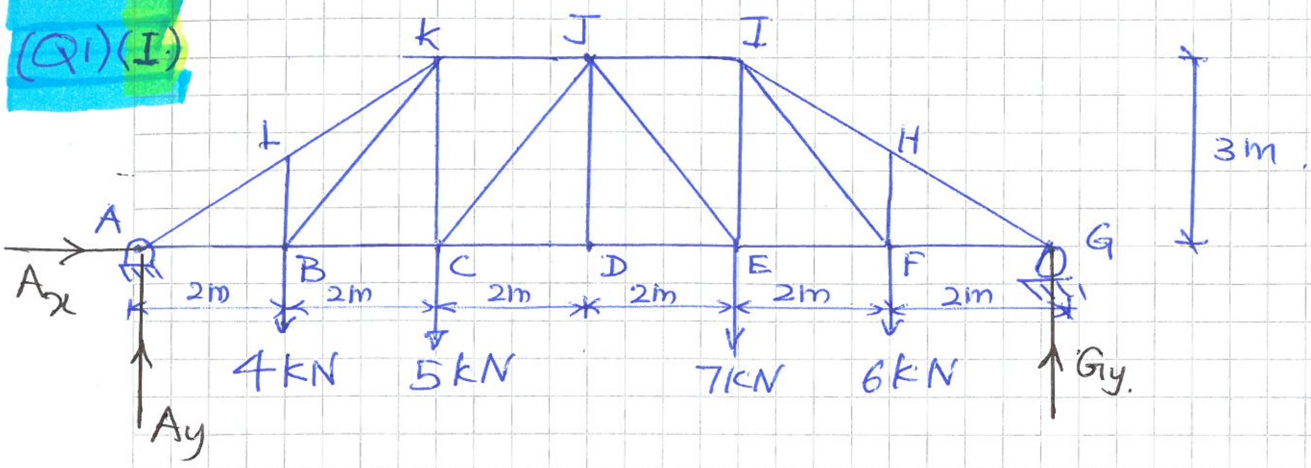
BYG 140 - Konstruksjonsmekanikk 1

May 19, 2014

(Q1)(I)



considering equilibrium of entire truss

$\rightarrow \sum F = A_x = 0$

$\uparrow \sum F = A_y + G_y - 22 = 0$

$A_y + G_y = 22$

$\curvearrowleft M_A = (G_y \times 12) - (6 \times 10) - (7 \times 8) - (5 \times 4) - (4 \times 2) = 0$

$G_y = 12 \text{ kN}$

$A_y = 22 - G_y$

$A_y = 10 \text{ kN}$

(II) Using the method of Joints.

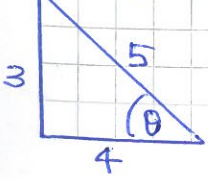
Joint G ;

considering equilibrium of the Joint G ;

$\uparrow F_{GH} \sin\theta + 12 = 0$

$F_{GH} = \dfrac{-12}{\sin\theta} = \dfrac{-12}{3/5}$

$F_{GH} = 20 \text{ kN}$ (compression)



$\sin\theta = \dfrac{3}{5}$

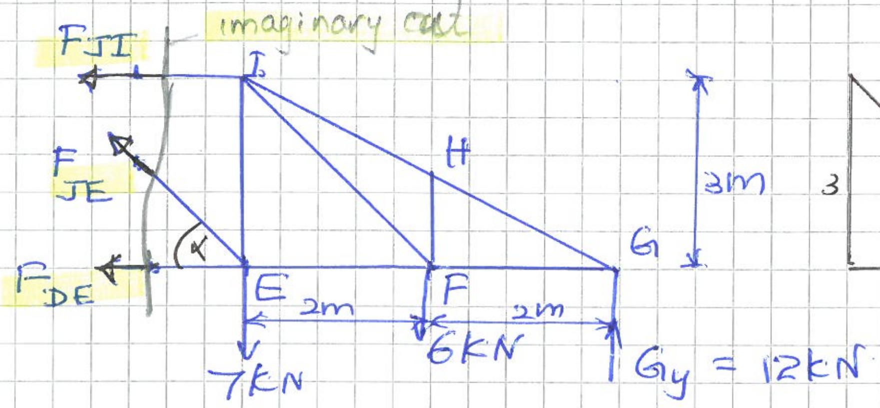
$\cos\theta = \dfrac{4}{5}$

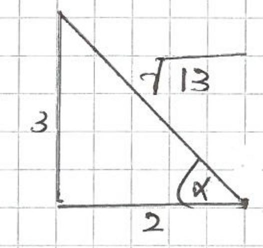
$\leftarrow \quad F_{GF} + F_{GH} \cos\theta = 0$

$$F_{GF} = -(-20) \times \frac{4}{5}$$

$$\underline{F_{GF} = 16\,kN \text{ (tension)}}$$

(III) Using the method of sections





$\sin\alpha = \frac{3}{\sqrt{13}}$

$\cos\alpha = \frac{2}{\sqrt{13}}$

Considering the equilibrium of the segment;

$\curvearrowleft M_E = (F_{JI} \times 3) - (6 \times 2) + (12 \times 4) = 0$

$$F_{JI} = -12\,kN$$

$$\underline{F_{JI} = 12\,kN \text{ (compression)}}$$

$\uparrow \Sigma F = F_{JE} \sin\alpha - 7 - 6 + 12 = 0$

$$F_{JE} = \frac{1}{3/\sqrt{13}} = 1.2\,kN$$

$$\underline{F_{JE} = 1.2\,kN \text{ (tension)}}$$

$\leftarrow \Sigma F = F_{DE} + F_{JI} + F_{JE} \cos\alpha = 0$

$$F_{DE} = 12 - \left(1.2 \times \frac{2}{\sqrt{13}}\right) kN$$

$$\underline{F_{DE} = 11.33\,kN \text{ (tension)}}$$

(IV) Zero-force members;

JD, LB, HF

(V) Cross sectional area

for member JI

$$F_{JI} = -12kN$$

allowable normal stress $\sigma_{all} \geq \frac{F_{JI}}{area}$

$$120 \times 10^6 \, Pa \geq \frac{12 \times 10^3 N}{area}$$

$$area \geq \frac{12 \times 10^3 N}{120 \times 10^6 N/m^2} = 100 \times 10^{-6} m^2$$

$$area \geq 100 \, mm^2$$

$$\underline{\underline{area \; JI = 100 mm^2}}$$

For member DE

$$F_{DE} = 11.33 \, kN$$

$$\sigma_{all} \geq \frac{F_{DE}}{area}$$

$$area \geq \frac{11.33 \times 10^3}{120 \times 10^6 N/m^2} = 94.4 \times 10^{-6} m^2$$

$$area \geq 94.4 mm^2$$

$$\underline{\underline{area \; DE = 95 mm^2}}$$

(vi) Change in length JI ; Δ

From Hooke's law

$$\frac{F}{A} = E\,\frac{\Delta}{L}$$

$F_{JI} = -12\,kN$

$L = 2m$

$A = 100mm^2$

$E = 200\,GPa.$

$$\Delta = \frac{FL}{AE}$$

$$\Delta = \frac{(-12\times10^{3})N \times 2m}{100\times10^{-6}\,m^2 \times 200\times10^{9}\,Pa} = -12\times10^{-4}\,m$$

Change in length JI:

$\Delta = 1.2mm.$ (Member is contracted.)

Change in length $\Delta = \dfrac{FL}{AE}$

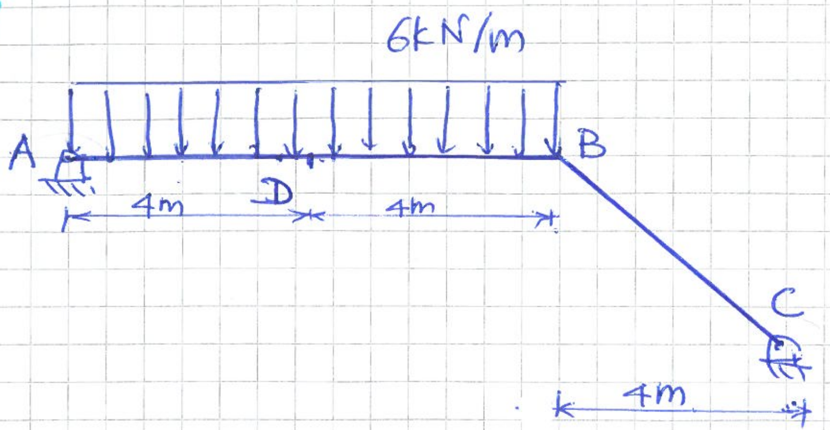
in this problem; F is constant
L is constant

Therefore, to decrease the change in length, A & E should be increased.

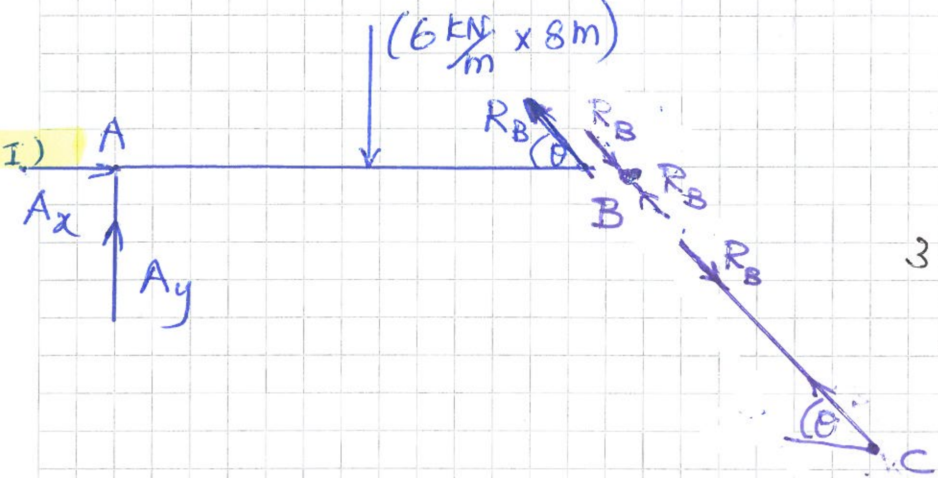
Suggestions;

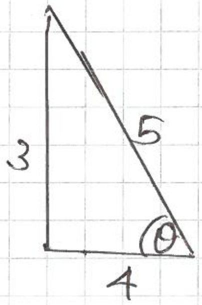
- increase the area of the cross section (A)
- Use another material with higher Modulus of Elasticity (E) than the used material.

(Q2)



(I)





$\cos\theta = \frac{4}{5}$

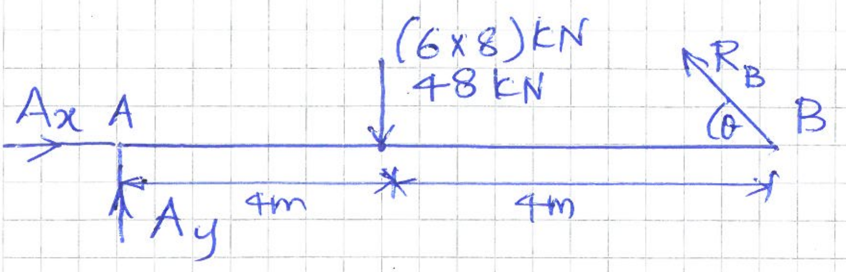
$\sin\theta = \frac{3}{5}$

BC member doesn't have any external forces except reaction of joint C & reaction of joint B.

Therefore BC is a two-force member.

- For the equilibrium of member BC, those two forces should be along the memeber

(II) Free body diagram for AB



Considering equilibrium of member AB

$$\circlearrowleft M_A = (R_B \sin\theta \times 8) - (48 \times 4) = 0$$

$$R_B = \frac{(48 \times 4)}{8 \times 3/5} = 40\,kN$$

$$\underline{\underline{R_B = 40\,kN}}$$

$$\rightarrow \Sigma F = A_x - R_B \cos\theta = 0$$

$$A_x = 40 \times \frac{4}{5} = 32\,kN$$

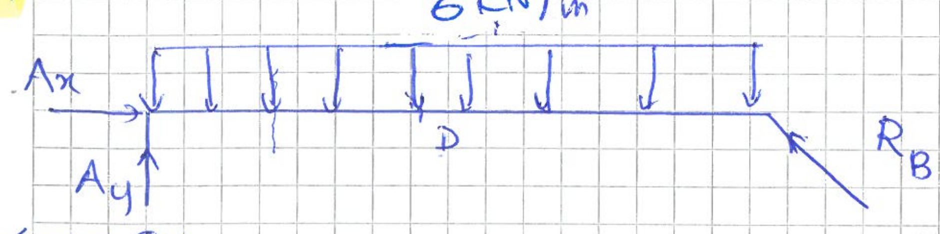
$$\underline{\underline{A_x = 32\,kN}}$$

$$\uparrow \Sigma F = A_y + R_B \sin\theta - 48 = 0$$

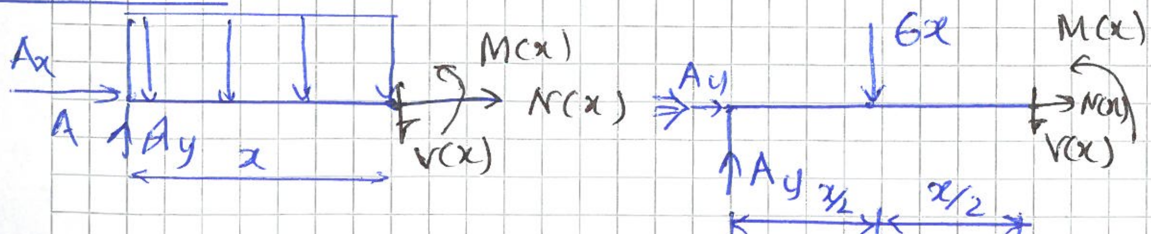
$$A_y = 48 - \left(40 \times \frac{3}{5}\right) = 24\,kN$$

$$\underline{\underline{A_y = 24\,kN}}$$

(III)



$\underline{0 \leq x \leq 8m}$



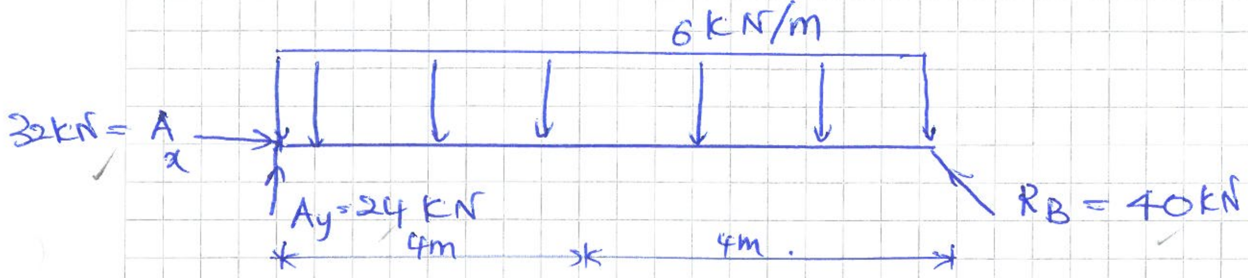
Considering equilibrium;

$\uparrow\Sigma F \quad A_y - 6x - V(x) = 0$

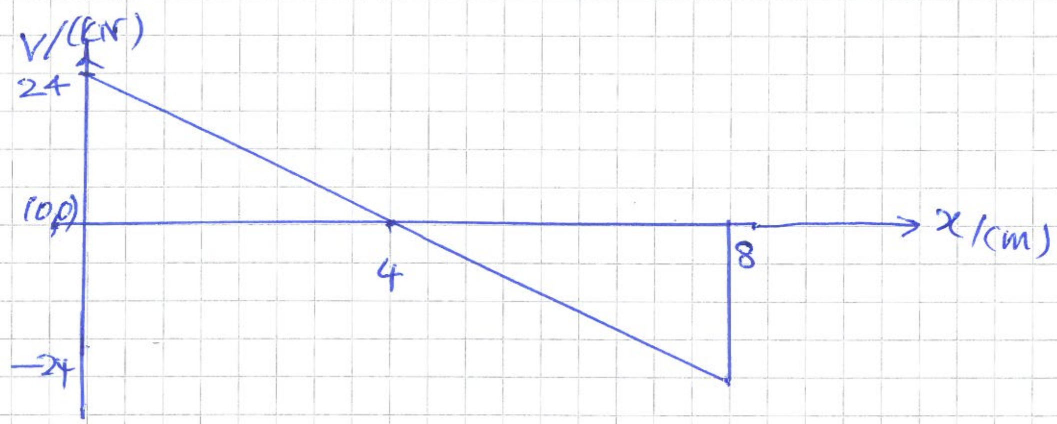
$$V(x) = 24 - 6x.$$

$\curvearrowleft M \quad -(A_y \times x) + \left(6x \times \frac{x}{2}\right) + M(x) = 0$

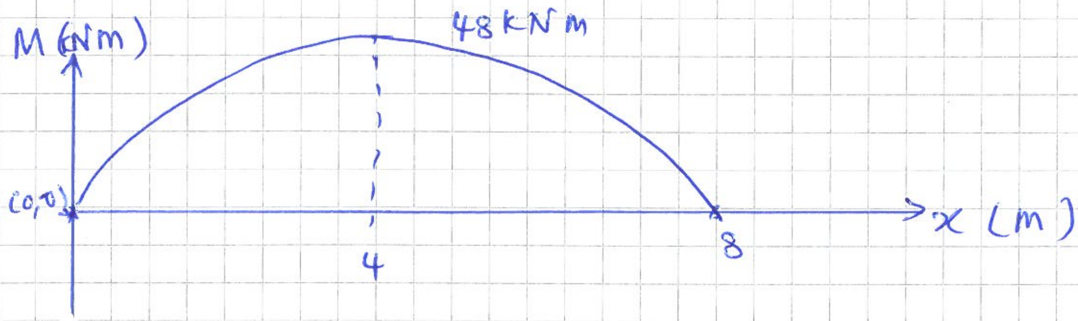
$$M(x) = 24x - 3x^2.$$



SFD



BMD



(iv) Cross sectional area (A)

$$A = (2 \times 15 \times 100) + (7.5 \times 230) \text{ mm}^2$$

$$A = 4725 \text{ mm}^2$$

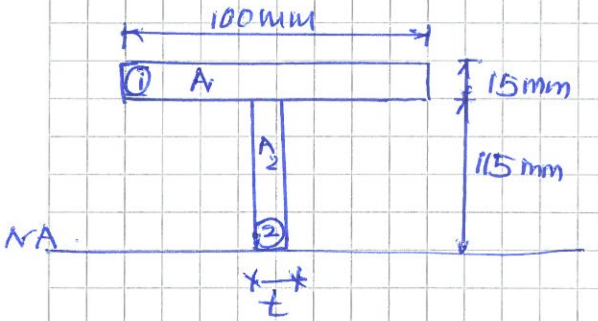
moment of inertia (I_x)

$$I_x = \left(\frac{1}{12} \times 100 \times 260^3\right) - \left(\frac{1}{12} \times (100 - 7.5) \times 230^3\right) \text{ mm}^4$$

$$I_x = 52679375 \text{ mm}^4$$

$$I_x = 52.7 \times 10^6 \text{ mm}^4$$

(v) maximum shear stress (τ_{max})



shear stress τ;

$$\tau = \frac{VQ}{It}$$

$$\tau_{max} = \frac{V_{max} Q_{max}}{It}$$

$I = 52.7 \times 10^6 \text{ mm}^4$

$t = 7.5 \text{ mm}$

$V_{max} = 24 \times 10^3 \text{ N}$ (from SFD)

$$Q_{max} = \sum A' \bar{y}' = A_1 \bar{y}_1' + A_2 \bar{y}_2'$$

$$= \left(15 \times 100 \times (115 + 7.5)\right) + \left(115 \times 7.5 \times \frac{115}{2}\right)$$

$$= 233343.8 \text{ mm}^3$$

The $\tau_{max} = \dfrac{V_{max} Q_{max}}{It} = \dfrac{24 \times 10^3 \text{ N} \times 233343.8 \text{ mm}^3}{52.7 \times 10^6 \text{ mm}^4 \times 7.5 \text{ mm}}$

$$\tau_{max} = 14.174 \text{ N/mm}^2$$

Maximum normal Stress (σ)

Normal stress due to axial force.

$$\sigma_N = \frac{P}{A} = \frac{-32\times10^{3}\ N}{4725\times10^{-6}\ m^2}$$

$$= -6.77\ MPa$$

Normal stress due to bending

$$\sigma_M = -\frac{My}{I} = \frac{-48\times10^{3}\ Nm \times 130\times10^{-3}\ m}{52.7\times10^{6}\times10^{-12}\ m^4}$$

$$= -118.41\ MPa$$

Maximum normal stress

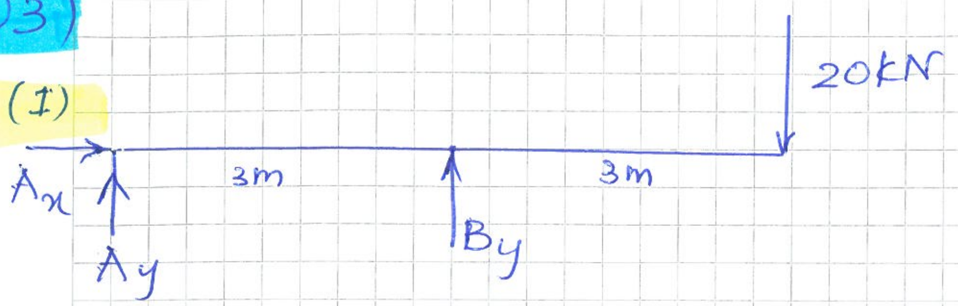
$$\sigma = \sigma_N + \sigma_M = -(6.77 + 118.41)\ MPa$$

$$\underline{\sigma_N = -125.18\ MPa}$$

(VI) When A becomes a fixed support, Axial force of member BC will decrease as fixed support can carry bending moment.

(Q3)

(I)



considering equilibrius of the beam

$\rightarrow \sum F = A_x = 0$

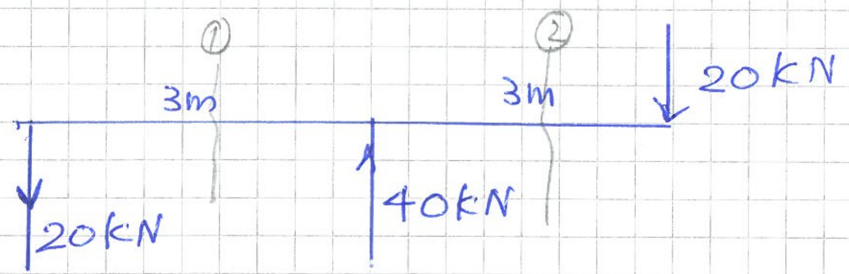
$\uparrow \sum F = A_y + B_y - 20 = 0$

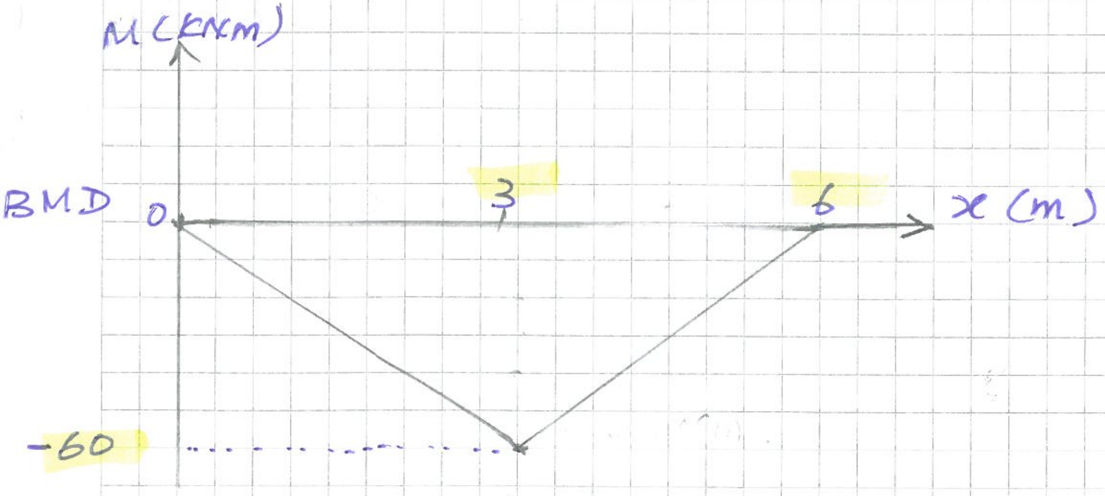
$\curvearrowright M_A = 3B_y - (20 \times 6) = 0$

$B_y = 40\,kN$

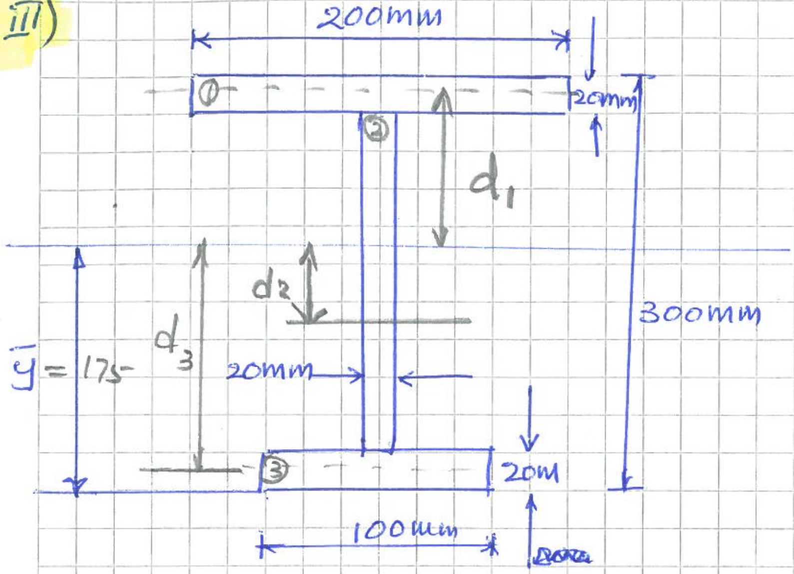
$A_y = -20\,kN$

(II)





(II)



$\bar{y}$ - location of neutral axis of c/s

I - moment of inertia of the c/s

$$\bar{y} = \frac{\sum A_i \bar{y}_i}{\sum A_i} = \frac{A_1 \bar{y}_1 + A_2 \bar{y}_2 + A_3 \bar{y}_3}{A_1 + A_2 + A_3}$$

$$= \frac{(200\times20)(300-10) + (260\times20)(150) + (100\times20)\times(10)\ mm^3}{(200\times20) + (260\times20) + (100\times20)\ mm^2}$$

$$\bar{y} = 175\,mm$$

Moment of inertia I

$$I = (I_1 + A_1 d_1^2) + (I_2 + A_2 d_2^2) + (I_3 + A_3 d_3^2)$$

$A_1 = (200\times20)\ mm^2$

$d_1 = (290-175)\ mm$

$I_1 = \left(\frac{1}{12}\times200\times20^3\right) mm^4 = 133333.3\ mm^4$

$A_2 = (260\times20)\ mm^2$

$d_2 = (175-150)\ mm$

$I_2 = \left(\frac{1}{12}\times20\times260^3\right) mm^4 = 29293333\ mm^4$

$A_3 = (100\times20)\ mm^2$

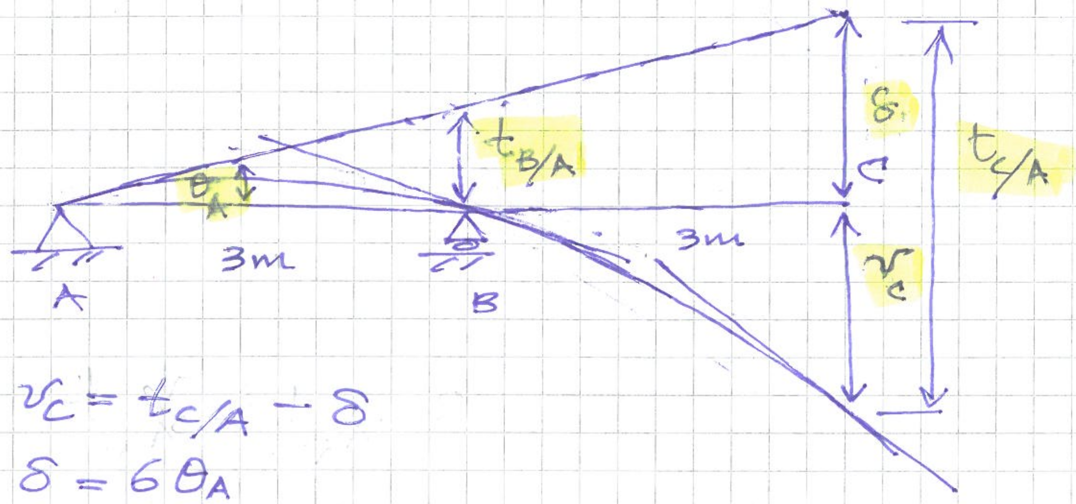
$d_3 = (175-10)\ mm$

$I_3 = \left(\frac{1}{12}\times100\times20^3\right) mm^4 = 6666.67\ mm^4$

$$I = \left[\left(\frac{1}{12}\times200\times20^3\right) + (200\times20)(290-175)^2\right] + \left[\left(\frac{1}{12}\times20\times260^3\right) + (260\times20)(175-150)^2\right]$$
$$+ \left[\left(\frac{1}{12}\times100\times20^3\right) + (100\times20)(175-10)^2\right] = 1.401\times10^8\ mm^4$$

(iv) Deflection at end C (v_C)

Deflected shape of the beam



$v_C = t_{C/A} - \delta$

$\delta = 6\theta_A$

$\theta_A = \dfrac{t_{B/A}}{3}$

$\therefore v_C = t_{C/A} - \dfrac{6\,t_{B/A}}{3} = t_{C/A} - 2t_{B/A}$

From moment area theorem ②;

$t_{C/A} = 3m \times \left(\frac{1}{2} \times 6m \times \frac{60\,kNm}{EI}\right) = \frac{540}{EI}\,kNm^3$

$t_{B/A} = \frac{1}{3} \times 3m \times \left(\frac{1}{2} \times 3m \times \frac{60\,kNm}{EI}\right) = \frac{90}{EI}\,kNm^3$

Then

$$v_C = \left(\frac{540}{EI} - 2\times\frac{90}{EI}\right) = \frac{360}{EI}$$

$$= \frac{360 \times 10^3\ Nm^3}{200 \times 10^9 \frac{N}{m^2} \times 1.401 \times 10^8 \times 10^{-12}\,m^4}$$

$= 0.01285m$

$\underline{\underline{v_C = 12.85\,mm}}$

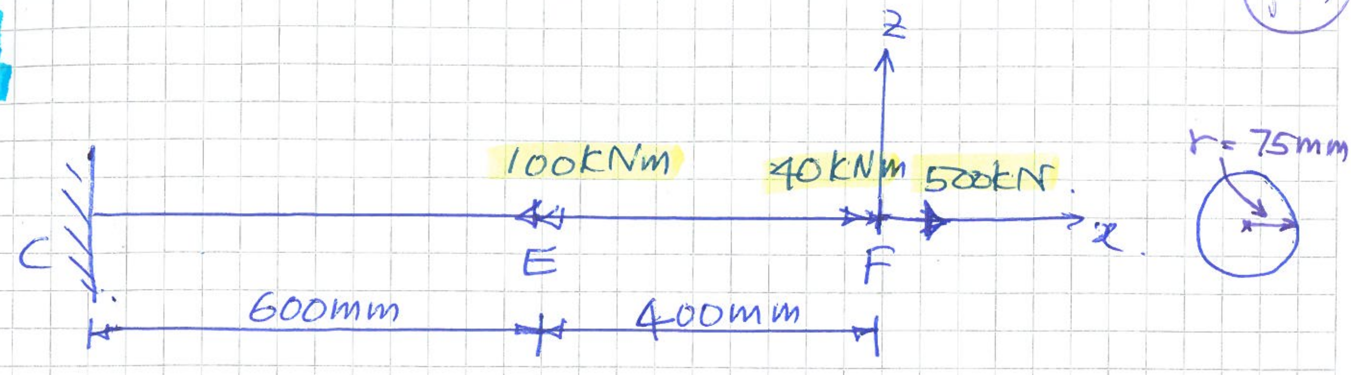
(v) To decrease deflection at end C

- increase second moment of inertia of the cross section.
- Use a material with higher modulus of elasticity.

$$v_c \propto \frac{1}{EI}$$

if $EI \uparrow$ then. $v_c \downarrow$

(Q4)



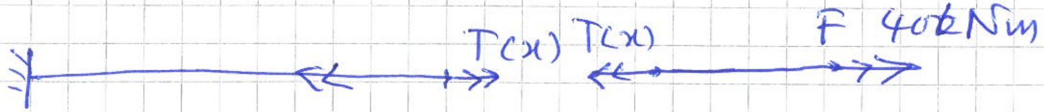
(i) Angle of twist at F

$$\phi_{F/C} = \phi_{F/E} + \phi_{E/C}.$$

$$= \frac{T_{EF} L_{EF}}{JG} + \frac{T_{EC} C_{EC}}{JE.}$$

$$T(x) + 100 - 40 = 0$$

$$T_{CE} = T(x) = -60\,kNm.$$



$$T(x) - 40 = 0$$

$$T_{EF} = T(x) = 40\,kNm.$$

Then

$$\phi_{F/C} = \frac{40 \times 10^{3} Nm \times 0{\cdot}4\,m}{JG} + \frac{(-60 \times 10^{3} Nm) \times 0{\cdot}6}{JG}$$

$$\phi_{F/C} = \frac{\left[(40 \times 0{\cdot}4) - (60 \times 0{\cdot}6)\right] \times 10^{3}}{JG}$$

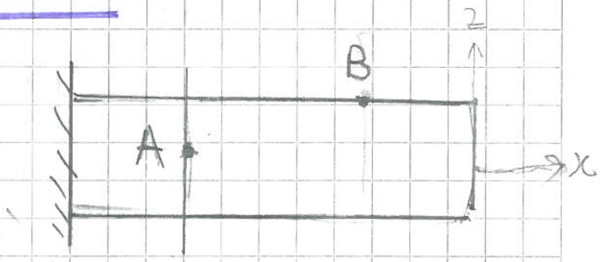
polar moment of inertia

$$J = \frac{\pi}{2} \times (75 \times 10^{-3})^4$$

$$G = 75 \times 10^9 \ N/m^2 \ (Pa)$$

Then
$$\phi_{F/C} = \frac{-20 \times 10^3}{\frac{\pi}{2}(75 \times 10^{-3})^4 \ m^4 \times 75 \times 10^9 \ N/m^2}$$

$$\phi_F = \phi_{F/C} = -0.0054 = -0.31^\circ$$

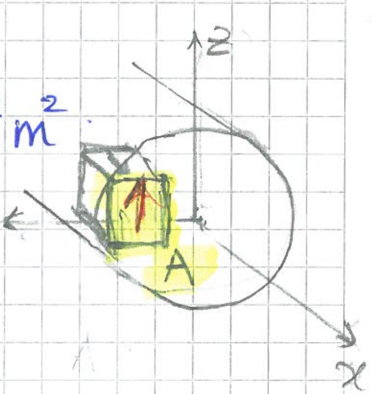


(II) stress component at A

Normal stress due to axial force }
$$\sigma_x = \frac{500 \times 10^3 \ N}{\pi \times (75 \times 10^{-3})^2 \ m^2}$$

$$\sigma_x = 0.028 \times 10^9 \ Pa$$

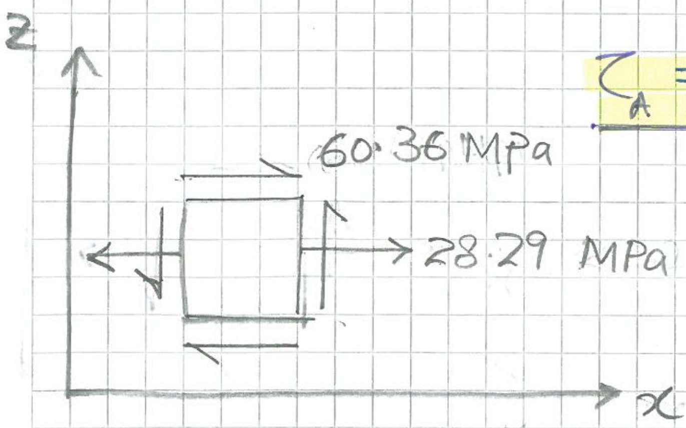
$$\sigma_x = 28.29 \ MPa$$



Shear stress due to torque }
$$\tau_A = \frac{T\rho}{J}$$

$$= \frac{60 \times 10^3 \times 50 \times 10^{-3}}{\frac{\pi}{2} \times (75 \times 10^{-3})^4}$$

$$\tau_A = 60.36 \ MPa$$

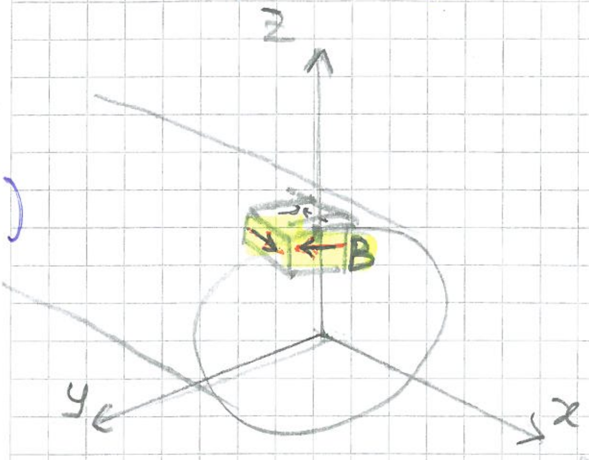


At point B

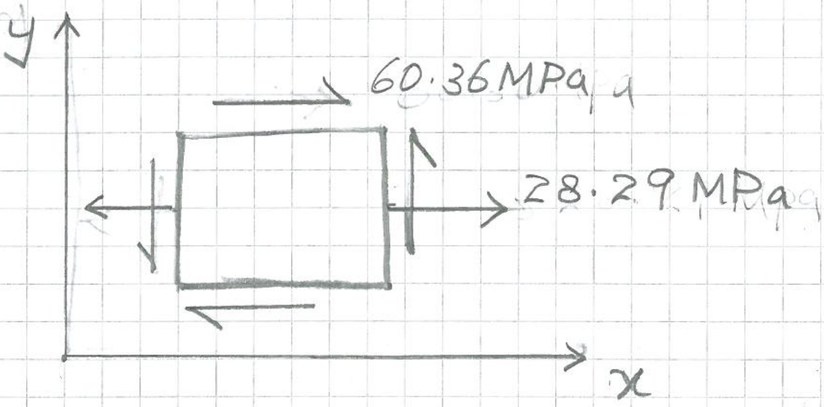
shear stress at B due to torque $\Big\} = \dfrac{T\rho}{J}$

$$= \frac{40\times10^{3} \times 75\times10^{-3}}{\frac{\pi}{2}\times(75\times10^{-3})^{4}}$$

(iii)



$$= 60.36 \text{ MPa}$$



(iv) Principle stress at point B

$\sigma_x = 28.29$ MPa

$\sigma_y = 0$

$\tau_{xy} = 60.36$ MPa

$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

$$\sigma_{1,2} = \left(\frac{28.29 + 0}{2}\right) \pm \sqrt{\left(\frac{28.29 - 0}{2}\right)^2 + (60.36)^2}$$

$$= 14.145 \pm 61.995$$

$$\sigma_1 = 76.14\ MPa$$

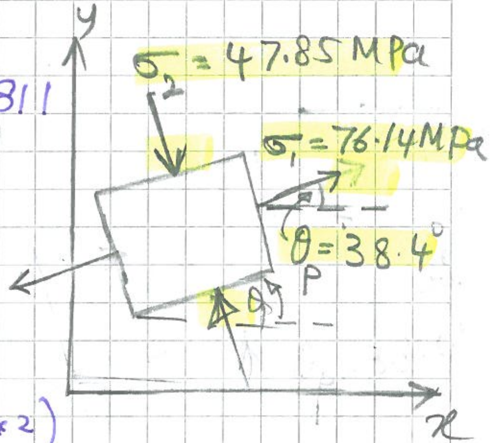
$$\sigma_2 = -47.85\ Mpa$$

Orientation of principle axis;

$$\tan 2\theta_p = \frac{\tau_{xy}}{\left(\frac{\sigma_x - \sigma_y}{2}\right)} = \frac{60.36}{\left(\frac{28.29 - 0}{2}\right)}$$

$$2\theta_p = \tan^{-1}(4.27) = 76.811$$

$$\theta_p = 38.4^\circ$$



check; when $\theta_p = 38.4^\circ$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{28.29}{2} + \frac{28.29}{2}\cos(2 \times 38.4) + 60.36\sin(38.4^\circ \times 2)$$

$$\sigma_{x'} = 76.14$$

(V.) Maximum in plane shear stress.

$$\tau_{max\ in\ plane} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{28.29 - 0}{2}\right)^2 + (60.36)^2}$$

$$\tau_{max\ in\ plane} = 61.995\ Mpa$$

Principal stresses are 76.14 MPa, 0, −47.85 MPa

$$\sigma_{max} = 76.14\ MPa$$

$$\sigma_{int} = 0$$

$$\sigma_{min} = -47.85\ MPa.$$

Absolute maximum shear stress

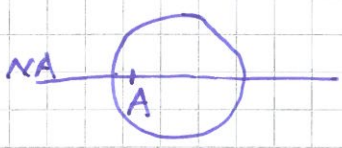
$$\tau_{abs\,max} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{76.14 - (-47.85)}{2}$$

$$\underline{\underline{\tau_{abs\,max} = 61.995 \text{ MPa}}}$$

(vi) If the shaft is subjected to vertical downward load;

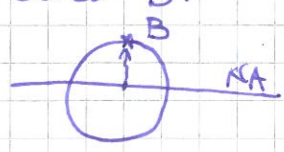
At point 'A'

- Shear stress (τ_A) - decrease.
  As bending induced shear stress acting downward direction
  $$\downarrow \tau_{bending} = \left(\frac{VQ}{It}\right)$$
- Normal stress (σ_x) - No change
  As bending induced normal stress is zero at neutral axis



At point 'B'

- shear stress (τ_B) - No change
  As bending induced shear stress is zero at point B.



$$Q_B = 0 \qquad \tau_{bending} = \frac{VQ_B}{It}$$

- Normal stress (σ_x) - increase
  as bending increases the normal stress.
  $\sigma = \frac{M}{I} y_{max}$ $\quad (y - y_{max}$ at B)