Universitetet i Stavanger

Det teknisk- naturvitenskapelige fakultet

SUBJECT:	**BYG 140 KONSTRUKSJONSMEKANIKK 1**
DATE:	May 22, 2018
TIME:	09:00 – 13:00 (4 hours)
AID:	**Authorized calculator, Dictionary (English-Norwegian) and drawing instruments.**

THE EXAM CONSISTS OF 4 QUESTIONS AND 13 PAGES (including the front page)
Norwegian translation of each question is attached

REMARKS: All the **Four** questions carry **equal marks** and answer **all** the questions.

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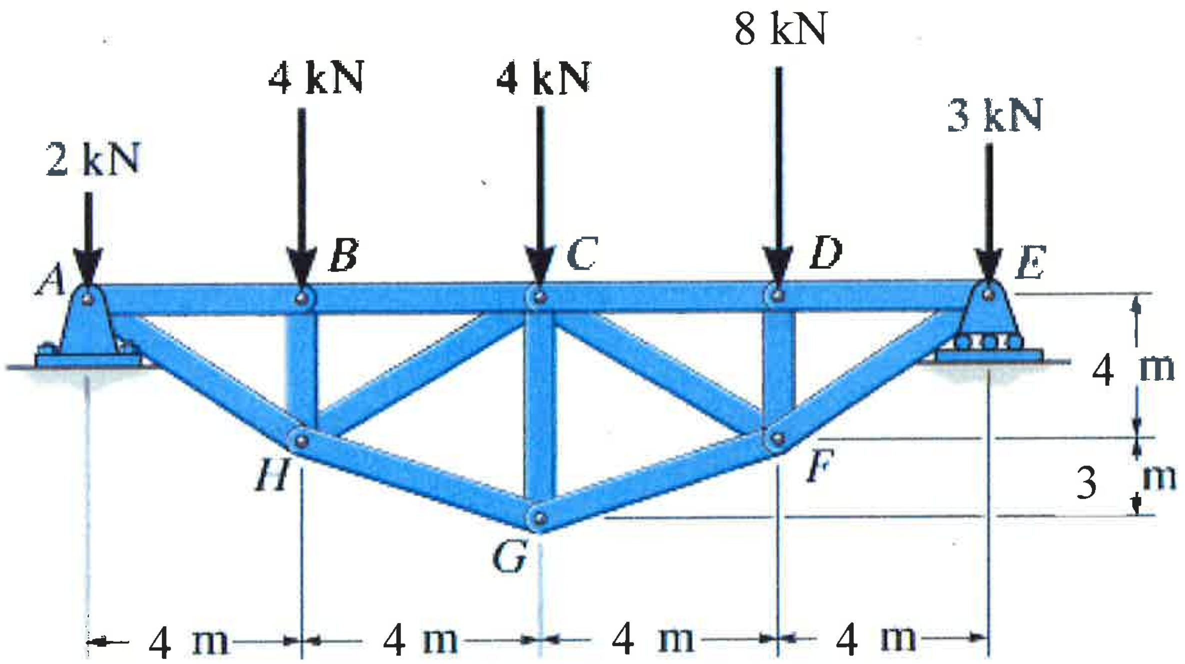
QUESTION (1): (25 %)

A truss subjected to forces at joints *A, B,C,D* and *E* as shown in Figure 1. The truss is supported by a pin support at *A* and a roller support at *E*. The truss members are connected by pin joints which cannot transfer moments. The truss members are made up of steel which has Modulus of Elasticity (*E*) 200GPa. The allowable normal stress of the material is 155 MPa.



(All forces are in kilonewtons and lengths are in meters)

Figure 1: Truss structure

a) Determine the support reaction forces at *A* and *E*. **(3 points)**

b) Determine the axial forces of the members *AB, AH, BH* and *BC* using the method of joints. Clearly state whether the members are in compression or tension. **(5 points)**

c) Identify and list zero-force members of the truss if there are any. **(2 points)**

d) Determine the axial forces of the members *CD, CF* and *GF* using the method of sections. Clearly state whether the members are in compression or tension. **(5 points)**

e) Determine required cross-sectional areas for the members *AB* and *BH.* Do you think that it is suitable to use the same cross-sectional area for *DE* and *DF*? **(4 points)**

f) Determine the change of length of the member *BH.* Do you think that this value is equal to the vertical displacement of point *H*? **(3 points)**

g) If the truss is subjected to a horizontal external force at point *E* to the direction of *EA* in addition to above loadings, what changes (i.e. increase, decrease or no change) can be observed in the magnitudes of the axial forces of the members *DE* and *EF*? State logical reasons for your answer without calculations. **(3 points)**

OPPGAVE (1): (25%)

Et fagverk er utsatt for krefter ved forbindelsene *A, B, C, D* og *E* som vist i Figur 1. Fagverket er lagret opp med et boltelager i *A* og et rullelager i *E*. Stavene i fagverket er satt sammen med bolter som ikke kan overføre momenter. Stavene er av stål, som har elastisitetsmodul (*E*) på 200 GPa. Tillatt normalspenning for materialet er 155 MPa.

a) Bestem opplagerkreftene ved *A* og *E*. **(3 poeng)**

b) Bestem aksialkreftene for stavene *AB,AH, BH* og *BC* vha. knutepunktsmetoden. Angi tydelig hvorvidt stavene er i trykk eller strekk. **(5 poeng)**

c) Identifiser og list opp nullstavene i fagverket, dersom der er noen. **(2 poeng)**

d) Bestem aksialkreftene for stavene *CD, CF* og *GF* vha. snittmetoden. Angi tydelig hvorvidt stavene er i trykk eller strekk. **(5 poeng)**

e) Bestem nødvendig tverrsnittsareal for stavene *AB* og *BH*. Tror du det vil være akseptabelt å bruke samme tverrsnittsareal for *DE* og *DF*? **(4 poeng)**

f) Bestem lengdeendring for staven *BH*. Tror du denne verdien vil være lik vertikal forskyvning av punkt *H*? **(3 poeng)**

g) Dersom fagverket blir utsatt for en horisontal ekstern kraft i punkt *E*, i retning *EA*, i tillegg til de ovenstående kreftene, hvilke endringer (i.e. økning, avtagning eller ingen endring) vil man se i aksialkreftene for stavene *DE* og *EF*? Angi logiske resonnementer for svaret, uten beregninger. **(3 poeng)**

QUESTION (2): (25%)

Figure 2(a) shows a 6 m long *AB* beam which carries triangular loads along its length from *A* to *D* and from *D* to *B* with a maximum value of 20 kN/m. The beam *AB* is supported with a member BC which is connected using a pin joint at *B*. The member *AB* and *BC* are supported at *A* and *C* by pin supports respectively as shown in Figure 2(a). *Hint*: Pin joints transfer zero moments. The cross-section of the beam *AB* is as shown in Figure 2(b).

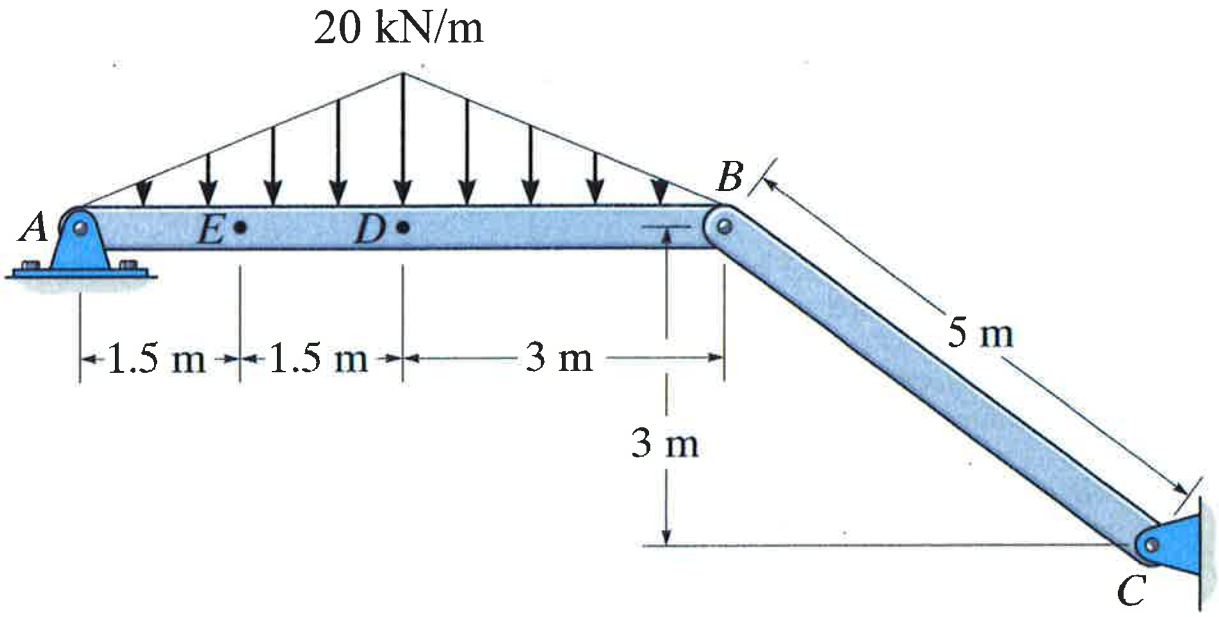


Figure 2(a): Structure (Beam *AB* and member *BC*)

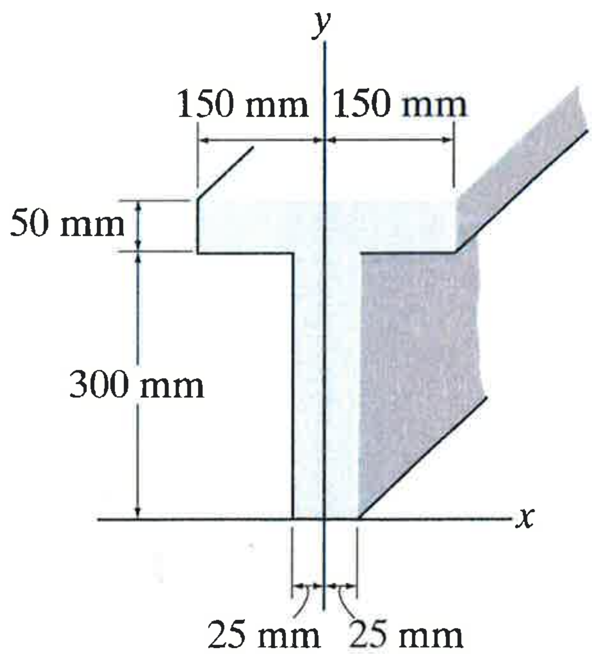


Figure 2(b): Cross section of the beam *AB*

a) Clearly indicate the two-force member/members in the structure. **(2 points)**

b) Draw a free-body diagram of the beam *AB* and determine reactions at the support *A* and the joint *B*. **(3 points)**

c) Draw shear force and moment diagrams for the beam *AB*. **(8 points)**

d) Determine cross-sectional area and moment of inertia about neutral axis for the cross-section of the beam *AB* shown in Figure 2(b). **(3 points)**

e) Determine the maximum shear stress and the normal stress of the beam *AB*. **(6 points)**

f) If the *ABC* is a rigid frame (i.e. *AB* and *BC* are connected rigidly and there is no pin joint at *B*), what will happen to the magnitude of maximum normal stress (i.e. calculated in section (e))? State logical reasons for your answer without calculations. **(3 points)**

OPPGAVE (2): (25%)

Figur 2(a) viser en 6 m lang bjelke *AB* som bærer en triangulær last langs lengden fra *A* til *D*, og fra *D* til *B* med en maksimumsverdi på 20 kN/m. Bjelken *AB* bæres av en bjelke *BC* som er forbundet med et boltelager i *B*. Bjelkene *AB* og *BC* er festet ved *A* og *C* vha. boltelagre som vist i Figur 2(a). *Hint:* Boltelagre overfører null moment. Tverrsnittet for bjelken *AB* er som vist i Figur 2(b).

a) Angi tydelig to-kraft stavene (i.e. staver kun utsatt for aksialkraft) i konstruksjonen. **(2 poeng)**

b) Tegn et fritt legeme-diagram for bjelken *AB* og bestem opplagerreaksjonene ved festet *A*, og i leddet *B*. **(3 poeng)**

c) Tegn skjærkraft- og momentdiagrammer for bjelken *AB*. **(8 poeng)**

d) Bestem tverrsnittsareal og treghetsmoment om nøytralaksen for tverrsnittet for bjelken *AB*, vist i Figur 2(b). **(3 poeng)**

e) Bestem maksimal skjærspenning og normalspenningen for bjelken *AB*. **(6 poeng)**

f) Dersom *ABC* var en stiv ramme (i.e. *AB* og *BC* er fast forbundet, slik at det ikke er noe boltelager i *B*), hva ville skje med størrelsen på maksimal normalspenningen (i.e. beregnet under punkt **e)**)? Angi logiske resonnementer for svaret, uten beregninger. **(3 poeng)**

QUESTION (3): (25%)

Figure 3(a) shows a simply supported beam of length 7.2 m which is subjected to concentrated loads 10 kN, 20 kN at C and D respectively and a moment of 6 kNm at end B. The beam is pinned supported at A and roller supported at B. The material used for the beam is a type of steel which has Modulus of Elasticity (E) 200GPa. The cross section of the beam is as shown in the Figure 3(b) and the beam is bending about x-x axis.

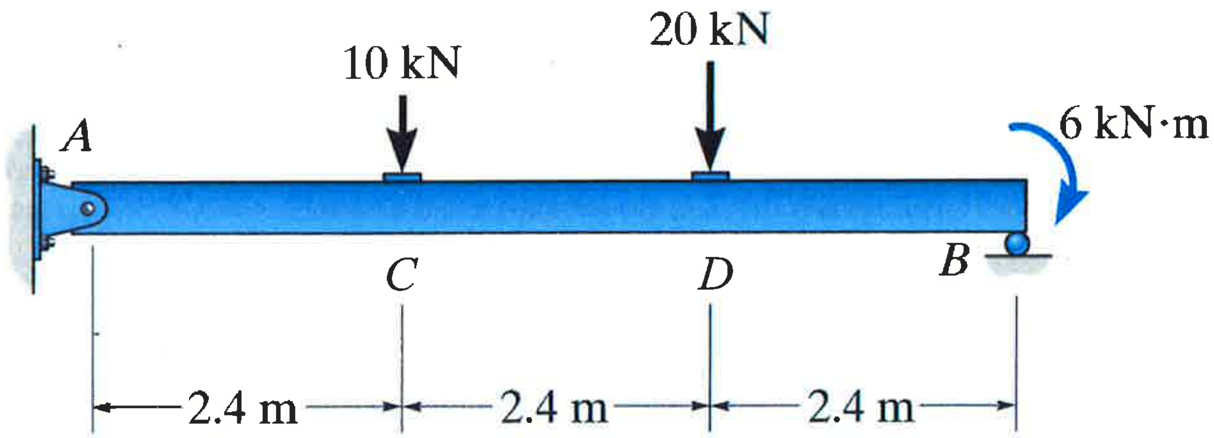
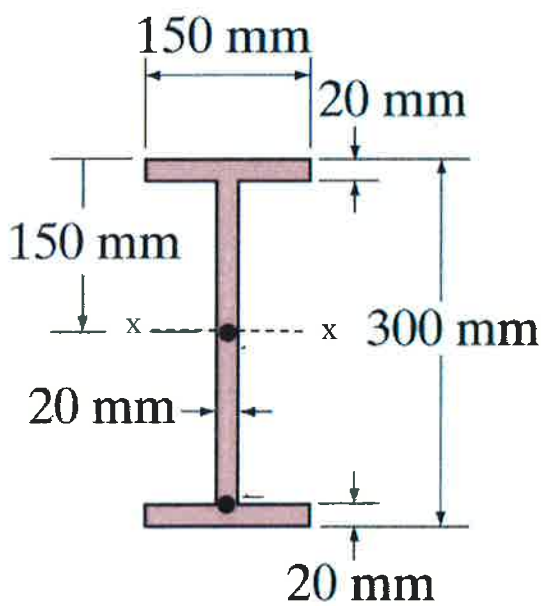


Figure 3 (a): Simply supported beam

All dimensions are in meters



All dimensions are in milimeters

Figure 3(b): Cross section of the beam *AB*

a) Determine the support reactions. **(3 points)**

b) Draw the bending moment diagram for the beam *AB* shown in Figure 3(a). **(6 points)**

c) Determine moment of inertia about the neutral axis (i.e. parallel to the *x-x* axis) of the cross section shown in Figure 3(b). **(3 points)**

d) Determine the deflection and slope at *C*. **(10 points)**

e) If the moment of 6 kNm at end *B* is removed while keeping other loadings are as it is, what will happen to the magnitude of the deflection at point *C*? State logical reasons for your answer without calculations. **(3 points)**

OPPGAVE (3): (25%)

Figur 3(a) viser en fritt opplagret bjelke med lengde 7.2 m, utsatt for punktlaster på hhv. 10 kN og 20 kN i punkt *C* og *D*, samt et moment på 6 kNm ved enden *B*. Bjelken bæres av et boltelager ved *A* og et rullelager ved *B*. Materialet i bjelken er en ståltype som har elastisitetsmodul (*E*) på 200GPa. Tverrsnittet for bjelken er som vist i Figur 3(b), og bjelken bøyes om *x-x* aksen.

a) Bestem opplagerreaksjonene. **(3 poeng)**

b) Tegn bøyemomentdiagrammet for bjelken *AB* vist i Figur 3(a). **(6 poeng)**

c) Bestem treghetsmomentet om nøytralaksen (i.e. parallell med *x-x* aksen) for tverrsnittet vist i Figur 3(b). **(3 poeng)**

d) Bestem nedbøyningen og helningen ved *C*. **(10 poeng)**

e) Dersom momentet på 6 kN ved enden *B* fjernes, men de andre lastene forblir slik de er, hva vil skje mhp. størrelsen på nedbøyningen ved punkt *C*? Angi logiske resonnementer for svaret, uten beregninger. **(3 poeng)**

QUESTION (4): (25%)

A signboard is subjected to uniform loading as shown in Figure 4(a). The supporting post of the signboard is fixedly supported to the ground as shown in the Figure 4(a). The cross section of the supporting post is solid circular section of diameter 100 mm. The supporting post is made up of steel which has Shear Modulus (G) 75GPa. Self-weights of the supporting post and the signboard are negligible.

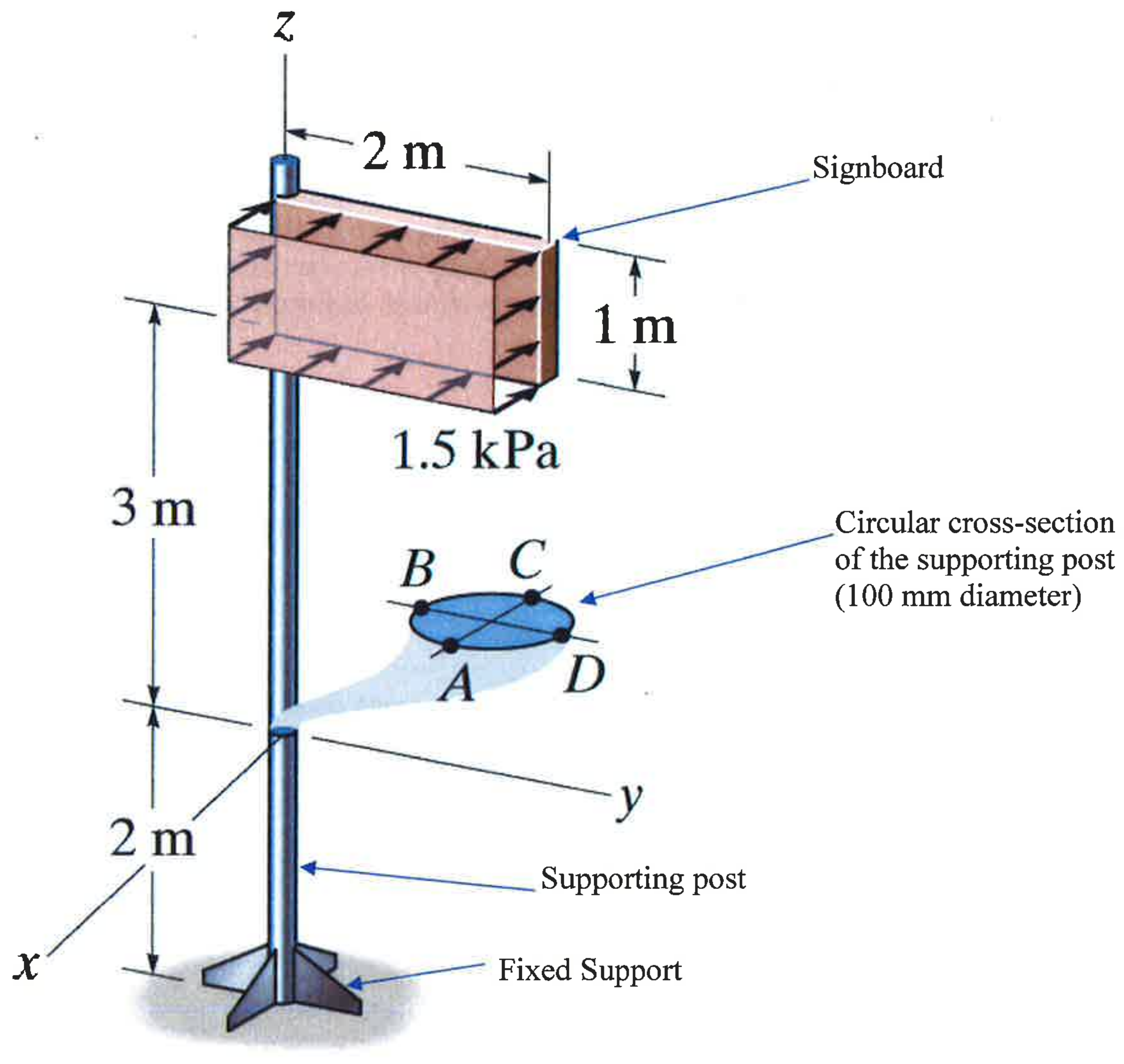


Figure 4 (a): A sign board.

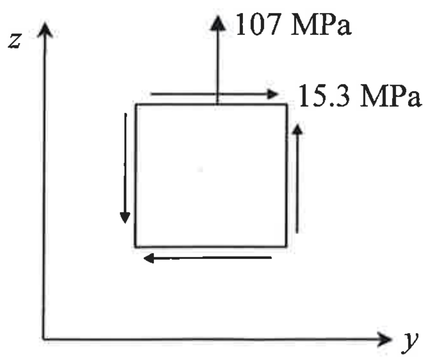


Figure 4 (b): State of stress at point *A*.

a) Determine the internal forces acting at the *ABCD* cross-section of the post. **(6 points)**

b) Determine the stress components at the points *A* and *B* of the supporting post. **(8 points)**

c) Show that the state of stress at the point *A* can be illustrated as in Figure 4 (b). **(3 points)**

d) Determine the principle stresses at the point *A* and the orientation of the principle axis (i.e. planes of minimum and maximum normal stress) relative to the given axes of Figure 4(b). **(4 points)**

e) Determine the absolute maximum shear stress at the point *A*. **(2 points)**

f) If self-weight of the supporting post is considered as significant, what changes (i.e. increase, decrease or no change) can be observed in the magnitudes of the principle stresses at the point *A*? State the logical reasons for your answer without calculations. **(2 points)**

OPPGAVE (4): (25%)

Et skilt er utsatt for en jevnt fordelt belastning som vist i Figur 4(a). Stolpen som bærer skiltet er fast forbundet med bakken som vist i Figur 4(a). Stolpens tverrsnitt er massivt og sirkulært med diameter 100 mm. Stolpen er lagd av stål, som har skjærmodul (G) på 75 GPa. Egenvekten av både stolpen og skiltet er neglisjerbare.

a) Bestem de interne kreftene som virker i *ABCD* tverrsnittet på stolpen. **(6 poeng)**

b) Bestem spenningskomponentene ved punkt *A* og *B* for stolpen. **(8 poeng)**

c) Vis at spenningstilstanden ved punkt *A* kan illustreres som i Figur 4(b). **(3 poeng)**

d) Bestem hovedspenningene ved punkt *A*, samt hovedaksenes orientering (i.e. planene for minimum- og maksimum normalspenning), relativt til aksene gitt i Figur 4(b). **(4 poeng)**

e) Bestem absolutt maksimal skjærspenning ved punkt *A*. **(2 poeng)**

f) Dersom egenvekten av stolpen antas signifikant, hvilke endringer (i.e. økning, avtagning eller ingen endring) vil man se mhp. størrelsen av hovedspenningene ved punkt *A*? Angi logiske resonnementer for svaret, uten beregninger. **(2 poeng)**

Fundamental Equations of Statics

Cartesian Vector

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

Magnitude

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Directions

$$\mathbf{u}_A = \frac{\mathbf{A}}{A} = \frac{A_x}{A}\mathbf{i} + \frac{A_y}{A}\mathbf{j} + \frac{A_z}{A}\mathbf{k}$$
$$= \cos\alpha\mathbf{i} + \cos\beta\mathbf{j} + \cos\gamma\mathbf{k}$$
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

Dot Product

$$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$$
$$= A_xB_x + A_yB_y + A_zB_z$$

Cross Product

$$\mathbf{C} = \mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Cartesian Position Vector

$$\mathbf{r} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k}$$

Cartesian Force Vector

$$\mathbf{F} = F\mathbf{u} = F\left(\frac{\mathbf{r}}{r}\right)$$

Moment of a Force

$$M_o = Fd$$
$$\mathbf{M}_o = \mathbf{r}\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Moment of a Force About a Specified Axis

$$M_a = \mathbf{u}\cdot\mathbf{r}\times\mathbf{F} = \begin{vmatrix} u_x & u_y & u_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Simplification of a Force and Couple System

$$\mathbf{F}_R = \Sigma\mathbf{F}$$
$$(\mathbf{M}_R)_O = \Sigma\mathbf{M} + \Sigma\mathbf{M}_O$$

Equilibrium

Particle

$$\Sigma F_x = 0,\ \Sigma F_y = 0,\ \Sigma F_z = 0$$

Rigid Body-Two Dimensions

$$\Sigma F_x = 0,\ \Sigma F_y = 0,\ \Sigma M_O = 0$$

Rigid Body-Three Dimensions

$$\Sigma F_x = 0,\ \Sigma F_y = 0,\ \Sigma F_z = 0$$
$$\Sigma M_{x'} = 0,\ \Sigma M_{y'} = 0,\ \Sigma M_{z'} = 0$$

Friction

Static (maximum) $F_s = \mu_s N$

Kinetic $F_k = \mu_k N$

Center of Gravity

Particles or Discrete Parts

$$\bar{r} = \frac{\Sigma\tilde{r}W}{\Sigma W}$$

Body

$$\bar{r} = \frac{\int \tilde{r}\,dW}{\int dW}$$

Area and Mass Moments of Inertia

$$I = \int r^2\,dA \qquad I = \int r^2\,dm$$

Parallel-Axis Theorem

$$I = \bar{I} + Ad^2 \qquad I = \bar{I} + md^2$$

Radius of Gyration

$$k = \sqrt{\frac{I}{A}} \qquad k = \sqrt{\frac{I}{m}}$$

Virtual Work

$$\delta U = 0$$

Fundamental Equations of Mechanics of Materials

Axial Load

Normal Stress

$$\sigma = \frac{P}{A}$$

Displacement

$$\delta = \int_0^L \frac{P(x)dx}{A(x)E}$$

$$\delta = \Sigma \frac{PL}{AE}$$

$$\delta_T = \alpha \, \Delta T L$$

Bending

Normal stress

$$\sigma = \frac{-My}{I}$$

Unsymmetric bending

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}, \qquad \tan \alpha = \frac{I_z}{I_y} \tan \theta$$

Section modulus

$$S_{\text{req'd}} = \frac{M_{\max}}{\sigma_{\text{allow}}}$$

Shear

Average direct shear stress

$$\tau_{\text{avg}} = \frac{V}{A}$$

Transverse shear stress

$$\tau = \frac{VQ}{It}$$

Shear flow

$$q = \tau t = \frac{VQ}{I}$$

Relations Between *w, V, M*

$$\frac{dV}{dx} = -w(x), \qquad \frac{dM}{dx} = V$$

Elastic Curve

$$\frac{1}{\rho} = \frac{M}{EI}$$

$$EI \frac{d^4 v}{dx^4} = -w(x)$$

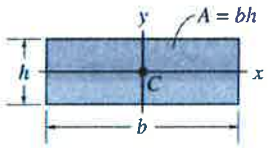
$$EI \frac{d^3 v}{dx^3} = V(x)$$

$$EI \frac{d^2 v}{dx^2} = M(x)$$

Slope and displacement with the Moment-Area Method

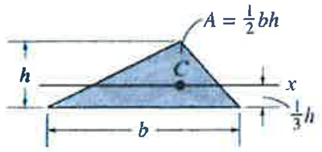
$$\theta_{B/A} = \int_A^B \frac{M}{EI} dx \qquad t_{A/B} = \bar{x} \int_A^B \frac{M}{EI} dx$$

Geometric Properties of Area Elements



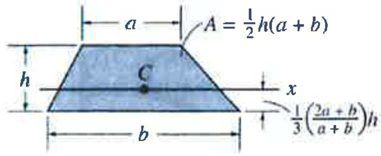
Rectangular area

$I_x = \frac{1}{12} bh^3$
$I_y = \frac{1}{12} hb^3$

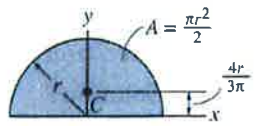


Triangular area

$I_x = \frac{1}{36} bh^3$



Trapezoidal area



Semicircular area

$I_x = \frac{1}{8} \pi r^4$
$I_y = \frac{1}{8} \pi r^4$

Torsion

Shear stress in circular shaft

$$\tau = \frac{T\rho}{J}$$

where

$$J = \frac{\pi}{2}c^4 \text{ solid cross section}$$

$$J = \frac{\pi}{2}(c_o^4 - c_i^4) \text{ tubular cross section}$$

Power

$$P = T\omega = 2\pi f T$$

Angle of twist

$$\phi = \int_0^L \frac{T(x)dx}{J(x)G}$$

$$\phi = \Sigma \frac{TL}{JG}$$

Average shear stress in a thin-walled tube

$$\tau_{avg} = \frac{T}{2tA_m}$$

Shear Flow

$$q = \tau_{avg} t = \frac{T}{2A_m}$$

Stress Transformation Equations

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

Principal Stress

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Maximum in-plane shear stress

$$\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$

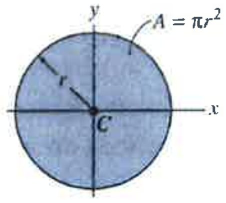
$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

Absolute maximum shear stress

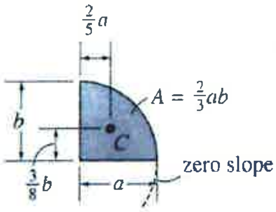
$$\tau_{\substack{abs\\max}} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

$$\sigma_{avg} = \frac{\sigma_{max} + \sigma_{min}}{2}$$

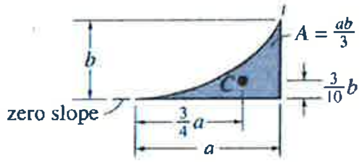


$$I_x = \frac{1}{4}\pi r^4$$
$$I_y = \frac{1}{4}\pi r^4$$

Circular area



Semiparabolic area



Exparabolic area

Material Property Relations

Poisson's ratio

$$\nu = -\frac{\epsilon_{lat}}{\epsilon_{long}}$$

Generalized Hooke's Law

$$\epsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$\epsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$$

$$\epsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy}, \quad \gamma_{yz} = \frac{1}{G}\tau_{yz}, \quad \gamma_{zx} = \frac{1}{G}\tau_{zx}$$

where

$$G = \frac{E}{2(1 + \nu)}$$

Strain transformation equations

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

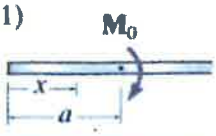
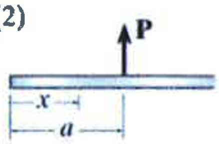
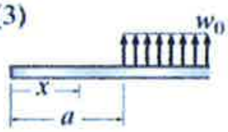
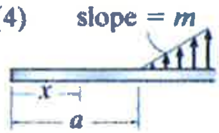
$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\epsilon_x - \epsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\frac{\gamma_{\substack{max\\in\text{-}plane}}}{2} = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\epsilon_{avg} = \frac{\epsilon_x + \epsilon_y}{2}$$

Discontinuity functions

Loading	Loading Function $w = w(x)$	Shear $V = \int w(x)dx$	Moment $M = \int V dx$
(1) 	$w = M_0\langle x-a\rangle^{-2}$	$V = M_0\langle x-a\rangle^{-1}$	$M = M_0\langle x-a\rangle^{0}$
(2) 	$w = P\langle x-a\rangle^{-1}$	$V = P\langle x-a\rangle^{0}$	$M = P\langle x-a\rangle^{1}$
(3) 	$w = w_0\langle x-a\rangle^{0}$	$V = w_0\langle x-a\rangle^{1}$	$M = \frac{w_0}{2}\langle x-a\rangle^{2}$
(4) 	$w = m\langle x-a\rangle^{1}$	$V = \frac{m}{2}\langle x-a\rangle^{2}$	$M = \frac{m}{6}\langle x-a\rangle^{3}$