Universitetet i Stavanger

Det teknisk- naturvitenskapelige fakultet

SUBJECT: **BYG 140 KONSTRUKSJONSMEKANIKK 1**

DATE: September 13, 2016

TIME: 09:00 – 13:00 (4 hours)

AID: Authorized calculator, Dictionary (English-Norwegian) and drawing instruments.

THE EXAM CONSISTS OF <u>4 QUESTIONS</u> AND <u>12 PAGES</u> (including the front page)
Norwegian translation of each question is attached

REMARKS: All the **Four** questions carry **equal marks** and answer **all** the questions.

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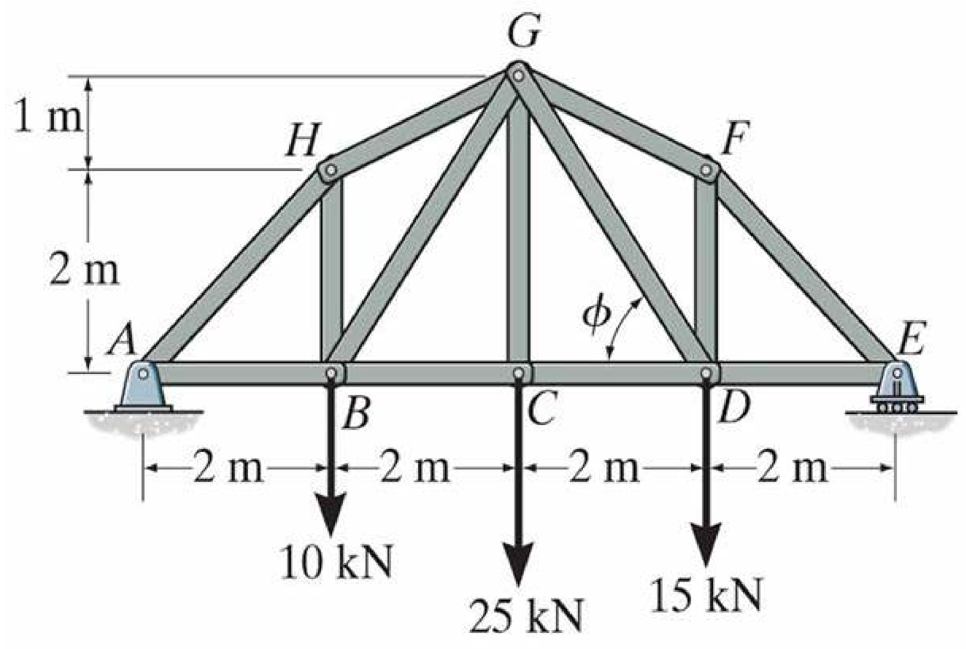
QUESTION (1): (25 %)

A truss subjected to vertical forces at joints *B*, *C* and *D* as shown in Figure 1. The truss is supported by a pin support at *A* and a roller support at *E*. The truss members are connected at each connection using pin joints which cannot transfer moments. The truss members are made up of steel which has Modulus of Elasticity (E) 200GPa.



(All forces are in kilonewtons and lengths are in meters)

Figure 1: Truss structure

I. Determine the support reaction forces at *A* and *E*. **(2 points)**

II. Determine the axial forces of the members *AB* and *AH* using the method of joints. Clearly state whether the members are in compression or tension. **(4 points)**

III. Determine the axial forces of the members *GF*, *GD* and *CD* using the method of sections. Clearly state whether the members are in compression or tension. **(8 points)**

IV. Are there zero-force members of the truss? If available, list the zero-force members of the truss without any calculation. **(2 points)**

V. Determine the required cross-sectional areas for the member *GC* if the allowable normal stress of the material is 250 MPa. **(4 points)**

VI. Determine the change in length (i.e. axial displacement) of the member *GC*. State whether the member is elongated or contracted. **(3 points)**

VII. If the member *FD* is removed from the truss, what will happen to the overall structure? State the reason/s for your answer in engineering viewpoint. **(2 points)**

OPPGAVE (1): (25 %)

Et fagverk er belastet med krefter ved forbindelsene *B*, *C* og *D*, som vist i Figure 1. Fagverket er lagret opp med et boltelager ved *A* og et rullelager ved *E*. Stavene i fagverket er satt sammen med bolter i hvert av forbindelsesleddene, som ikke kan overføre momenter. Stavene er av stål, som har elastisitetsmodul (*E*) på 200 GPa.

I. Bestem opplagerkreftene ved A og E. **(2 poeng)**

II. Bestem aksialkreftene for stavene *AB* og *AH* vha. knutepunktsmetoden. Angi tydelig hvorvidt stavene er i trykk eller strekk. **(4 poeng)**

III. Bestem aksialkreftene for stavene *GF*, *GD* og *CD* vha. snittmetoden. Angi tydelig hvorvidt stavene er i trykk eller strekk. **(8 poeng)**

IV. Er der nullkraft-staver i fagverket? Dersom slike finnes, list opp nullkraft-stavene i fagverket uten noen beregninger. **(2 poeng)**

V. Bestem det nødvendige tverrsnittsarealet for stav GC dersom tillatt normalspenning for materialet er 250 MPa **(4 poeng)**

VI. Bestem lengdeendring (i.e. aksiell forskyvning) for staven *GC*. Angi hvorvidt staven er forlenget eller forkortet. **(3 poeng)**

VII. Dersom stav FD fjernes fra fagverket, hva vil da skje med konstruksjonen? Begrunn svaret ut ifra en ingeniørs synspunkt. **(2 poeng)**

QUESTION (2): (25%)

ABCD structure carries a concentrated load of 400 N and a moment of 800 Nm as shown in the Figure 2(a). The structure *ABCD* is supported by pin supports at *A* and *D*. The frame *BCD* is connected to the member *AB* by using a pin joint at *B*. *Hint*: Pin joints transfer zero moments. The cross-section of the frame *BCD* is as shown in Figure 2(b) and the frame *BCD* is bending about *z-z* axis.

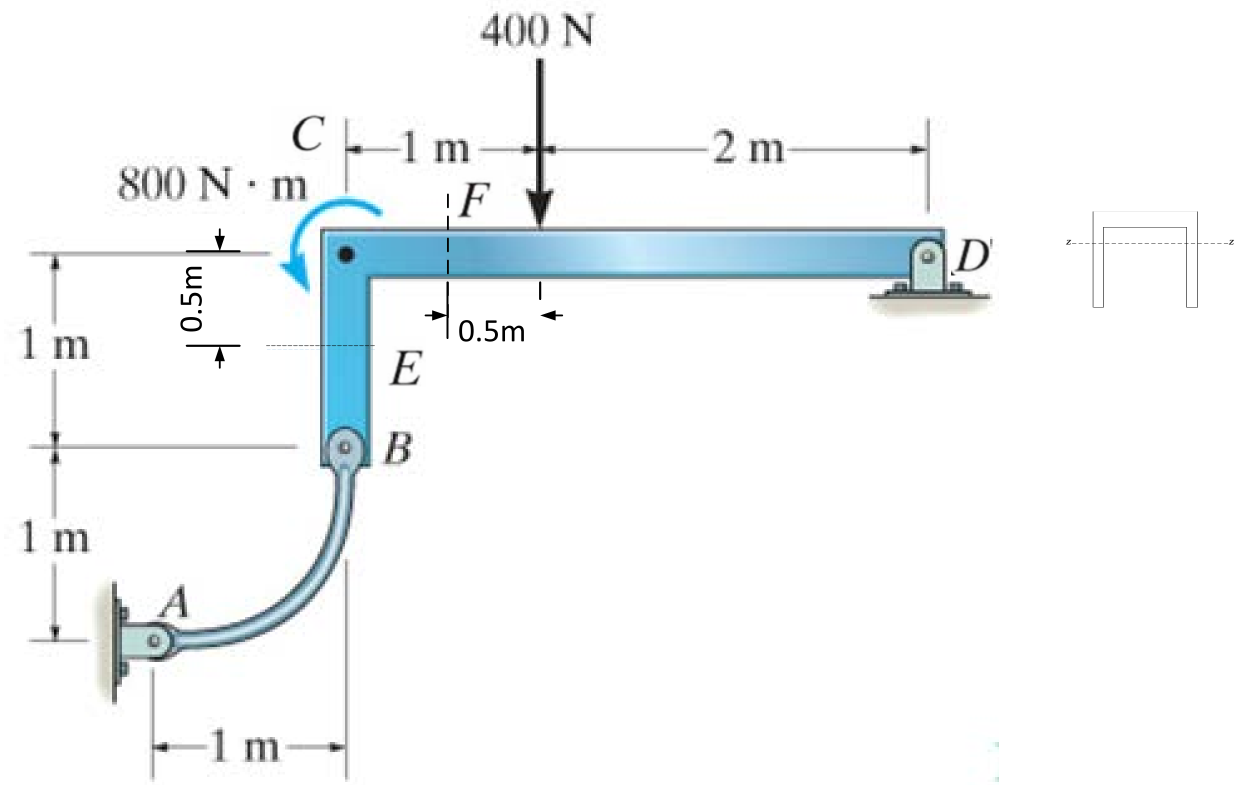


Figure 2(a): *ABCD* Structure

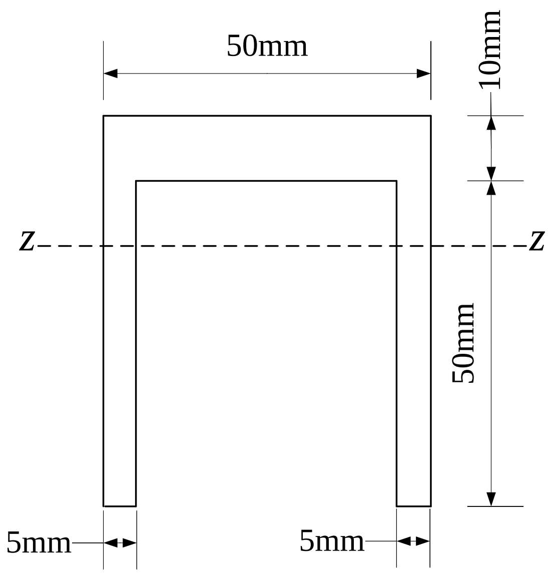


Figure 2(b): Cross section of the frame *BCD*

I. Draw a free-body diagram of the structure *ABCD* and determine the support reactions of the structure. **(6 points)**

II. Determine the internal normal axial forces, shear forces and bending moments at *E* and *F* cross sections. **(6 points)**

III. Determine cross sectional area and moment of inertia about neutral axis of the cross-section shown in Figure 2(b). **(4 points)**

IV. Determine the maximum shear stresses and the normal stresses of the cross sections at points *E* and *F*. **(6 points)**

V. If the concentrated load (i.e. 400 N) is removed, and magnitude of other loading, supports and the cross-section of the *BCD* frame remain same, what change can be seen of the magnitude of the maximum shear stresses and the normal stresses of the cross sections at the points *E* and *F*? State logical reason/s for your answer without calculations. (**3 points)**

OPPGAVE (2): (25%)

ABCD-strukturen utsettes for en konsentrert last på 400 N og et moment på 800 Nm, som vist i Figure 2(a). Strukturen ABCD er lagret opp med boltelagre ved A og D. Rammen BCD er forbundet med staven AB vha. et boltelager ved B. Hint: Boltelagre kan ikke overføre moment. Tverrsnittet for rammen BCD er som vist i Figure 2(b) og rammen BCD bøyes om z-z aksen

I. Tegn et fritt legeme-diagram for strukturen ABCD og bestem opplagerreaksjonene for strukturen. **(6 poeng)**

II. Bestem de interne normalkrefter, skjærkrefter og bøyemomenter ved E- og F tverrsnittene **(6 poeng)**

III. Bestem tverrsnittsarealet og treghetsmomentet om nøytralaksen for tverrsnittet for bjelken vist i Figure 2(b). **(4 poeng)**

IV. Bestem maksimale skjærspenninger, og normalspenninger for tverrsnittene ved punkt E og F. **(6 poeng)**

V. Dersom den konsentrerte lasten (i.e. 400 N) fjernes, mens størrelsen på de andre lastene, opplagringene, samt tverrsnittet for BCD-rammen forblir de samme, hvilken endring ser man i størrelsen på maksimale skjærspenninger og normalspenninger over tverrsnittene ved punkt *E* og *F*? Angi logiske resonnementer for svaret, uten beregninger. **(3 poeng)**

QUESTION (3): (25%)

Figure 3 (a) shows a cantilever beam, fixed at point A and free at point C. The total length of the beam is 4 m and it is subjected to a concentrated moment of 10 kN at point C and an uniform load between B and C, with a value of 5 kN/m. The beam's material has a modulus of Elasticity (E) of 25 GPa and its cross section is rectangular with a hole in its middle, as shown in Figure 3 (b). Assume that bending of this beam acts along the z-z axis.

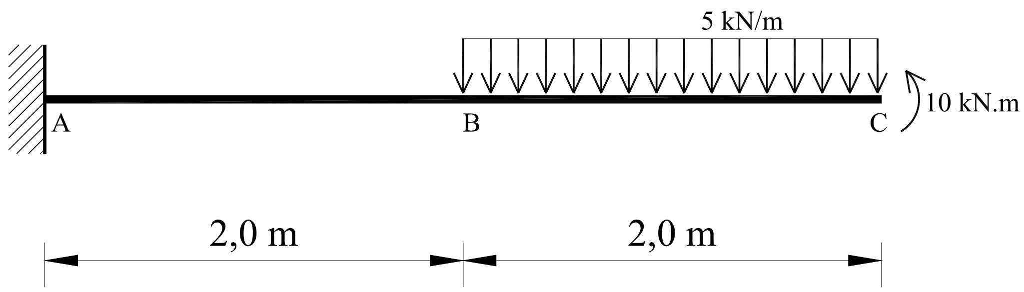


Figure 3 (a) – Simply supported beam (dimensions in m)

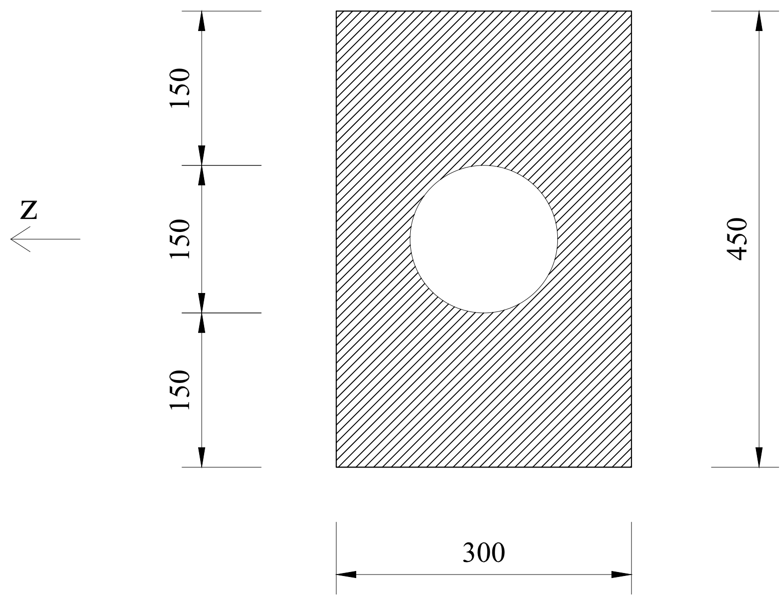


Figure 3 (b) – Beam´s cross section (dimensions in mm)

I. Determine the support reactions of the beam and draw the corresponding free body diagram **(4 points)**

II. Draw the shear force and bending moment diagrams. **(4 points)**

III. Determine the moment of inertia of the cross section along z-z **(4 points)**

IV. Determine the vertical displacement of the beam at point C **(6 points)**

V. Determine the rotation at point C **(4 points)**

VI. If the cross-section of the beam was a rectangle with the same dimensions and did not have the hole in the middle, would the magnitude of the vertical displacement increase, decrease, or remain the same. State logical reason/s for your answer without calculations. **(3 points)**

OPPGAVE (3): (25%)

Figure 3(a) viser en fritt opplagret bjelke, fast montert ved punkt *A*, og fritt hengende ved punkt *C*. Bjelkens totale lengde er 4 m, og den er utsatt for et konsentrert moment på 10 kNm ved punkt C, samt en uniformt fordelt last mellom B og C på 5 kN/m. Bjelkens materiale har en elastisitetsmodul (*E*) på 25 GPa, og tverrsnittet er rektangulært med et hull i midten, som vist i Figure 3(b). Anta at bjelkens bøying foregår om *z-z* aksen.

I. Bestem opplagerreaksjonene for bjelken og tegn det korresponderende fritt legemediagram-met. **(4 poeng)**

II. Tegn skjærkraft- og bøyemomentdiagrammene. **(4 poeng)**

III. Bestem treghetsmomentet for tverrsnittet, om *z-z*. **(4 poeng)**

IV. Bestem den vertikale nedbøyningen ved punkt C. **(6 poeng)**

V. Bestem bjelkens rotasjon ved punkt *C*. **(4 poeng)**

VI. Dersom tverrsnittet for bjelken var et rektangel med de samme dimensjonene, men uten hullet i midten, ville da størrelsen på den vertikale deformasjonen minke, øke, eller forbli den samme? Angi logiske resonnementer for svaret, uten beregninger. **(3 poeng)**

QUESTION (4): (25%)

The box wrench rod represented in Figure 4 (a) has a radius of 20 mm, is subjected to a load of 350 N and is under equilibrium. Consider that the material has a modulus of elasticity (E) of 200 GPa and a Poisson coefficient of ν=0.30.

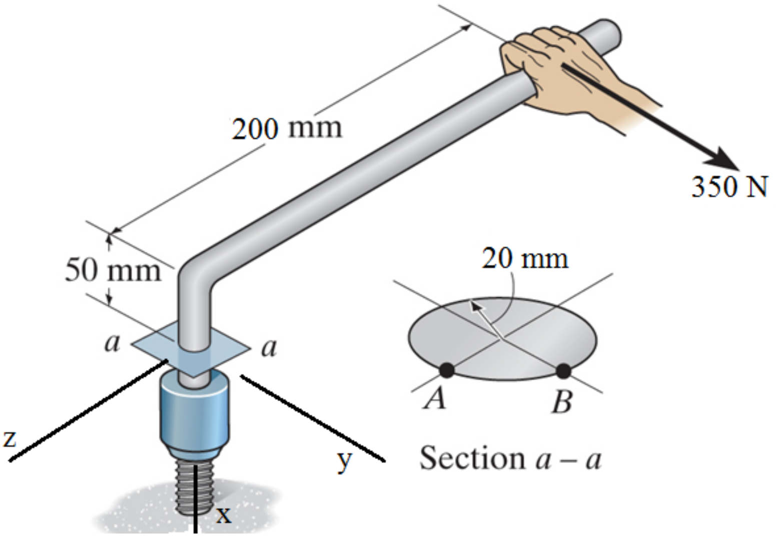


Figure 4 (a) – Metallic wrench subjected to 1 force load

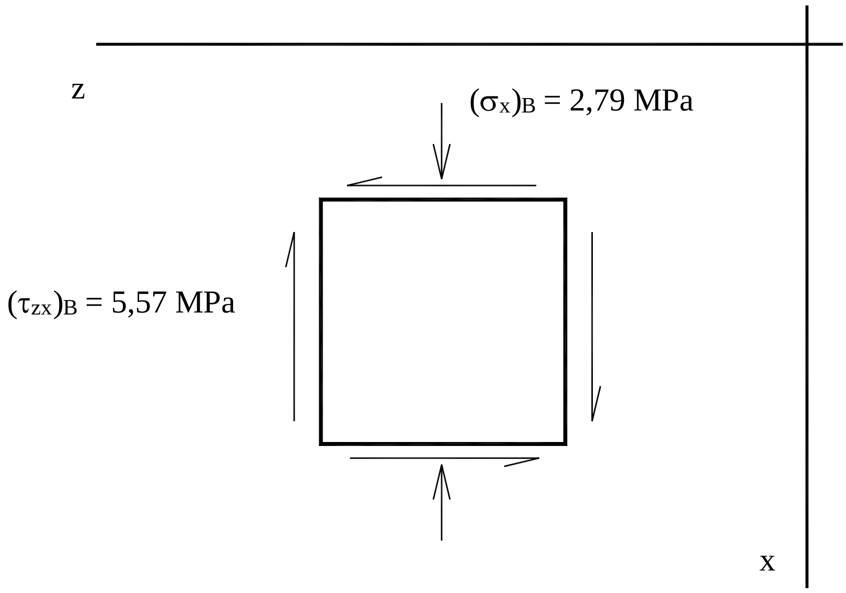


Figure 4 (b) – State of stress at Point B

I. Determine the internal forces (forces and moments) acting in the section **a-a** of the wrench. **(6 points)**

II. Prove that the state of stress at point B can be represented as shown in Figure 4 (b). **(6 points)**

III. Determine the principle stresses at point B and draw the corresponding orientation, relative to the axis represented. **(5 points)**

IV. Determine the absolute maximum shear stress at point B **(3 points)**

V. Calculate the normal and shear strains at point B considering the generalized Hooke's law. **(2 points)**

VI. If the wrench's section was smaller, while all other conditions would remain the same, to what magnitude (increase, decrease or no change) would the stresses change in the a-a section. State the logical reasons for your answer without calculations. **(3 points)**

OPPGAVE (4): (25%)

Armen på fastnøkkelen representert i Figure 4(a) har en radius på 20 mm, er utsatt for en last på 350 N, og er i likevekt. Anta at materialet har en elastisitetsmodul (E) på 200 GPa og en Poissonkoeffisient på $\nu = 0.30$.

I. Bestem de interne kreftene (kreftene og bøyemomenter) som virker i snittet ***a-a*** på fastnøkkelen. **(6 poeng)**

II. Vis at spenningstilstanden ved punkt *B* kan representeres som vist i Figure 4(b). **(6 poeng)**

III. Bestem hovedspenningene ved punkt *B*, og tegn den korresponderende orienteringen, relativt til aksene som er representert. **(5 poeng)**

IV. Bestem absolutt maksimal skjærspenning ved punkt *B*. **(3 poeng)**

V. Beregn normal og skjærtøyningene ved punkt *B*, tatt i betraktning den generaliserte Hooke's lov. **(2 poeng)**

VI. Dersom fastnøkkelens tverrsnitt var mindre, mens alle andre forhold var forblitt de samme, til hvilken størrelse (økning, reduksjon, eller ingen endring) ville spenningene endres i *a-a* snittet. Angi logiske resonnementer for svaret, uten beregninger. **(3 poeng)**

Fundamental Equations of Mechanics of Materials

Axial Load

Normal Stress

$$\sigma = \frac{P}{A}$$

Displacement

$$\delta = \int_0^L \frac{P(x)dx}{A(x)E}$$

$$\delta = \Sigma \frac{PL}{AE}$$

$$\delta_T = \alpha \Delta T L$$

Bending

Normal stress

$$\sigma = \frac{-My}{I}$$

Unsymmetric bending

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}, \qquad \tan\alpha = \frac{I_z}{I_y}\tan\theta$$

Section modulus

$$S_{\text{req'd}} = \frac{M_{\text{max}}}{\sigma_{\text{allow}}}$$

Shear

Average direct shear stress

$$\tau_{\text{avg}} = \frac{V}{A}$$

Transverse shear stress

$$\tau = \frac{VQ}{It}$$

Shear flow

$$q = \tau t = \frac{VQ}{I}$$

Relations Between w, V, M

$$\frac{dV}{dx} = -w(x), \qquad \frac{dM}{dx} = V$$

Elastic Curve

$$\frac{1}{\rho} = \frac{M}{EI}$$

$$EI\frac{d^4v}{dx^4} = -w(x)$$

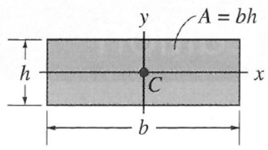
$$EI\frac{d^3v}{dx^3} = V(x)$$

$$EI\frac{d^2v}{dx^2} = M(x)$$

Slope and displacement with the Moment-Area Method

$$\theta_{B/A} = \int_A^B \frac{M}{EI}dx \qquad t_{A/B} = \bar{x}\int_A^B \frac{M}{EI}dx$$

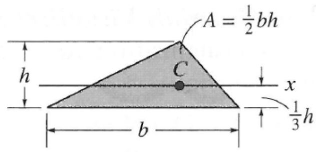
Geometric Properties of Area Elements



$I_x = \frac{1}{12}bh^3$

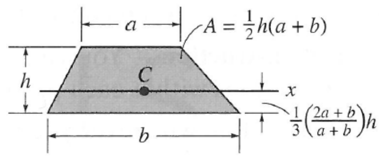
$I_y = \frac{1}{12}hb^3$

Rectangular area

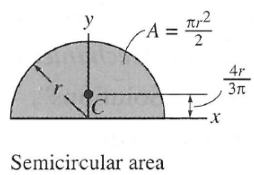


$I_x = \frac{1}{36}bh^3$

Triangular area



Trapezoidal area



$I_x = \frac{1}{8}\pi r^4$

$I_y = \frac{1}{8}\pi r^4$

Semicircular area

Torsion

Shear stress in circular shaft

$$\tau = \frac{T\rho}{J}$$

where

$$J = \frac{\pi}{2} c^4 \text{ solid cross section}$$

$$J = \frac{\pi}{2}(c_o^4 - c_i^4) \text{ tubular cross section}$$

Power

$$P = T\omega = 2\pi f T$$

Angle of twist

$$\phi = \int_0^L \frac{T(x)dx}{J(x)G}$$

$$\phi = \Sigma \frac{TL}{JG}$$

Average shear stress in a thin-walled tube

$$\tau_{avg} = \frac{T}{2tA_m}$$

Shear Flow

$$q = \tau_{avg} t = \frac{T}{2A_m}$$

Stress Transformation Equations

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

Principal Stress

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Maximum in-plane shear stress

$$\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$

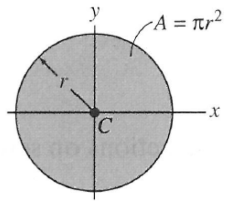
$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

Absolute maximum shear stress

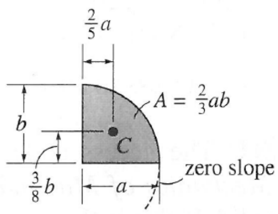
$$\tau_{\substack{abs\\max}} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

$$\sigma_{avg} = \frac{\sigma_{max} + \sigma_{min}}{2}$$

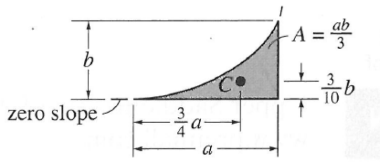


$$I_x = \tfrac{1}{4}\pi r^4$$
$$I_y = \tfrac{1}{4}\pi r^4$$

Circular area



Semiparabolic area



Exparabolic area

Material Property Relations

Poisson's ratio

$$\nu = -\frac{\epsilon_{lat}}{\epsilon_{long}}$$

Generalized Hooke's Law

$$\epsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$\epsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$$

$$\epsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy}, \quad \gamma_{yz} = \frac{1}{G}\tau_{yz}, \quad \gamma_{zx} = \frac{1}{G}\tau_{zx}$$

where

$$G = \frac{E}{2(1+\nu)}$$

Strain transformation equations

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\epsilon_x - \epsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\frac{\gamma_{\substack{max\\in\text{-}plane}}}{2} = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\epsilon_{avg} = \frac{\epsilon_x + \epsilon_y}{2}$$