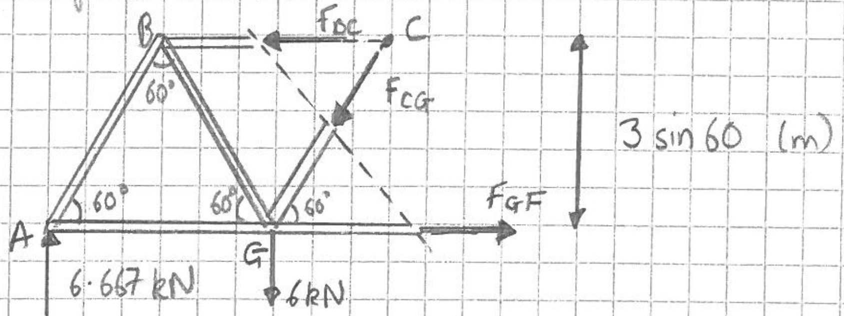
$\circlearrowright + \sum M_E = 0 \qquad 6(6) + 8(3) - A_y(9) = 0$

$\Rightarrow \quad A_y = \underline{6.667 \text{ kN}}$

$\xrightarrow{+} \sum F_x = 0 \qquad \underline{A_x = 0}$

$+\uparrow \sum F_y = 0 \qquad 6.667 - 6 - 8 + E_y = 0 \Rightarrow \underline{E_y = 7.33 \text{kN}}$

II Method of Sections



$\circlearrowright + \sum M_C = 0 \qquad F_{GF}(3\sin 60) - 6.667(4.5) + 6(1.5) = 0$

$\Rightarrow \quad F_{GF} = \underline{8.08 \text{ kN}} \quad \underline{T}$

$\circlearrowright + \sum M_G = 0 \qquad F_{BC}(3\sin 60) - 6.667(3) = 0$

$\Rightarrow \quad \underline{F_{BC} = 7.698 \text{ kN}} \quad \underline{C}$

$+\uparrow \sum F_y = 0 \qquad 6.667 - 6 - F_{CG}\sin 60° = 0$

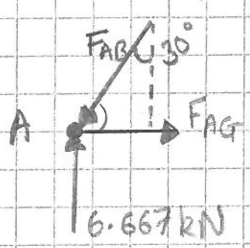
$\Rightarrow \quad \underline{F_{CG} = 0.770} \text{ kN} \quad \underline{C}$

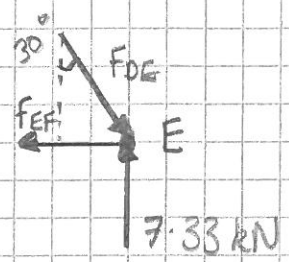
III There are 2 cases which result in zero force members

1) If only 2 members form a truss joint and no external load or support reaction is applied - then both members will be zero force members

2) If 3 members form a truss joint and if 2 of those members are colinear then the 3rd member must be a zero force member.

IV Method of Joints





at A $+\uparrow \sum F_y = 0$ $\qquad 6.667 - F_{AB}\cos 30° = 0$

$\Rightarrow \quad F_{AB} = \underline{7.70 \text{ kN}} \quad \underline{C}$

$\underset{+}{\rightarrow} \sum F_x = 0 \qquad F_{AG} - 7.70 \sin 30 = 0$

$F_{AG} = 3.85 \text{ kN} \quad \underline{T}$

at B $+\uparrow \sum F_y = 0 \qquad 7.33 - F_{DE}\cos 30° = 0$

$\Rightarrow \quad F_{DE} = 8.46 \text{ kN} \quad \underline{C}$

$\underset{+}{\rightarrow} \sum F_x = 0 \qquad F_{EF} - 8.46 \sin 30° = 0$

$\Rightarrow \quad F_{EF} = \underline{4.23 \text{ kN}} \quad \underline{T}$

V. cross-sec area of GF

$$\text{cross-sec area} = \frac{F_{GF}}{\sigma_{allow}} = \frac{8.08 \times 10^3 (N)}{150 \times 10^6 \ (Pa)} = 5.39 \times 10^{-5} m^2$$

OR $= 53.9 \ mm^2$

FOR CG

$$\text{area} = \frac{F_{CG}}{\sigma_{allow}} = \frac{0.77 \times 10^3 \ (N)}{150 \times 10^6 \ (Pa)} = 5.13 \times 10^{-6} \ m^2$$

OR $= 5.13 \ mm^2$

VI

For member GF

$$\delta_{GF} = \frac{PL}{AE}$$

GF $\quad P = 8.08 \times 10^3 \ N \quad L = 3m \quad A = 5.39 \times 10^{-5} \ m^2$

From table 1 $\quad E = 200 \ GPa$

$$\Rightarrow \quad \delta_{GF} = \frac{8.08 \times 10^3 \times 3}{5.39 \times 10^{-5} \times 200 \times 10^9} = 2.25 \times 10^{-3} \ m$$

(elongation)

For member CG

$P = 0.77 \times 10^3 \ N \quad L = 3m \quad A = 5.13 \times 10^{-6} m^2 \quad E = 200 \ GPa$

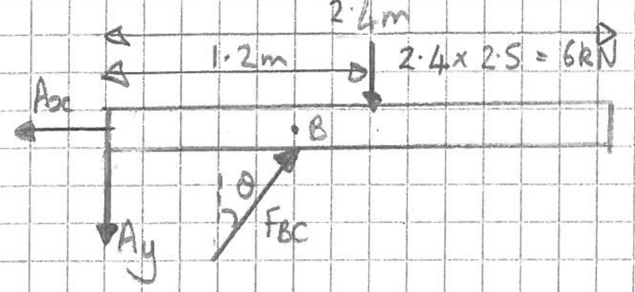
$$\Rightarrow \quad \delta_{CG} = \frac{0.77 \times 10^3 \times 3}{5.13 \times 10^{-6} \times 200 \times 10^9} = 2.25 \times 10^{-3} \ m$$

(contraction)

Q2

I member BC

II



$$\theta = \operatorname{atan}\left(\frac{0.9}{1.2}\right) = 36.9^\circ$$

$+\circlearrowleft \sum M_B = 0 \qquad A_y\,0.9 - 6 \times 0.3 = 0$

$\Rightarrow A_y = 2\,kN$

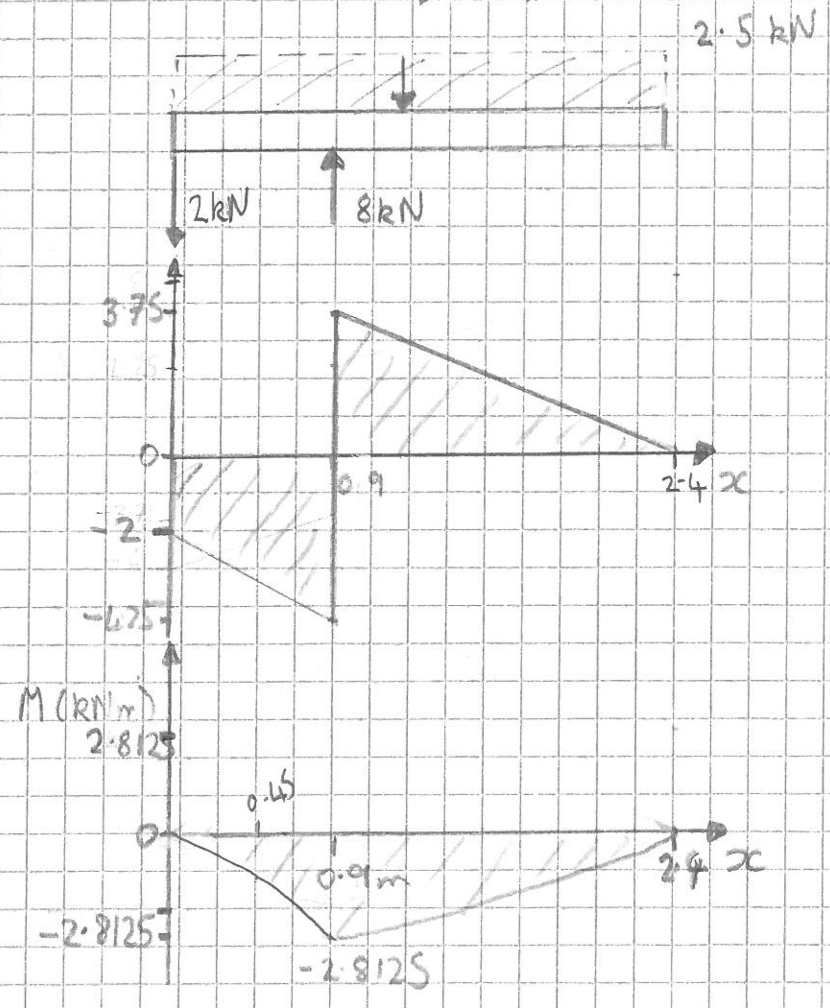
$\xrightarrow{+} \sum F_x = 0 \qquad F_{BC} \sin 36.9 - A_x = 0$

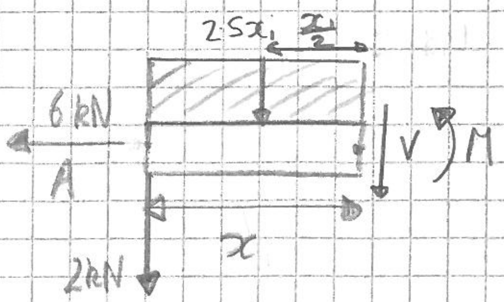
$+\uparrow \sum F_y = 0 \qquad F_{BC} \cos 36.9 - A_y - 6 = 0$

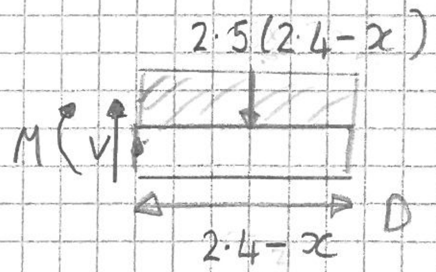
$\Rightarrow F_{BC} = 10\,kN$

$\Rightarrow A_x = 6\,kN$

III







$0 \leq x < 0.9m$

$+\circlearrowleft \sum M_x = 0 \qquad 2x + \dfrac{2.5x^2}{2} + M = 0$

$+\uparrow \sum F_y = 0 \qquad -2 - 2.5x - V = 0$

$V = -2 - 2.5x_1$

$0.9m \leq x \leq 2.4m$

$+\circlearrowleft \sum M_x = 0 \qquad -M - \dfrac{2.5}{2}(2.4 - x)^2 = 0$

$\Rightarrow \quad M = -\dfrac{2.5(2.4-x)^2}{2}$

$+\uparrow \sum F_y \qquad V - 2.5(2.4 - x) = 0$

$V = 2.5(2.4 - x)$

Evaluate at 0, 0.9m, 2.4m

$x = 0m \qquad V = -2kN \qquad M = 0$

$x = 0.9m \qquad V = -4.25 \qquad M = -2.8125$

AND $V = +3.75$ (using 2nd equation)

$x = 2.4 \qquad V = 0 \qquad M = 0$

IV centroid $\bar{x} = 0$

centroid $\bar{y} \Rightarrow$

$$\bar{y} = \frac{\Sigma \tilde{y}_i A_i}{\Sigma A_i} = \frac{1225(1000)(50) + 1000(400)(200) + 400(800)(100)}{(1000)(50) + (400)(200) + (800)(100)}$$

$$= 825 \text{ mm}$$

Find I_{xx} using parallel axis theorem

$$I_{x'} = I_x + Ad^2$$

also use MOI of a rectangle about its centroid is

$$I_x = \frac{1}{2} bh^3$$

$$I_{xx} = \left[\frac{1}{12}(1000)(50)^3 + 1000(50)(400)^2\right] + 2\left[\frac{1}{12}(50)(400)^3 + 50(400)(175)^2\right]$$

$$+ \left[\frac{1}{12}(100)(1200)^3 + 100(1200)(225)^2\right]$$

$$I_{xx} = 302 \cdot 44 (10^8) \text{ mm}^4$$

$$= \left[1 \cdot 042 \times 10^{-5} + 8 \times 10^{-3}\right] + 2\left[2 \cdot 666 \times 10^{-4} + 6 \cdot 125 \times 10^{-4}\right]$$

$$+ \left[0 \cdot 0144 + 6 \cdot 075 \times 10^{-3}\right]$$

OR in m^4 $\quad I_{xx} = 3 \cdot 024 \times 10^{-2} \text{ m}^4$

V.

Bending Moment at point E $\quad M_E = -1.153 \text{ kN.m}$

Shear force at point E $\quad V_E = -3.125 \text{ kN}$

Bending stress $= \dfrac{M_E}{I_{xx}} \times y$

$y = \bar{y} - 800 = 825 - 800 = 25 \text{ mm}$

$= 0.025 \text{ m}$

Bending stress $= \dfrac{-1.153 \times 10^3 \times 0.025}{3.024 \times 10^{-2}}$

$= -953.2 \text{ Pa}$

Axial stress $= \dfrac{P}{A} \leftarrow A_x$

$P = -6 \text{kN} \qquad A = 0.21 \text{m}^2$

Axial stress $= \dfrac{-6 \times 10^3}{0.21} = -28.57 \text{ kPa}$

Normal stress $= -0.953 \text{ kPa} - 28.57 \text{ kPa}$

$= -29.52 \text{ kPa}$

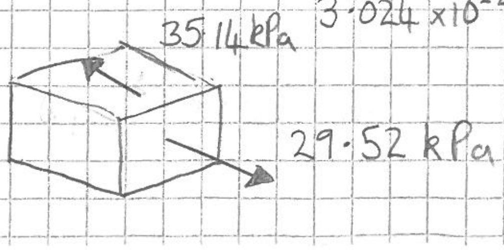
$Q_E = \Sigma \bar{y}' A' = 0.425 \times (0.8 \times 0.1) = 0.034 \text{ m}^3$

take t at 0.8 m to be 0.1 m thickness

$I_{xx} = 3.024 \times 10^{-2}$

Shear stress $= \dfrac{V_E Q_E}{I_{xx} t_E}$

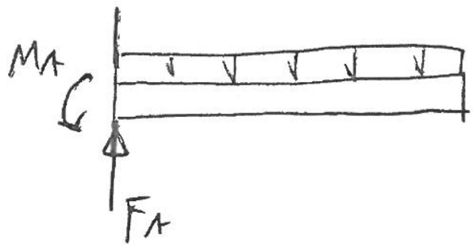
Shear stress $= \dfrac{-3.125 \times 10^3 \times 0.034}{3.024 \times 10^{-2} \times 0.1} = -35.14 \text{ kPa}$



Exam BYG 140;
21.05.2013
Answers, Q3 & 4

①

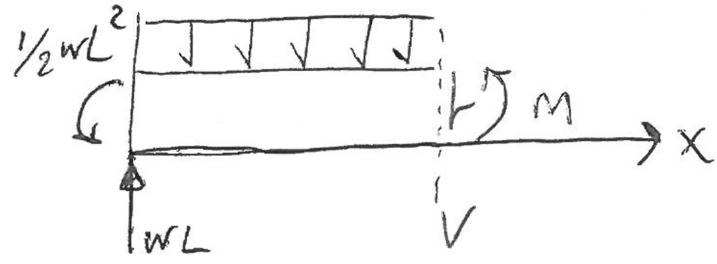
Q3 I



Support reactions at A:

$$F_A = W \cdot L$$

$$M_A = W \cdot L \cdot \frac{L}{2} = \frac{1}{2} WL^2$$



Internal shear force - and bending moment at section:

$$V + W \cdot X - WL = 0 \Rightarrow V(X) = W(L - X) \Rightarrow \underline{\underline{V_{max} = WL}}$$

(for $X = 0$)

$$M + W \cdot X \cdot \frac{X}{2} + \frac{1}{2} WL^2 - WL \cdot X = 0$$

$$\Rightarrow M(X) = -\frac{1}{2} wX^2 + wLX - \frac{1}{2} wL^2 \Rightarrow \underline{\underline{M_{max} = -\frac{1}{2} wL^2}}$$

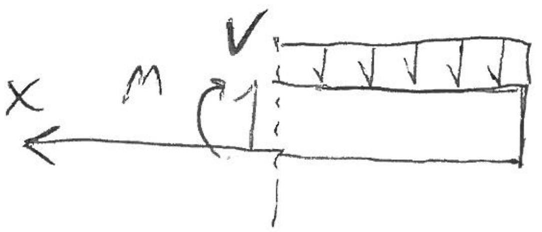
(for $X = 0$)

II 2 choices of X direction:

1 Origin at A $\Rightarrow$ $\underline{\underline{M(X) = -\frac{1}{2} wX^2 + wLX - \frac{1}{2} wL^2}}$

(you might have obtained the answer in I)

2 Origin at B:



$$M + wX \cdot \frac{X}{2} = 0 \Rightarrow \underline{\underline{M(X) = -\frac{1}{2} wX^2}}$$

III Elastic curve: $EI \cdot \frac{d^2 v}{dx^2} = M(x)$

Alt. 1, origin at A:

$$EI \frac{d^2 v}{dx^2} = w\left(-\frac{1}{2}x^2 + Lx - \frac{1}{2}L^2\right)$$

$$\Rightarrow EI \frac{dv}{dx} = w\left(-\frac{1}{2}\cdot\frac{1}{3}x^3 + L\cdot\frac{1}{2}x^2 - \frac{1}{2}L^2 \cdot x\right) + C_1$$

$$= w\left(-\frac{1}{6}x^3 + \frac{1}{2}Lx^2 - \frac{1}{2}L^2 x\right) + C_1$$

$$\Rightarrow EI \cdot v = w\left(-\frac{1}{6}\cdot\frac{1}{4}x^4 + \frac{1}{2}L\cdot\frac{1}{3}x^3 - \frac{1}{2}L^2\cdot\frac{1}{2}x^2\right) + C_1 x + C_2$$

$$= w\left(-\frac{1}{24}x^4 + \frac{1}{6}Lx^3 - \frac{1}{4}L^2x^2\right) + C_1 x + C_2$$

Border conditions:

Zero slope at $x=0$: $\left.\frac{dv}{dx}\right|_{x=0} = 0 \Rightarrow \underline{C_1 = 0}$

Zero deflection at $x=0$: $v(0) = 0 \Rightarrow \underline{C_2 = 0}$

$$\Rightarrow \underline{\underline{v(x) = \frac{w}{EI}\left(-\frac{1}{24}x^4 + \frac{1}{6}Lx^3 - \frac{1}{4}L^2x^2\right)}}$$

Alt. 2, origin at B:

$$EI \frac{d^2 v}{dx^2} = -\frac{1}{2}wx^2$$

$$\Rightarrow EI \frac{dv}{dx} = -\frac{1}{2}w\cdot\frac{1}{3}x^3 + C_1 = -\frac{1}{6}wx^3 + C_1$$

$$\Rightarrow EI\, v = -\frac{1}{6}w\cdot\frac{1}{4}x^4 + C_1 x + C_2 = -\frac{1}{24}wx^4 + C_1 x + C_2$$

Border conditions:

Zero slope at $X = L$: $\left.\frac{dv}{dx}\right|_{X=L} = 0$

$$\Rightarrow -\frac{1}{6} w L^3 + C_1 = 0 \quad \Rightarrow \quad \underline{C_1 = \frac{1}{6} w L^3}$$

Zero deflection at $X = L$: $v(L) = 0$

$$\Rightarrow -\frac{1}{24} w L^4 + \frac{1}{6} w L^3 \cdot L + C_2 = 0$$

$$\Rightarrow C_2 = \left(\frac{1}{24} - \frac{1}{6}\right) w L^4 = \underline{-\frac{1}{8} w L^4}$$

$$\Rightarrow \underline{\underline{v(X) = \frac{w}{EI}\left(-\frac{1}{24} X^4 + \frac{1}{6} L^3 X - \frac{1}{8} L^4\right)}}$$

IV Section- and material properties from tables:

Table 2: W 200 x 22: $I = 20 \cdot 10^6 \text{ mm}^4$

Table 1: A36: $E = 200 \text{ GPa} = 2 \cdot 10^5 \frac{N}{mm^2}$

Load: $12 \text{ kN/m} = 12 \text{ N/mm}$

$$\frac{w}{EI} = \frac{12 \text{ N/mm}}{2 \cdot 10^5 \frac{N}{mm^2} \cdot 2 \cdot 10^7 \text{ mm}^4} = \underline{3 \cdot 10^{-12} \frac{1}{mm^3}}$$

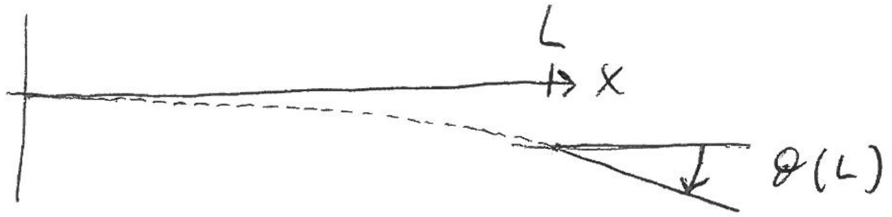
$$1: v(L) = v(2500 \text{ mm}) = 3 \cdot 10^{-12} \cdot \underbrace{\left(-\frac{1}{24} + \frac{1}{6} - \frac{1}{4}\right)}_{= -\frac{1}{8}} \cdot 2500^4$$

$$v(L) = -14.648 \approx \underline{\underline{-14.6 \text{ mm}}}$$ QED

2: $v(0) = 3 \cdot 10^{-12} \cdot -\frac{1}{8} \cdot 2500^{4} = \underline{\underline{-14.6 \text{ mm}}}$

V Slope angle at B:

1



$$\theta(L) = \arctan\left(\frac{dv}{dx}\bigg|_{x=L}\right) \approx \frac{dv}{dx}\bigg|_{x=L} \quad \text{(assume small angles)}$$

$$\frac{dv}{dx}\bigg|_{x=L} = \frac{w}{EI} \cdot \left(-\frac{1}{6}x^{3} + \frac{1}{2}Lx^{2} - \frac{1}{2}L^{2}x\right)$$

$$= 3 \cdot 10^{-12} \cdot \underbrace{\left(-\frac{1}{6} + \frac{1}{2} - \frac{1}{2}\right)}_{-\frac{1}{6}} \cdot 2500^{3} = \underline{-7.8125 \cdot 10^{-3}}$$

$$\underline{\underline{\theta = \begin{cases} -7.81 \cdot 10^{-3} \text{ Rad} \\ -0.45^{\circ} \end{cases}}}$$

2

θ (position, since axis direction is reversed)

$$\theta(0) \approx \frac{dv}{dx}\bigg|_{x=0} = C_1 = \frac{1}{6}wL^{3} = 7.8125 \cdot 10^{-3}$$

$$\underline{\underline{\theta = \begin{cases} 7.81 \cdot 10^{-3} \text{ Rad} \\ 0.45^{\circ} \end{cases}}}$$

VI Allowable deflection: $v_{max} = \frac{0.2}{100} \cdot 2500\ mm$

$$= 5\ mm$$

$$v_B = \frac{w}{EI} \cdot -\frac{1}{8} \cdot L^4 = -5\ mm$$

$$\Rightarrow \quad I_{min} = \frac{w \cdot L^4}{8\,E\,v_{B,max}} = \frac{12\,\frac{N}{mm} \cdot (2500\ mm)^4}{8 \cdot 2 \cdot 10^5\,\frac{N}{mm^2} \cdot 5\ mm}$$

$$= 58.593.750\ mm^4 = \underline{58.6 \cdot 10^6\ mm^4}$$

From Table 2: W 200 x 59, W 250 x 45, W 310 x 33 } will all do...

Q4 I: Torsion: $\tau_T = \frac{T \cdot \rho}{J}$

Bending: $\sigma_B = \frac{M \cdot y}{I}$

Transverse shear: $\tau_F = \frac{VQ}{It}$

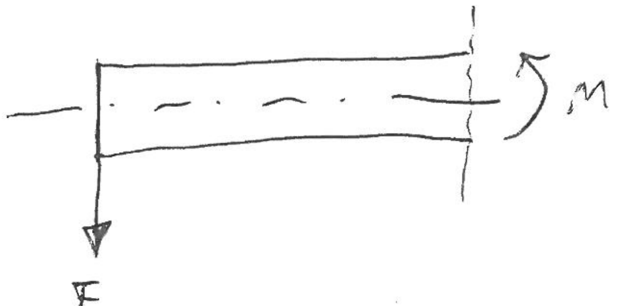
Torque, T is constant for all lengths; $T = 4$ kNm

$= 4 \cdot 10^6$ N mm

Shear force, V is also constant for all lengths; $V = 16$ kN

$= 16\,000$ N

Bending moment at $L = 145$ mm:



$M + F \cdot L = 0 \Rightarrow M = -F \cdot L$

(Directed as to give tension at the upper part)

$M = 16\,000\text{ N} \cdot 145\text{ mm} = 2.32 \cdot 10^6$ Nmm

Stress from torsion: $J = \frac{\pi}{2} c^4 = \frac{\pi}{2} \cdot (25\text{ mm})^4$

(c = radius) $= 613.592\text{ mm}^4$

(Same in A and B)

$\tau_T = \frac{4 \cdot 10^6 \cdot 25}{613.592} = 162.97\text{ N/mm}^2 \approx 163$ MPa

Stress from bending: $I = \frac{\pi}{4} r^4 = \frac{\pi}{4}(25\ mm)^4$

$= 306.796\ mm^4$

A: $y = r = 25\ mm$: $\sigma_{B,A} = \frac{2{,}32 \cdot 10^6 \cdot 25}{306.796} = 189.05\ N/mm^2$

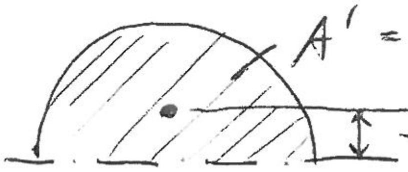
$= 189\ MPa$

B: $y = 0$: $\sigma_{B,B} = 0\ MPa$

Stress from transverse shear:

A: $Q = 0 \Rightarrow \tau_{F,A} = 0\ MPa$

B:



$A' = \frac{\pi}{2} r^2$

$y' = \frac{4r}{\pi 3}$

$\left.\right\}\ Q = A' \cdot y'$

$Q = \frac{\pi r^2}{2} \cdot \frac{4r}{3\pi} = \frac{2}{3} r^3 = \frac{2}{3} \cdot (25\ mm)^3 = 10.417\ mm^3$

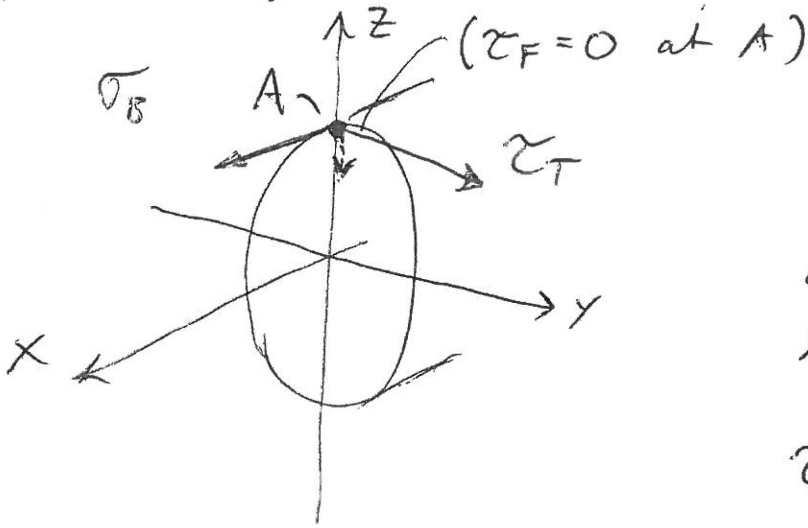
$t = 2r = 50\ mm$

$\tau_{F,B} = \frac{16\,000 \cdot 10417}{306.796 \cdot 50} = 10.87\ N/mm^2 = 11\ MPa$

⇒

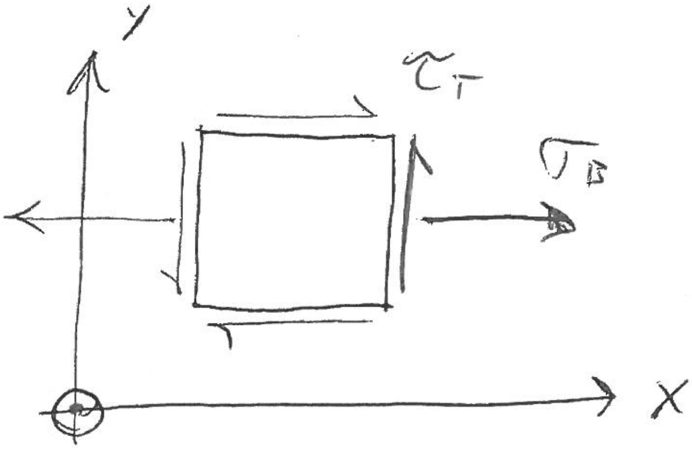
	A	B
τ_T	163	163
σ_B	189	0
τ_F	0	11

II: State of stress at pt. A:



σ_B is tension; in this case directed i positive X-direction.

τ_T is directed in positive Y-direction



In fig. 4b, the Z axis is directed normal to the paper-plane

III: Principle stresses:

$\sigma_X = 189$

$\sigma_Y = 0$

$\tau_{XY} = 163$

$$\sigma_{1,2} = \frac{\sigma_X + \sigma_Y}{2} \pm \sqrt{\left(\frac{\sigma_X - \sigma_Y}{2}\right)^2 + \tau_{XY}^2}$$

$$\Rightarrow \sigma_{1,2} = \frac{189}{2} \pm \sqrt{\left(\frac{189}{2}\right)^2 + 163^2} = 94.5 \pm 188.4$$

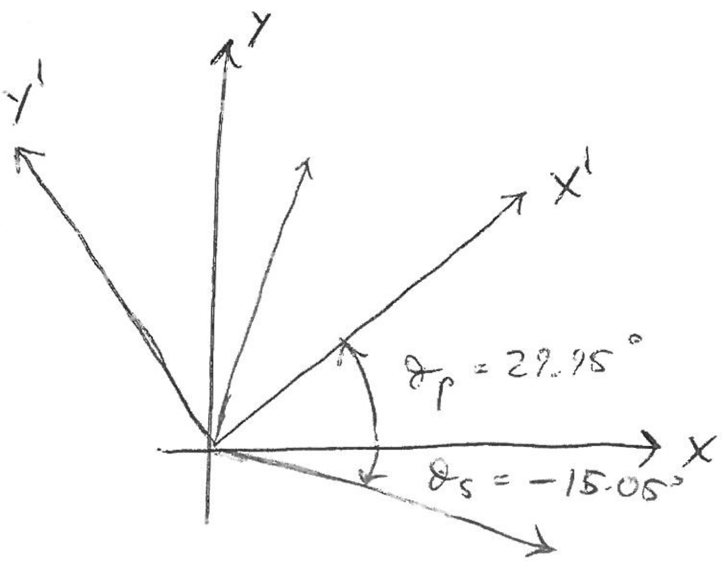
$\Rightarrow \sigma_1 = 282.9$ MPa

$\sigma_2 = -93.9$ "

Orientation of principle axis:

$$\tan 2\theta_p = \frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}} = \frac{163}{94.5} \Rightarrow 2\theta_p = 59.90^\circ$$

$$\Rightarrow \underline{\underline{\theta_p = 29.95^\circ}}$$



IV: Max. in-plane shear stress:

$$\tau_{ip,max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \underline{\underline{188.4 \text{ MPa}}}$$

Calculated in III

Orientation of max in-plane axis:

$$\tan 2\theta_s = \frac{-\frac{\sigma_x - \sigma_y}{2}}{\tau_{xy}} = -\frac{94.5}{163} \Rightarrow 2\theta_s = -30.10^\circ$$

$$\Rightarrow \underline{\underline{\theta_s = -15.05^\circ}}$$

These axis are rotated -45° relative to the principle axis

$$\Delta\theta = -15.05^\circ - 29.95^\circ = \underline{\underline{-45^\circ}}$$

V : Elastic properties: $\quad G = \dfrac{E}{2(1+\nu)}$

May read off graph : $\quad \sigma \approx 200\ \text{MPa}\ /\ \varepsilon \approx 0.0027$

$$E = \left.\frac{\Delta\sigma}{\Delta\varepsilon}\right|_{\text{Elastic}} = \frac{200}{0.0027} \approx \underline{\underline{74\,000\ \text{N/mm}^2}}$$

$$G = \frac{74\,000}{2\cdot(1+0.35)} \approx \underline{\underline{27\,400\ \text{N/mm}^2}}$$

VI : Normal- and shear strain; point A

$\sigma_x = 189\ \text{MPa}\ ,\ \sigma_y = 0\ ,\ \tau_{xy} = 163\ \text{MPa}$

$$\varepsilon_x = \frac{1}{74\,000}\cdot\left[189 - 0.35\cdot(0+0)\right] = \frac{189}{74\,000} = \underline{\underline{2.55\cdot 10^{-3}}}$$

$$\varepsilon_y = \frac{1}{74\,000}\cdot\left[0 - 0.35\cdot(189+0)\right] = -\frac{0.35\cdot 189}{74\,000} = \underline{\underline{-8.94\cdot 10^{-4}}}$$

$$\varepsilon_z = \varepsilon_y = \underline{\underline{-8.94\cdot 10^{-4}}}$$

$$\gamma_{xy} = \frac{1}{27\,400}\cdot 163 = \underline{\underline{5.95\cdot 10^{-3}}}$$

$$\gamma_{yz} = \gamma_{zx} = \underline{\underline{0}}$$