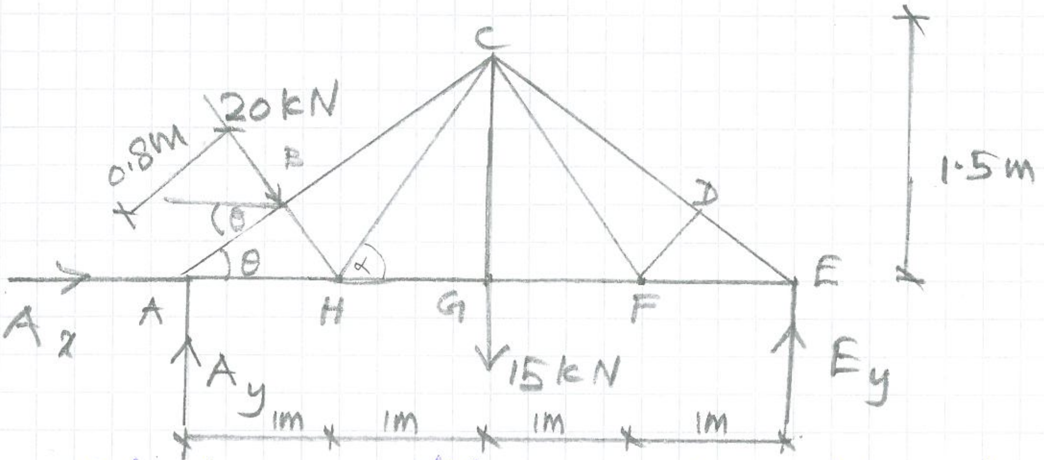
BYG 140 - Konstruksjonmekanikk 1

May 19, 2015

(Q1) (i)

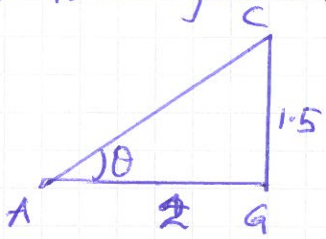


considering equilibrium of entire truss

$$\rightarrow \Sigma F_x = A_x + 20 \sin\theta = 0$$

$$A_x = -20 \sin\theta$$

From triangle AGC



$$\tan\theta = \frac{1.5}{2}$$

$$\theta = \tan^{-1}\left(\frac{1.5}{2}\right) = 36.87^\circ$$

$$\therefore \quad A_x = -20 \sin(36.87^\circ)$$

$$A_x = -12 \text{ kN}$$

$$\uparrow \Sigma F_y = A_y + E_y - 15 - 20\cos\theta = 0$$

$$A_y + E_y = 15 + 20\cos(36.87^\circ)$$

$$A_y + E_y = 31 \text{ kN}$$

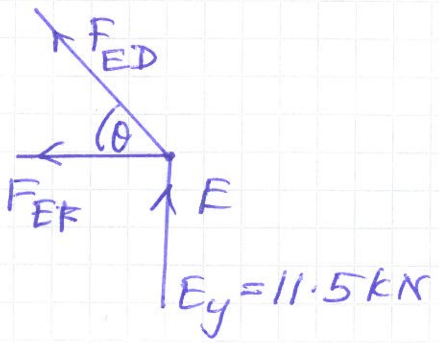
Taking moment about A,

$$\circlearrowleft \; 4E_y - (2 \times 15) - (20 \times 0.8) = 0$$

$E_y = 11.5$ kN $\qquad\qquad A_y = 19.5$ kN

(II) Axial forces of members using method of joints.

considering equilibrium of joint E



$\theta = 36.87^\circ$ (calculated)

$\uparrow \Sigma F_y = E_y + F_{ED}\sin\theta = 0$

$F_{ED} = \dfrac{-11.5}{\sin(36.87^\circ)} = -19.17\,kN$

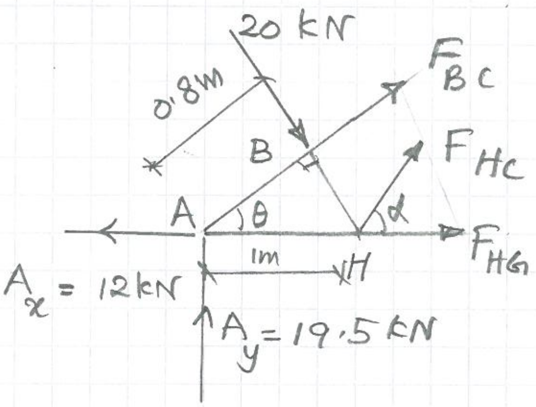
$F_{ED} = 19.17\,kN$ (compression)

$\rightarrow \Sigma F_x = -F_{EF} - F_{ED}\cos\theta = 0$

$F_{EF} = -(-19.17\cos 36.87^\circ)$

$F_{EF} = 15.33\,kN$ (tension)

(III) Axial forces of members using method of sections.



considering equilibrium of the segment

Taking moment about H,

$\circlearrowleft (A_y \times 1) + (F_{BC} \times 1\sin\theta) = 0$

$F_{BC} = \dfrac{-19.5}{\sin 36.87^\circ}$

$F_{BC} = -32.5\,kN$

$F_{BC} = 32.5\,kN$ (compression)

$\uparrow \Sigma F_y = A_y + F_{HC}\sin\alpha + F_{BC}\sin\theta - 20\cos\theta = 0$

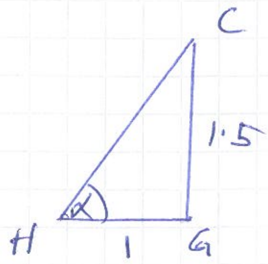
$F_{HC}\sin\alpha = 20\cos 36.87^\circ + 32.5\sin 36.87^\circ$

-19.5

$F_{HC} = \dfrac{16}{\sin\alpha}$

$$\tan\alpha = \frac{1.5}{1}$$

$$\alpha = 56.3^\circ$$



Then $$F_{HC} = \frac{16}{\sin 56.3^\circ}$$

$$\underline{\underline{F_{HC} = 19.23\ kN\ (\text{tension})}}$$

(iv) zero force members - FD, CF.

(v) Required cross sectional area.

$F_{EF} = 15.33\ kN$ (calculated (iii))

$F_{ED} = 19.17\ kN$ (")

For member EF

$$\sigma_{all} \geq \frac{F_{EF}}{\text{Area}}$$

$$155\times10^6 \geq \frac{15.33\times10^3}{\text{area}}$$

$$\text{Area} \geq \frac{15.33\times10^3\ N}{155\ N/mm^2} = 98.9\ mm^2$$

$$\underline{\underline{\text{Area EF} = 100\ mm^2}}$$

For member ED

$$\sigma_{all} \geq \frac{19.17\times10^3}{\text{area}}$$

$$\text{area} \geq \frac{19.17\times10^3\ N}{155\ N/mm^2} = 123.68\ mm^2$$

$$\underline{\underline{\text{Area ED} = 125\ mm^2}}$$

(vi) change in length in EF (Δ)

$$\frac{F}{A} = E\frac{\Delta}{L}$$

$$\Delta = \frac{15.33\times10^3\ N}{100\times10^{-6}\ m^2} \times \frac{1m}{200\times10^9} = 0.000767\ m.$$

$$\underline{\underline{\Delta = 0.767\ mm}}$$ Member elongated.

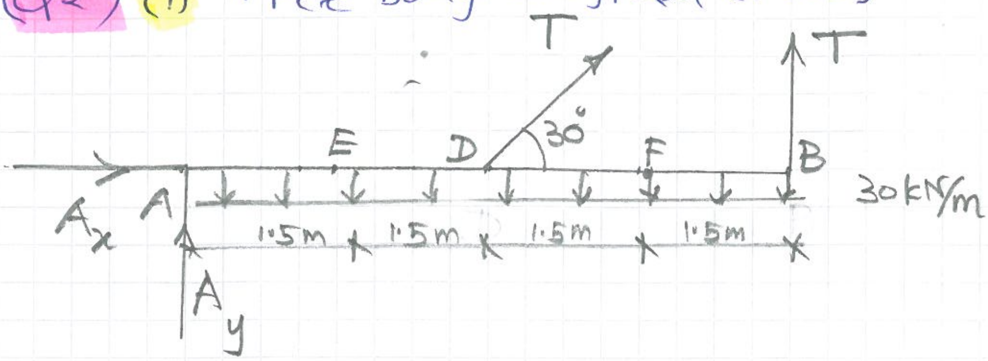
(VII) If the material changes to Aluminium.
(ie: E $\downarrow$)
But loading & support are same.

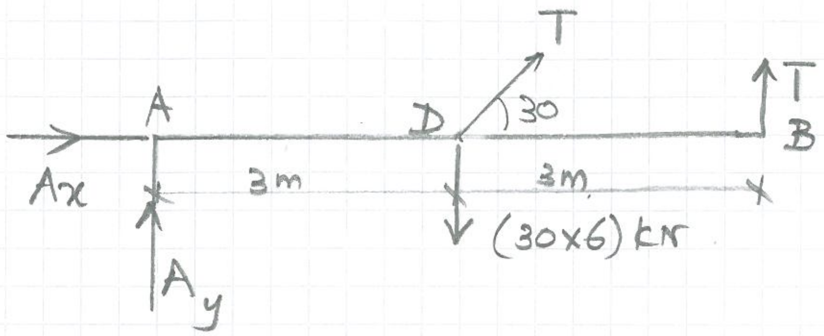
- Therefore axial forces are same; There is no any infuence from material properties.

- displacement $\Delta = \dfrac{FL}{AE}$

when $E\downarrow$ $\Delta\uparrow$ - increases.

Q2) (i) Free body diagram (FBD)





considering equilibrium,

$\rightarrow \Sigma F_x = A_x + T\cos 30 = 0$ —— ①

$\uparrow \Sigma F_y = A_y + T + T\sin 30 - (30 \times 6) = 0$ —— ②

taking moment about D.

$3A_y - 3T = 0$

$A_y = T$ —— ③

solving equation ①, ②, & ③.

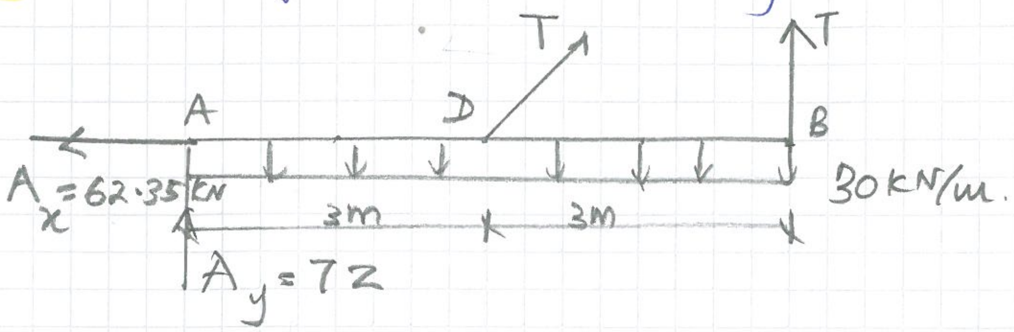
② $\Rightarrow$ $T(1 + 1 + \sin 30) = (30 \times 6)$

$T = \dfrac{30 \times 6}{(2 + \sin 30)} = 72 \text{ kN}$

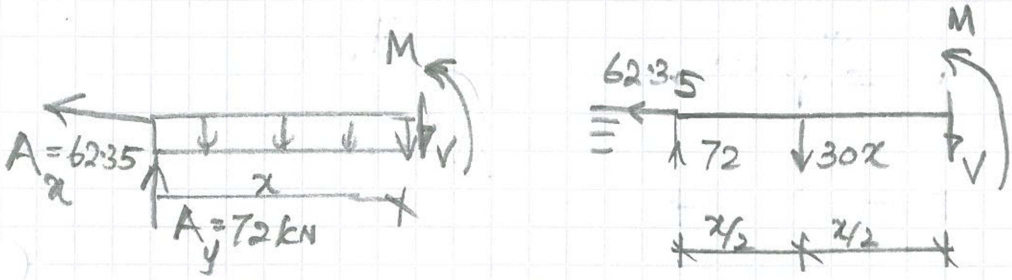
① $\Rightarrow$ $A_x = -72\cos 30 = -62.35 \text{ kN}$

③ $\Rightarrow$ $A_y = 72 \text{ kN}$

(ii) Shear force and bending moment diagrams.



$0 \leqslant x < 3$



considering equilibrium of the segment

$$\uparrow \; -V - 30x + 72 = 0$$
$$V = 72 - 30x.$$

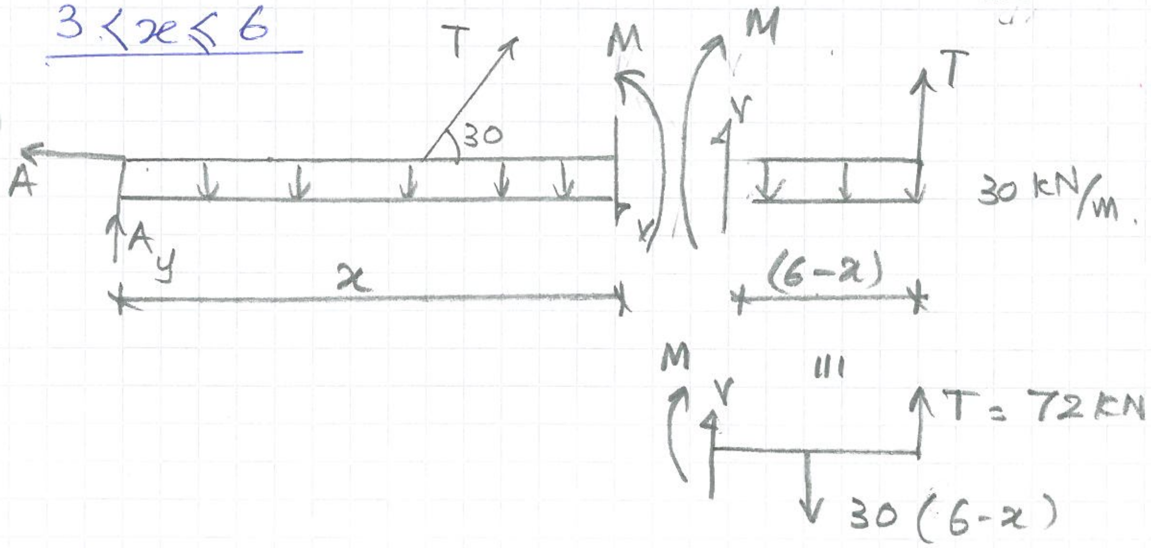
x	V
0	72
3	-18

$$\curvearrowleft \; M + \frac{30x^2}{2} - 72x = 0$$
$$M = 72x - 15x^2$$

$\left[\frac{dM}{dx} = 0 \text{ for maximum } M, \; \frac{dM}{dx} = 72 - 30x = 0 \; ; \; x = 2.4\right]$

x	M
0	0
3	81
2.4	86.4

$3 < x \leqslant 6$



$$\uparrow \; V + 72 - 30(6-x) = 0$$
$$V = 30(6-x) - 72$$
$$V = -30x + 108$$

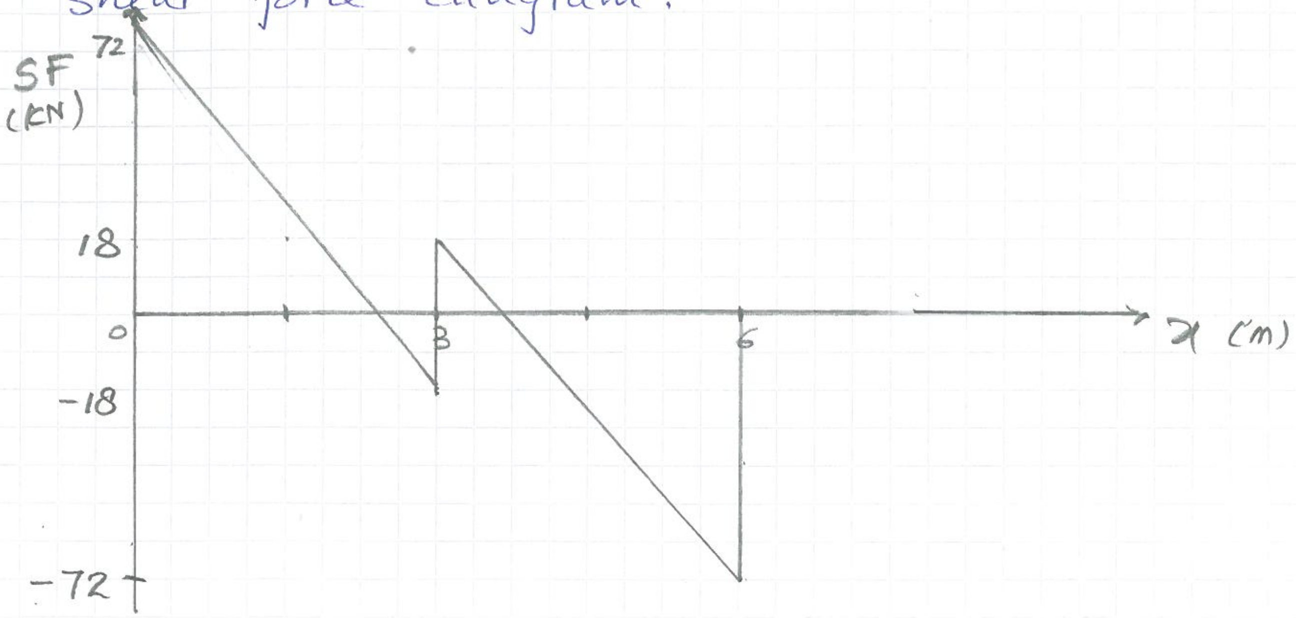
x	V
3	18
6	-72

$$\curvearrowright \; M + \frac{30(6-x)^2}{2} - 72(6-x) = 0.$$
$$M = 72(6-x) - 15(6-x)^2$$

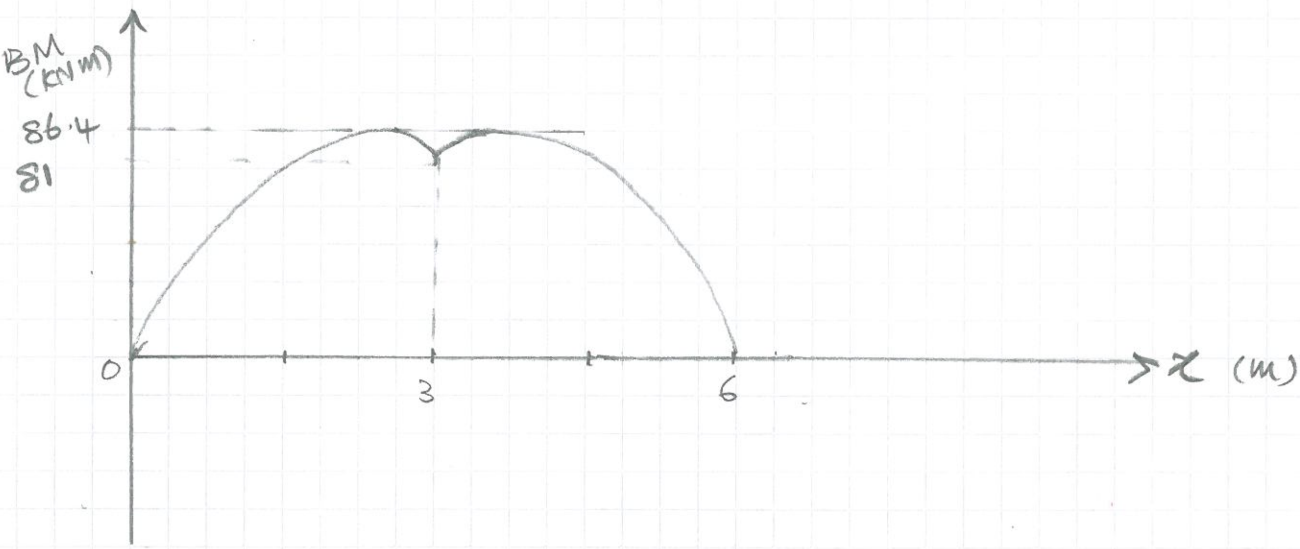
$\left[\frac{dM}{dx} = \right.$

x	M
3	81
6	0

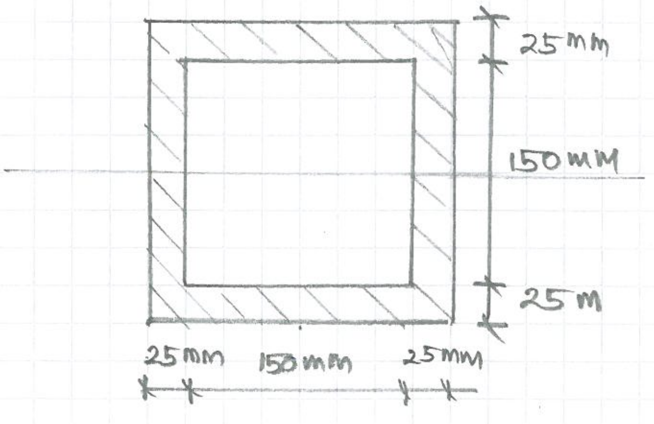
shear force diagram.



Bending moment diagram.



(iii) Cross sectional area



$$\text{Area} = (200 \times 200) - (150 \times 150)$$

$$= 17500 \, mm^2$$

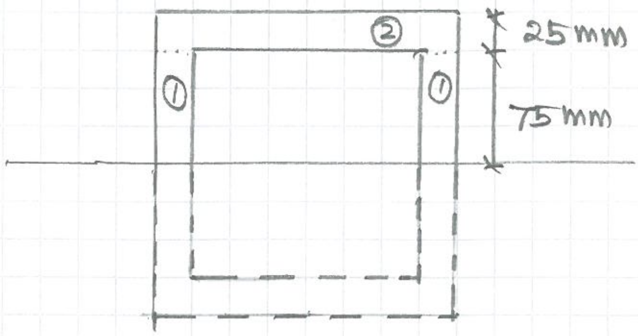
Moment of inertia.

$$I = \left(\frac{1}{12} \times 200 \times 200^3\right) - \left(\frac{1}{12} \times 150 \times 150^3\right)$$

$$= 91145833.30 \, mm^4$$

$$I = 91.15 \times 10^{-6} \, m^4$$

(IV) Maximum shear stress (τ_{max})



Shear stress τ;

$$\tau = \frac{VQ}{It}$$

$I = 91.15 \times 10^6\ mm^4$

$t = 50\ mm$

$V_{max} = 72 \times 10^3\ N$ (From SFD (V))

$$Q_{max} = \Sigma A'\bar{y}' = 2(A_1 \bar{y}_1) + A_2 \bar{y}_2$$

$$= 2\underbrace{\left[(75 \times 25) \times \frac{75}{2}\right]}_{140625} + \underbrace{\left[(25 \times 200)\left(75 + \frac{25}{2}\right)\right]}_{437500}$$

$$= 578125\ mm^3$$

Then $\tau_{max} = \dfrac{V_{max} Q_{max}}{It}$

$$= \frac{72 \times 10^3\ N \times 578125\ mm^3}{50\ mm \times 91.15 \times 10^6\ mm^4}$$

$$\tau_{max} = 9.133\ N/mm^2\ (MPa)$$

Maximum normal stress (σ)

Normal stress due to axial force.

$$\sigma_N = \frac{P}{A} = \frac{62.35 \times 10^3\ N}{17500\ mm^2} \quad (P = A_x)$$

$$= 3.56\ N/mm^2$$

Normal stress due to bending

$$\sigma_M = \frac{My}{I} = \frac{86.4 \times 10^3 \times 10^3\ Nmm \times 100\ mm}{91.15 \times 10^6\ mm^4}$$

$$= 94.79\ N/mm^2$$

Then maximum normal stress $= \sigma_N + \sigma_M$

$= 3.56 + 94.79$

$= 98.35\ N/mm^2$

$= 98.35\ MPa$

(V) Normal stressesat E and F are not same.

at F - Normal stress is only due to bending.
Stress due to axial force is zero.

at E - Normal stress is due to both axial stress and bending stress.

$$\sigma_E = \frac{M_E y}{I} + \frac{N_E}{A}$$

$M_E = M_F$ (from BMD)

but $N_E \neq N_F$

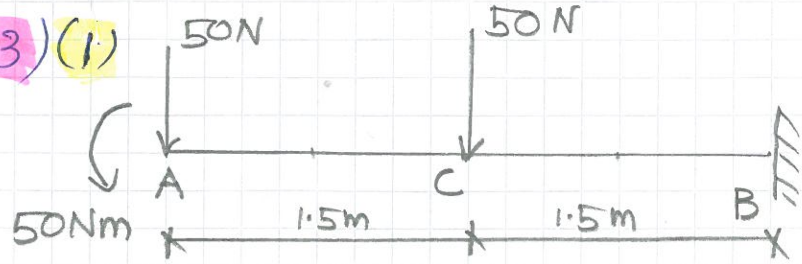
N_E has a value.

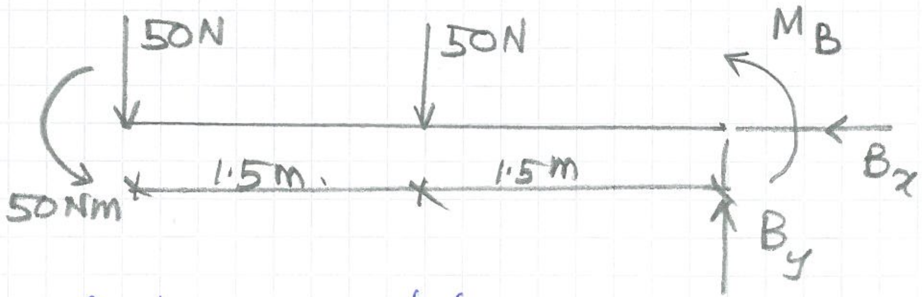
$N_F = 0$

$$\sigma_F = \frac{M_F y}{I} + \frac{N_F}{A}$$

Therefor $\sigma_E \neq \sigma_F$

(Q3) (I)





considering equilibrium.

$\rightarrow \sum F_x = -B_x = 0$ $\boxed{B_x = 0}$

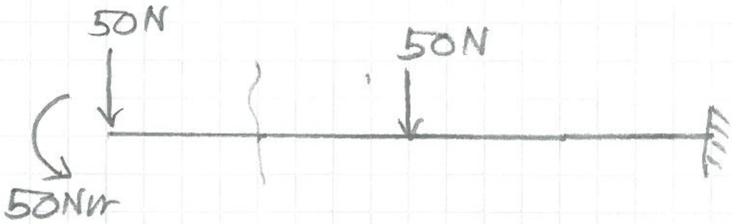
$\uparrow \sum F_y = B_y - 50 - 50 = 0$

$\boxed{B_y = 100\ N}$

$\curvearrowleft\ M_B + (50 \times 1.5) + (50 \times 3) + 50 = 0$

$\boxed{M_B = -275\ Nm}$

(II)



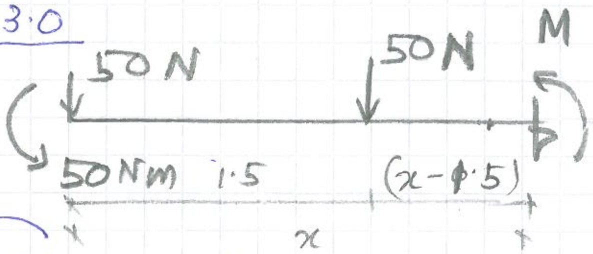
$0 \leq x < 1.5$

$\curvearrowleft\ M + 50x + 50 = 0$

$M = -50x - 50$

x	M
0	−50
1.5	−125

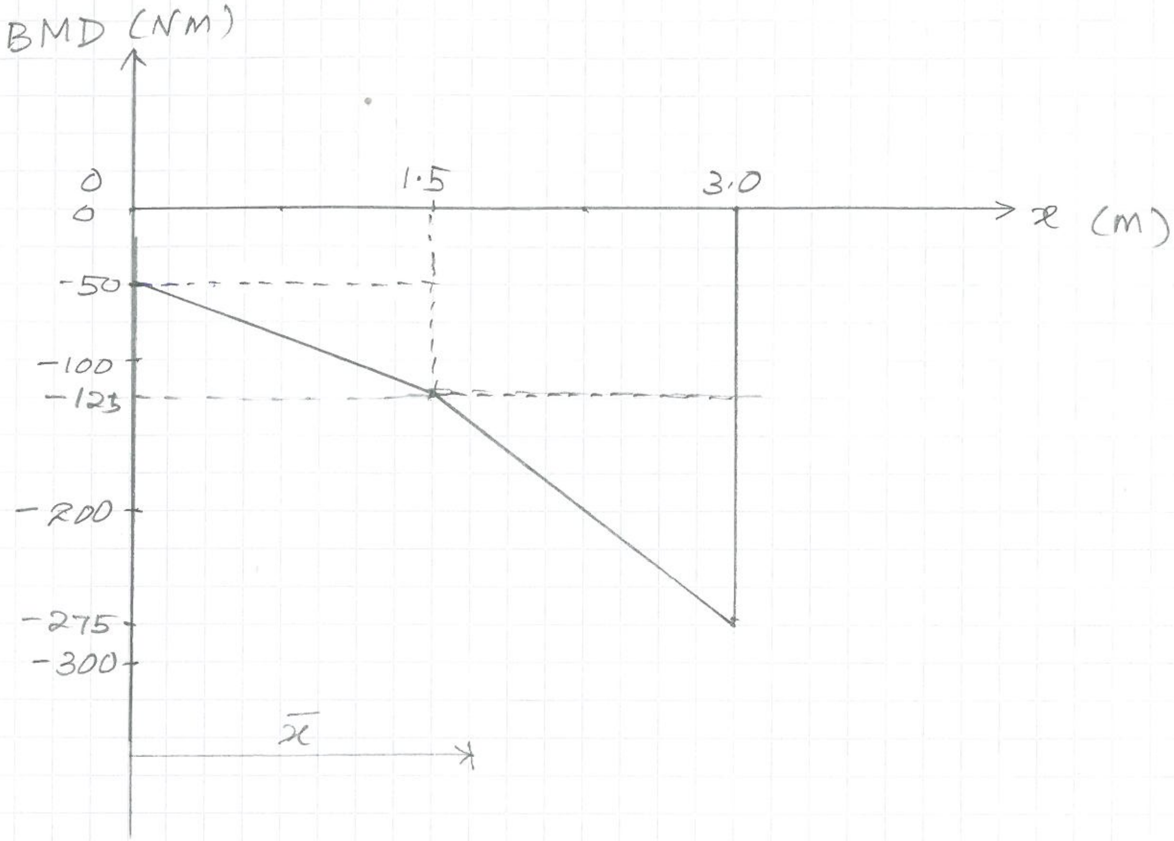
$1.5 \leq x \leq 3.0$



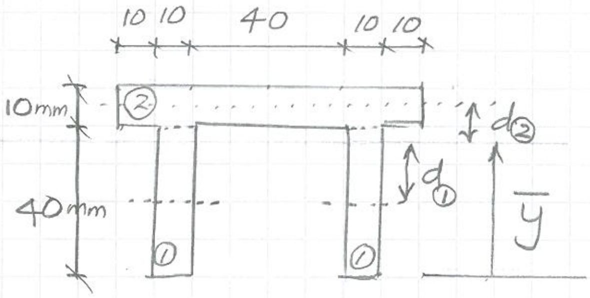
$\curvearrowleft\ M + 50(x - 1.5) + 50x + 50 = 0$

$M = -100x + 25$

x	M
1.5	−125
3	−275



(iii)



$d_① = \bar{y} - 20 = 32.5 - 20 = 12.5$ mm

$d_② = 5 + (40 - 32.5) = 12.5$ mm

$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{A_①\bar{y}_① + A_①\bar{y}_① + A_②\bar{y}_②}{A_① + A_① + A_②}$$

$$= \frac{2[(10\times 40)\times 20] + (80\times 10)(40 + \frac{10}{2})}{(2\times 40\times 10) + (80\times 10)}$$

$$\bar{y} = 32.5 \text{ mm}$$

moment of inertia.

$$I = \Sigma(I_i + A_i d_i^2)$$

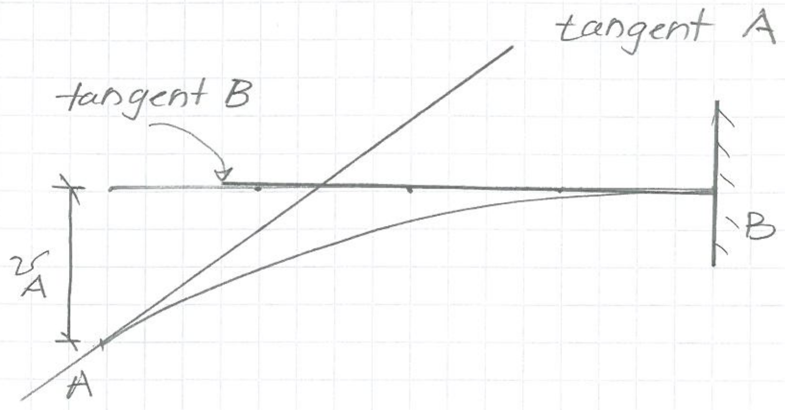
$$= 2(I_① + A_① d_①^2) + (I_② + A_② d_②^2)$$

$$= 2\left[\left(\frac{1}{12}\times 10\times 40^3\right) + \left(40\times 10\times 12.5^2\right)\right] + \left[\left(\frac{1}{12}\times 80\times 20^3\right) + \left(80\times 10\times 12.5^2\right)\right]$$

$$I = 363333.3 \text{ mm}^4$$

$$I = 3.633\times 10^{-7} \text{ m}^4$$

(iv) Deflection at end A – by moment area method.



deflection at A $= v_A = t_{A/B}$

$t_{A/B} = \begin{pmatrix}\text{area of the} \\ \frac{M}{EI}\text{ diagram} \\ \text{between A \& B}\end{pmatrix} \times \begin{pmatrix}\text{distance to} \\ \text{the centroid} \\ \text{from A}\end{pmatrix}$

area $\frac{M}{EI}$ diagram $= \left\{\left[\left(\frac{50+125}{2}\right)\times 1.5\right] + \left[\left(\frac{125+275}{2}\right)\times 1.5\right]\right\}\frac{1}{EI}$

$= \frac{431.25}{EI}$ Nm

distance to the centroid from A

$= \frac{\Sigma A\bar{x}}{\Sigma A}$

$$= \frac{\left\{\left[(50\times 1.5)\times\frac{1.5}{2}\right] + \left[\left(\frac{1}{2}\times 75\times 1.5\right)\times\frac{2\times 1.5}{3}\right] + \left[(125\times 1.5)\times\left(\frac{1.5}{2}+1.5\right)\right] + \left[\left(\frac{1}{2}\times 150\times 1.5\right)\left(1.5+\frac{2\times 1.5}{3}\right)\right]\right\}}{(50\times 1.5) + \left(\frac{1}{2}\times 75\times 1.5\right) + (125\times 1.5) + \left(\frac{1}{2}\times 150\times 1.5\right)}$$

$= \frac{815.625}{431.25} = 1.891$ m

Then

$t_{A/B} = \frac{431.25}{EI}\times 1.891 = \frac{431.25\text{ Nm} \times 1.891\text{ m}}{200\times 10^9\text{ N/m}^2 \times 3.633\times 10^{-7}\text{ m}^4}$

$t_{A/B} = 0.011225$ m

$\underline{\underline{v_A = 11.225\text{ mm}}}$

Deflection at A - by integration.

$0 \leq x \leq 1.5$

$$EI \frac{d^2v}{dx^2} = M(x)$$

$$EI \frac{d^2v}{dx^2} = -50x - 50$$

integrating

$$EI \frac{dv}{dx} = -\frac{50x^2}{2} - 50x + C_1 \quad \text{——— (1)}$$

$$EI\, v = -\frac{50x^3}{6} - \frac{50x^2}{2} + C_1x + C_2 \quad \text{——— (2)}$$

$1.5 < x < 3.0$

$$EI \frac{d^2v}{dx^2} = -100x + 25$$

$$EI \frac{dv}{dx} = -\frac{100x^2}{2} + 25x + C_3 \quad \text{——— (3)}$$

$$EI\, v = -\frac{50}{3}x^3 + \frac{25x^2}{2} + C_3x + C_4 \quad \text{——— (4)}$$

Boundary conditions

For $0 \leq x < 1.5$

For $1.5 < x < 3.0$ $\quad x = 3 \quad v = 0$

$x = 3 \quad \frac{dv}{dx} = 0$

For both $\quad x = 1.5 \quad \frac{dv}{dx}$ are same for bothside

$x = 1.5 \quad v$ are same from bothside.

eq (3) $(-50 \times 3^2) + (25 \times 3) + C_3 = 0$

$C_3 = 375$

eq (4) $\left(-\frac{50}{3} \times 3^3\right) + \left(\frac{25 \times 3^2}{3}\right) + (375 \times 3) + C_4 = 0$

$C_4 = -787.5$

① = ③ for $x = 1.5$

$$(-50 \times 1.5^2) + (25 \times 1.5) + 375 = (-25 \times 1.5^2) - (50 \times 1.5) + C_1$$

$$C_1 = 431.25$$

② = ④ for $x = 1.5$

$$\left(\frac{-50 \times 1.5^3}{3}\right) + \left(\frac{25 \times 1.5^2}{2}\right) + (375 \times 1.5) - 787.5 =$$

$$\left(-\frac{25}{3} \times 1.5^3\right) - (25 \times 1.5^2) + (431.25 \times 1.5) + C_2$$

$$C_2 = -815.625.$$

From equation ②

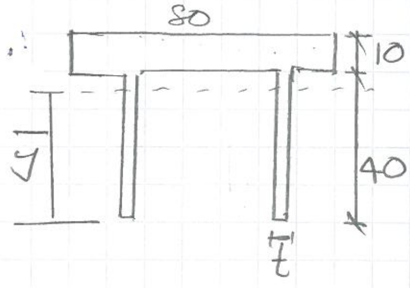
$$EI\,v = -\frac{50}{6}x^3 - \frac{50x^2}{2} + 431.25x - 815.625.$$

when $x = 0$; $v = -\dfrac{815.625}{EI}$

$$= \frac{815.625}{200 \times 10^9 \times 3.633 \times 10^{-7}} = 0.011225\,m$$

$$v_A = 11.225\,mm$$

(v) When web thickness decreases (ie $t < 10\,mm$) and length, support conditions and loading remain same.

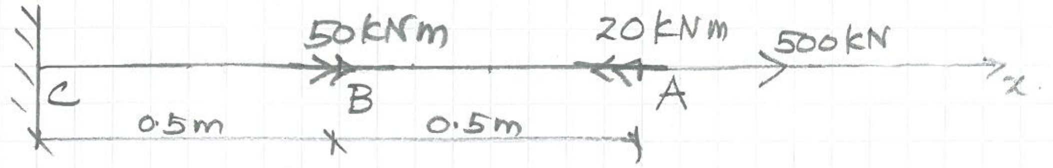


$\bar{y}\uparrow$ - increases.

$$I = 2\left[\left(\frac{1}{12} \times t \times 40^3\right) + \left(40 \times t \times \left(\bar{y} - \frac{40}{2}\right)^2\right)\right] + \left[\left(\frac{1}{12} \times 80 \times 10^3\right) + \left(80 \times 10 \times (45 - \bar{y})^2\right)\right]$$

I changes, increase or decrease depends on $\bar{y}$ value

(Q 4) (1)



Angle of twist at end A $-(\phi_{A/C})$

$$\phi_{A/C} = \phi_{A/B} + \phi_{B/C}$$

$$= \frac{T_{AB} L_{AB}}{J_{AB} G} + \frac{T_{BC} L_{BC}}{J_{BC} G}$$

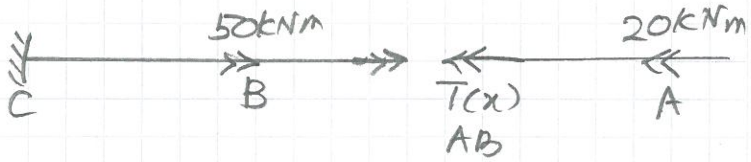
From given data.

$$G = 75\,GPa.$$

$$L_{AB} = L_{BC} = 0.5\,m.$$

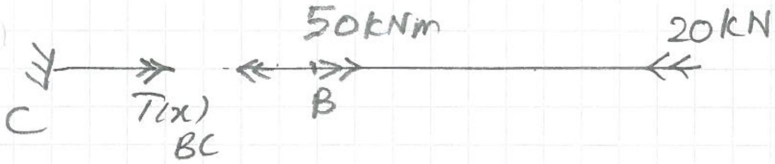
$$J_{AB} = \frac{\pi}{2}(100^4 - 80^4)10^{-12} = 9.274 \times 10^{-5}\,m^4$$

$$J_{BC} = \frac{\pi}{2}(100 \times 10^{-3})^4 = 1.57 \times 10^{-4}\,m^4$$



$$\leftarrow \; T_{AB}(x) + 20\,kNm = 0$$

$$T_{AB}(x) = -20\,kNm$$



$$\rightarrow \; -T_{BC}(x) + 50 - 20 = 0$$

$$T_{BC}(x) = 30\,kNm.$$

Then $\phi_{A/C} = \frac{(-20 \times 10^3) \times 0.5}{9.274 \times 10^{-5} \times 75 \times 10^9} + \frac{(30 \times 10^3) \times 0.5}{1.57 \times 10^{-4} \times 75 \times 10^9}$

$$\phi_{A/C} = -0.000167$$

$$\phi_{A/C} = -0.0096^{\circ}$$

(ii) Maximum shear stress.

$$\tau = \frac{T\rho}{J}$$

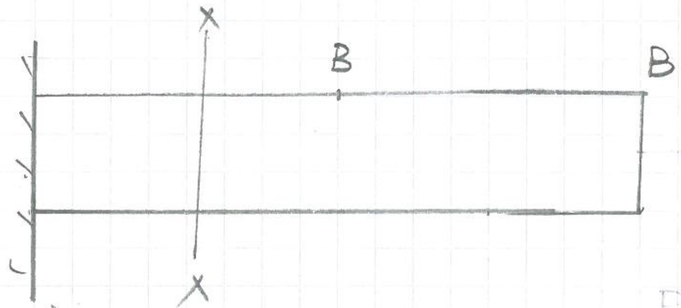
For AB ; $\tau_{AB} = \frac{-20\times10^{3}\times100\times10^{-3}}{9.274\times10^{-5}} = -21.6 \text{ MPa}.$

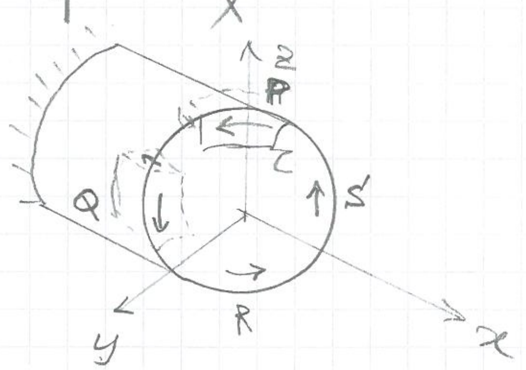
$\tau_{BC} = \frac{30\times10^{3}\times100\times10^{-3}}{1.57\times10^{-4}} = 19.1 \text{ MPa}$

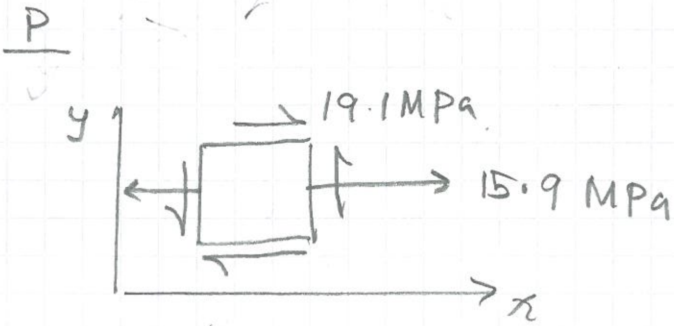
$\therefore$ Maximum shear stress $\underline{\tau_{AB} = 21.6 \text{ MPa}}$

(iii) Normal stress for BC $= \frac{500\times10^{3}}{(\pi\times100^{2})} = 15.9 \text{ MPa}.$

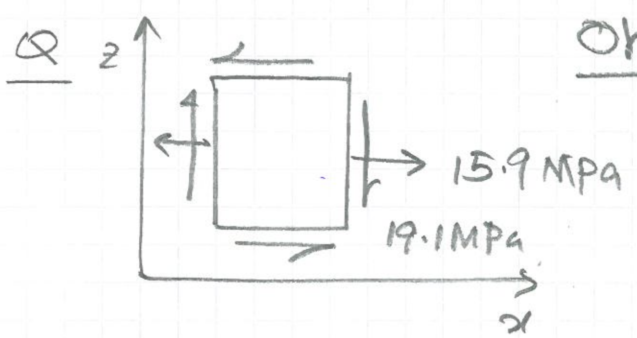
shear stress $= 19.1 \text{ MPa}$ (calculated - (ii))







Or



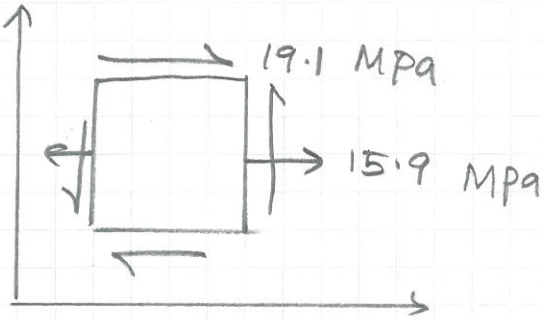
(iv) principle stress at outer surface of BC part.

by using equations.

$\sigma_x = 15.9$ Mpa

$\sigma_y = 0$

$\tau_{xy} = 19.1$ Mpa.



$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \left(\frac{15.9+0}{2}\right) \pm \sqrt{\left(\frac{15.9-0}{2}\right)^2 + 19.1^2}$$

$$= 7.95 \pm 20.69$$

$\sigma_1 = 28.64$ Mpa

$\sigma_2 = -12.73$ Mpa.

Orientation

$$\tan 2\theta_p = \frac{\tau_{xy}}{\left(\frac{\sigma_x-\sigma_y}{2}\right)} = \frac{19.1}{(15.9/2)}$$

$$2\theta_p = \tan^{-1} 2.40 = 67.40^\circ$$

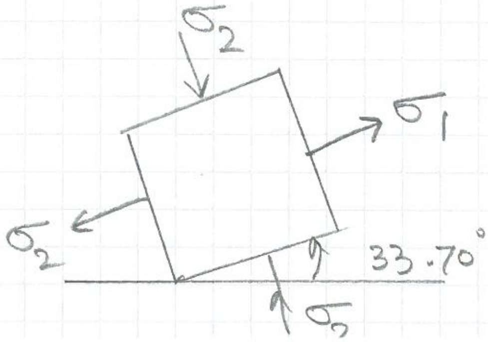
$\theta_p = 33.70^\circ$

check when $\theta_p = 33.70^\circ$

$$\sigma_x = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta_p + \tau_{xy}\sin 2\theta_p$$

$$= \frac{15.9}{2} + \frac{15.9}{2}\underbrace{\cos 67.40}_{3.055} + \underbrace{19.1\sin 67.40^\circ}_{17.63}$$

$= 28.64$ Mpa.

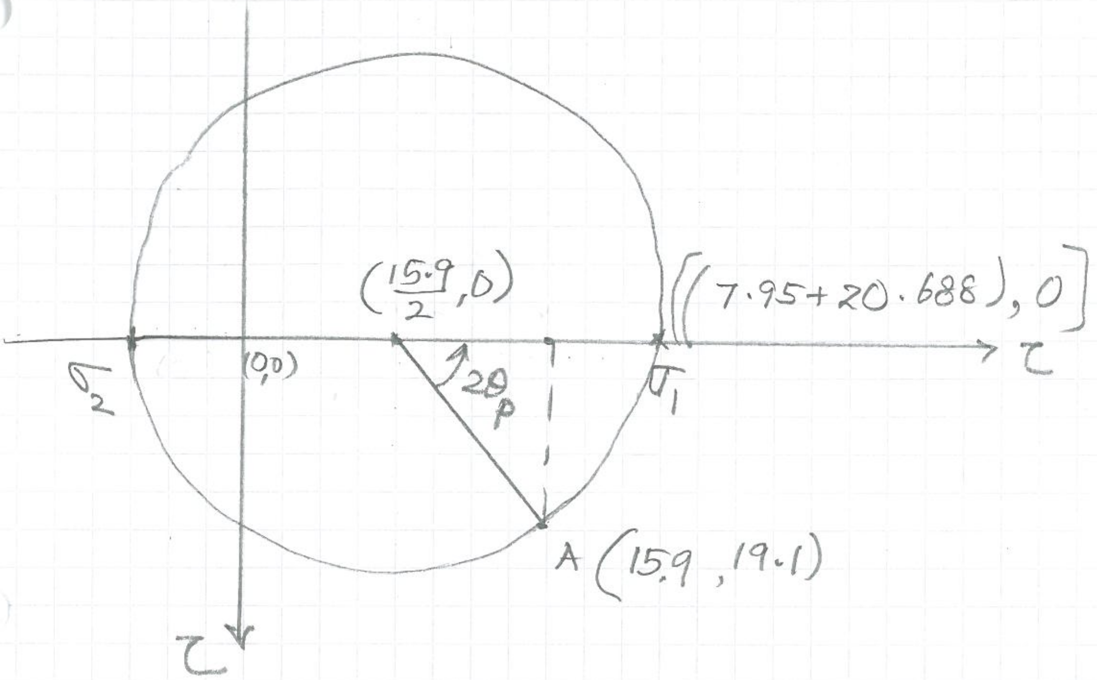


principle stresses by Mohr's circle.

$\sigma_x = 15.9$

$\sigma_y = 0$

$\tau_{xy} = 19.1$



radius of the Mohr's circle $= \sqrt{\left(15.9 - \frac{15.9}{2}\right)^2 + (19.1-0)^2}$

$= 20.688$

principle stresses.

$\sigma_1 = \frac{15.9}{2} + 20.688 = \underline{\underline{28.64 \text{ Mpa.}}}$

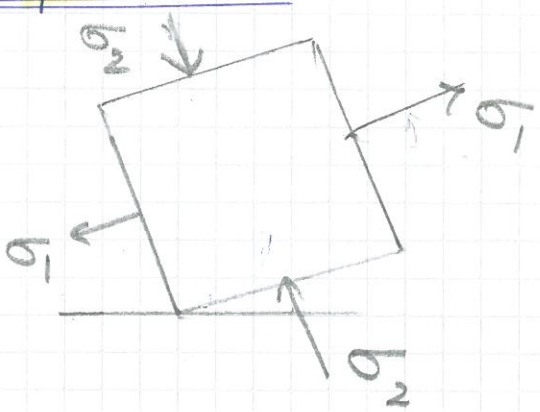
$\sigma_2 = -\left(20.688 - \frac{15.9}{2}\right) = \underline{\underline{-12.73 \text{ Mpa}}}$

Orientation

$\tan 2\theta_p = \frac{19.1}{(15.9/2)}$

$2\theta_p = \tan^{-1}(2.40) = 67.40^\circ$

$\underline{\theta_p = 33.70^\circ}$



(v) Absolute maximum shear shress

$$\tau_{abs\,max} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

$\sigma_{max} = 28.64$ MPa

$\sigma_{int} = 0$

$\sigma_{min} = -12.73$ MPa

$$= \frac{28.64 - (-12.73)}{2}$$

$$= 20.69 \text{ MPa.}$$

(vi) when ABC shaf is solid and applied torques and type of matirial are same,

G - same.

T_{AB} - same as part (i)

T_{BC} - same as part (i)

J_{BC} is same but J_{AB} changes

$J_{AB} \uparrow$ - increases.

angle of twist $\phi_{A/C}$

$$\phi_{A/C} = \frac{T_{AB}(x) L_{AB}}{J_{AB} G} + \frac{T_{BC}(x) L_{BC}}{J_{BC} G}$$

$$= \phi_{A/B} + \phi_{B/C}$$

$$= \uparrow + \text{same.}$$

but $\phi_{A/B} = (-)$ value. $\downarrow$

Then $\phi_{A/C} \uparrow$ increasing