10/02

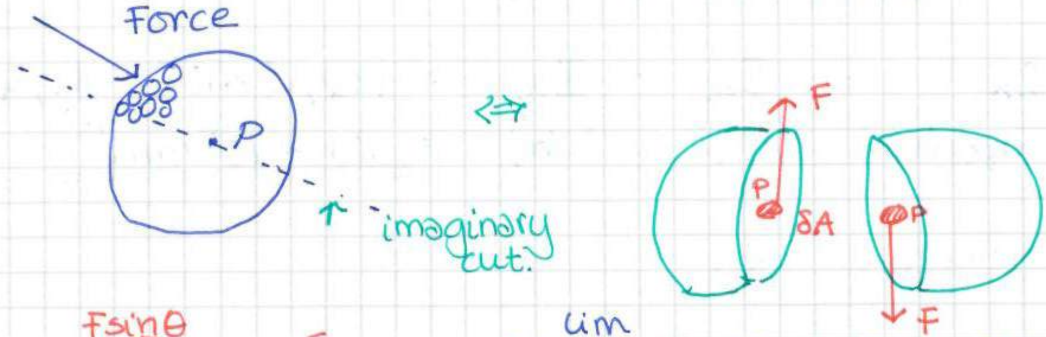
MECHANICS OF MATERIALS

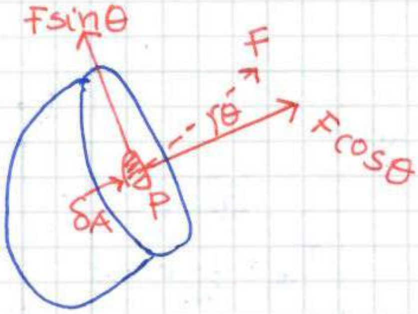
Chapter 1 8 Stress

1.3 - Distribution of internal loading.

- This intensity of internal forces acting on a specific plane passing through a point is called stress.

Assumtion : continous , cohesive(connected together) homogeneous





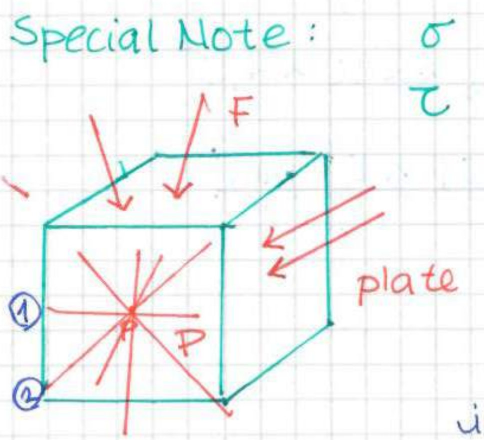
$\lim_{\delta A \to 0} = \frac{F\cos\theta}{\delta A}$ = Normal stress(at point P (⊥ the plane

$\lim_{\delta A \to 0} \frac{F\sin\theta}{\delta A} =$ shear stress (τ) at point P (tangent to the plan

units : N/m^2 or N/mm^2 Pascal (Pa) mega pas (MPa)

Special Note : σ = Normal stress

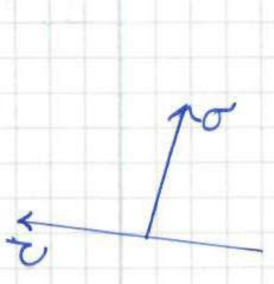
τ = shear stress.



imaginary cut I

$\tau = 5\ N/mm^2$

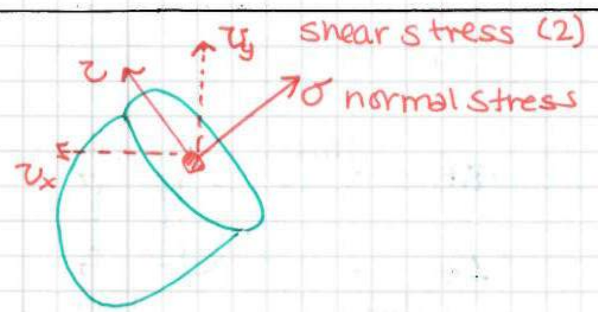
$\sigma = 10\ N/mm^2$

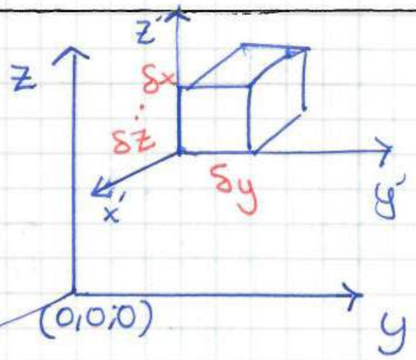


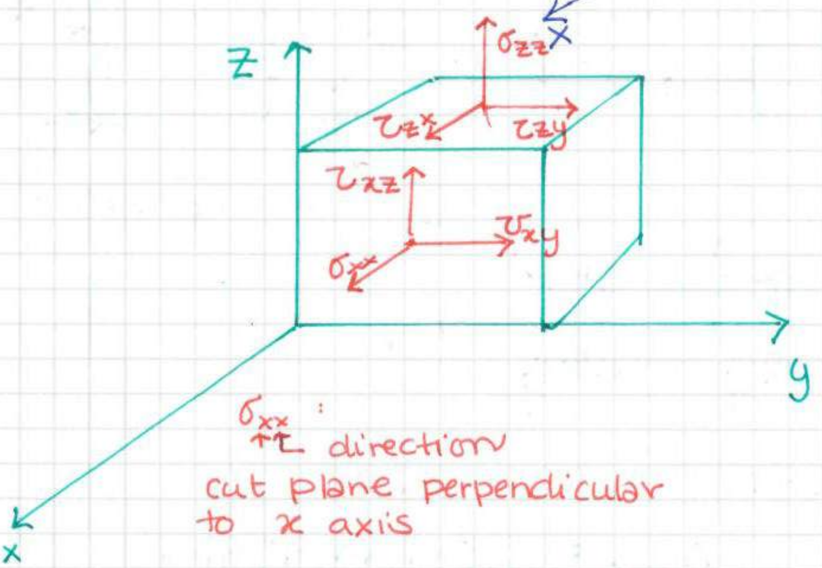
imaginary cut II } different value in each cut.

$\tau_2 \neq \tau_1$

$\sigma_1 \neq \sigma_2$



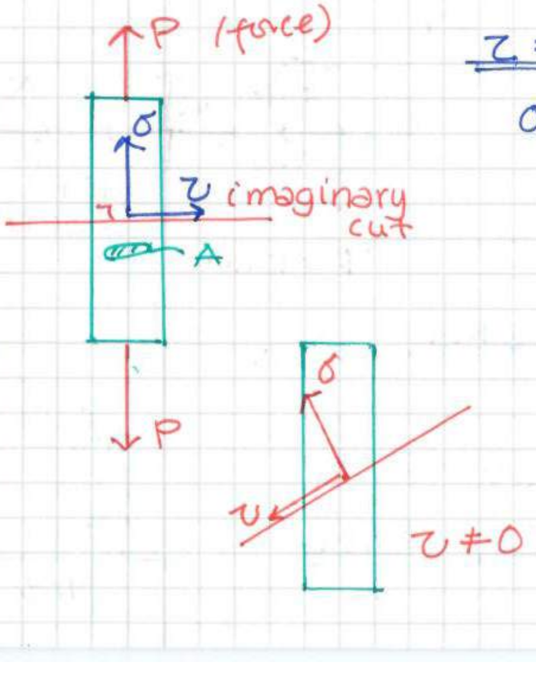




σ_{xx} :
↑ ⊥ direction
cut plane perpendicular to x axis

1,4 : Average normal stress in an axially loaded bar:

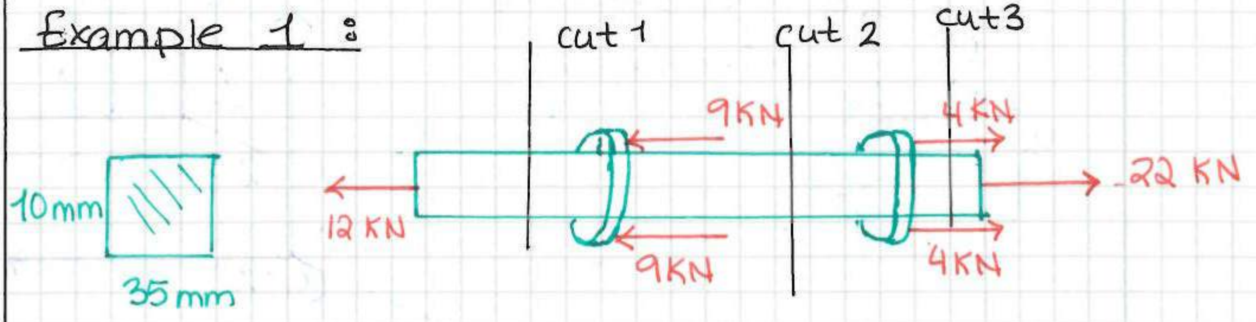
→ only subjected to normal stress
→ stress assumed averaged over the area.

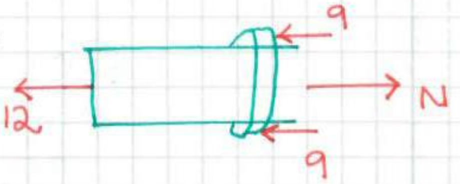


$\underline{\underline{\tau = 0}}$ (as V=0)

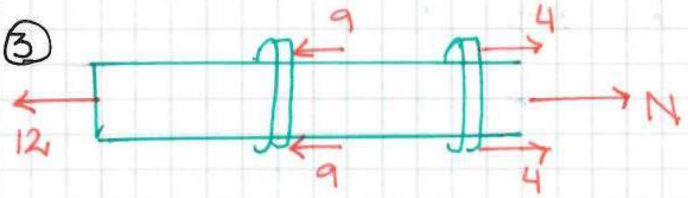
$\sigma_{avg} = \frac{P}{A}$ = average normal stress

Example 1 :

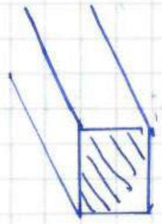




$N = 12+9+9 = 30\ KN$



$N = 12+9+9-4-4 = 22\ KN$



Area = 35 mm × 10 mm

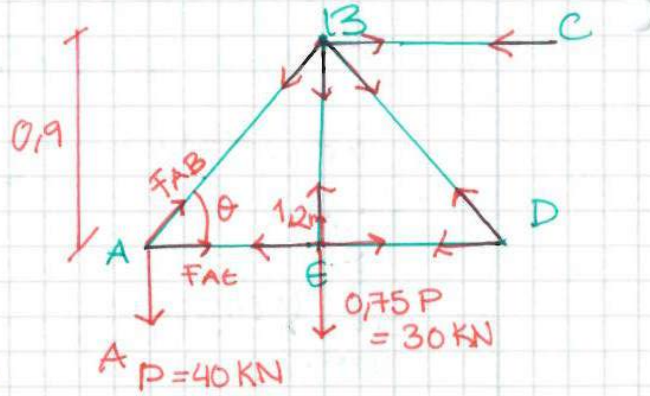
F = 30 KN

$\sigma\ avg = \frac{F}{A} = \frac{30}{350\ mm^2} = \underline{85{,}7\ MPa}$

Example 2 :

▨ = 780 mm²

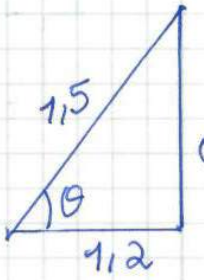
$\sigma_{avg} = \frac{\text{Axial force}}{\text{Area}}$



joint A :

$+\uparrow\ \Sigma F_y = F_{AB} \cdot \sin\theta - P = 0$

$F_{AB} = \frac{40}{(0{,}9/1{,}5)} = \underline{66{,}66\ KN}$



$\rightarrow \Sigma F_x = F_{AE} + F_{AB} \cos\theta = 0$

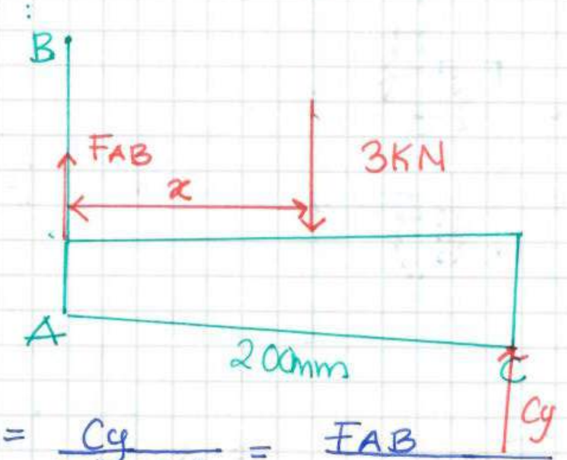
$\underline{F_{AE} = -53{,}38\ KN}$

$$\sigma_{AB} = \frac{F_{AB}}{A} = \frac{66,667 \times 10^{3}\ N}{(780 \times 10^{-6})\ m^2} = \underline{\underline{-85,47\ MPa(T)}}$$

$$\sigma_{AE} = \frac{F_{AE}}{A} = \frac{-53,33 \times 10^{3}\ N}{(780 \times 10^{-6}) m^2} = -\underline{68,376\ MPa(C)}$$

example 3

FBD :



cross sectional area of AB rod = 400mm

C/S area of contact at C = 650 mm²

$$\sigma_{avg} = \frac{C_y}{\text{area at } C} = \frac{F_{AB}}{\text{area of } AB} \quad ①$$

$$\Rightarrow \frac{C_y}{650} = \frac{F_{AB}}{400} \quad ①$$

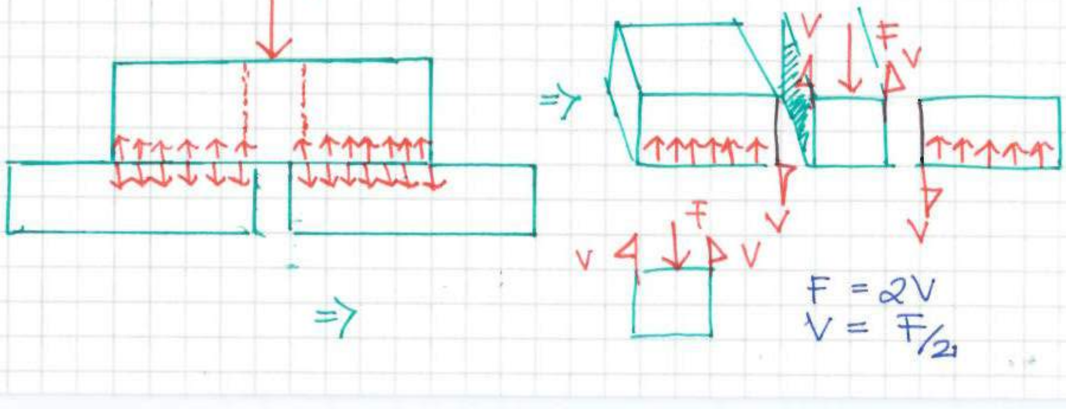
$$+\uparrow \Sigma F_y = F_{AB} - 3 + C_y = 0 \quad ②$$

$$+\overset{\curvearrowleft}{M_A} = -3x + C_y \times 200 = 0 \quad ③$$

solve for x, C_y, F_{AB}

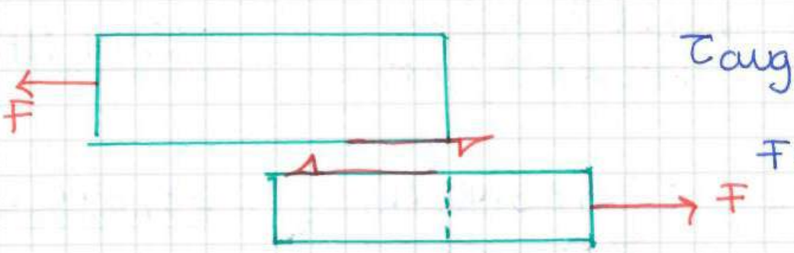
$\underline{\underline{x = 124\ mm}}$

1;5 Average shear stress



F = 2V

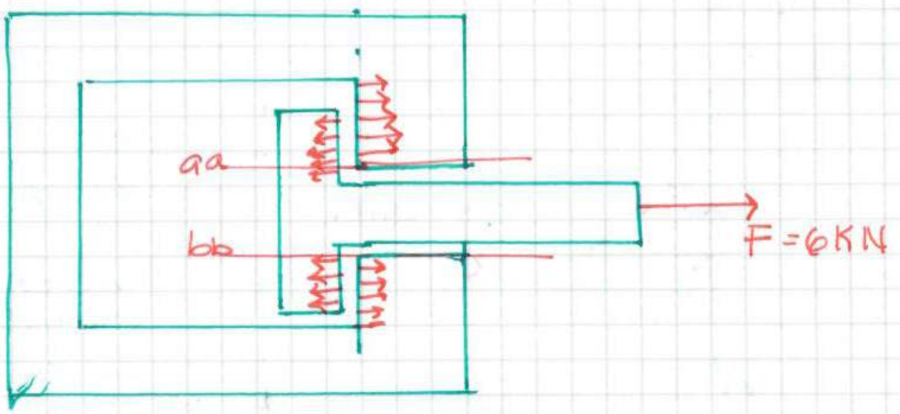
V = F/2

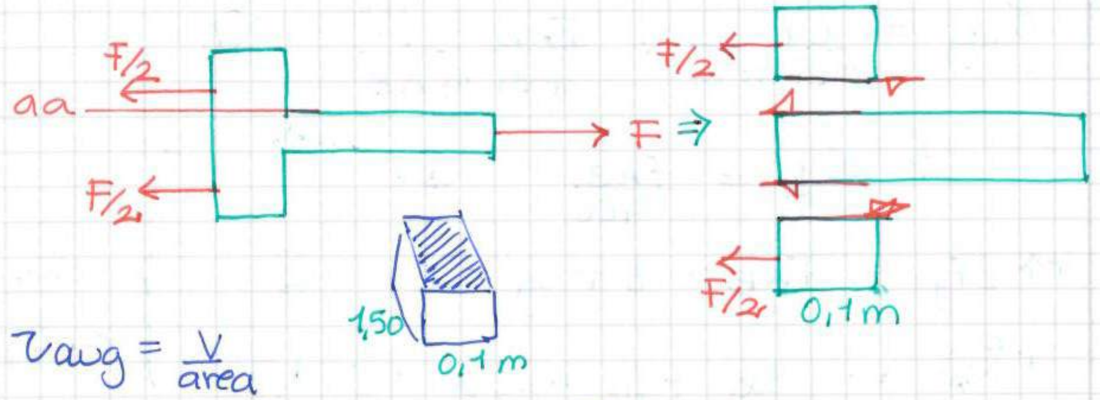


$$\tau_{avg} = \frac{V}{area}$$

$$F = V$$

example 4.





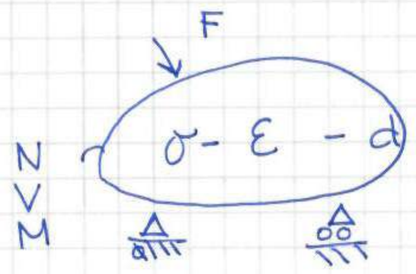
$$V = F/2$$

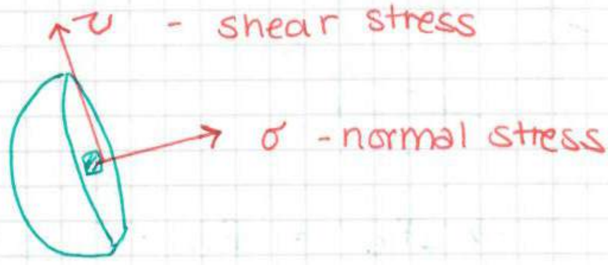
$$\tau_{avg} = \frac{V}{area}$$

$$= \frac{V}{(150\times100)} = \frac{3\times10^{3}\,N}{(150\times100)} = \underline{\underline{200\ KPa}}$$

14/02.

Chapter 1 : Stress , Summary





units - N/m^2 = Pa

N/mm^2 = MPa

* Average Normal stress

axially loaded bar

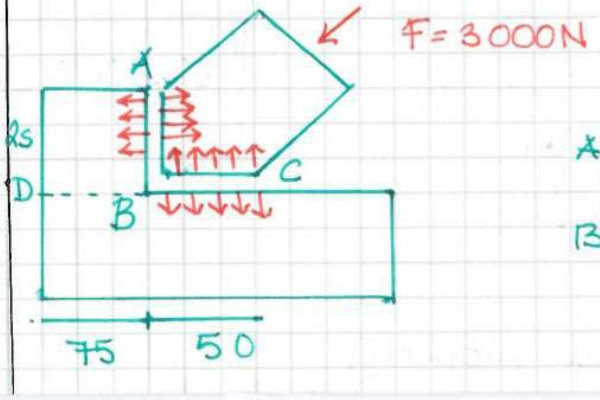
$$\sigma_{avg} = \frac{F}{Area}$$

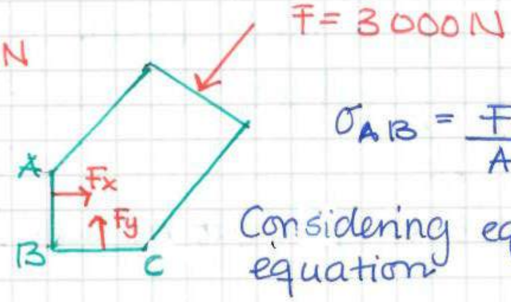
* Average shear stress

- try to understand the behaviour
- imaginary cut & Shearforce V

$$\tau_{avg} = \frac{V}{\text{Shear area}}$$

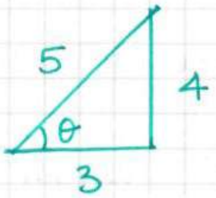
Example 4 & 5





$$\sigma_{AB} = \frac{F_{AB}}{Area}$$

Considering equlibrium equation



$\sin\theta = 4/3$

$\cos\theta = 3/5$

$\rightarrow \Sigma F_x = F_x - F\cos\theta = 0$

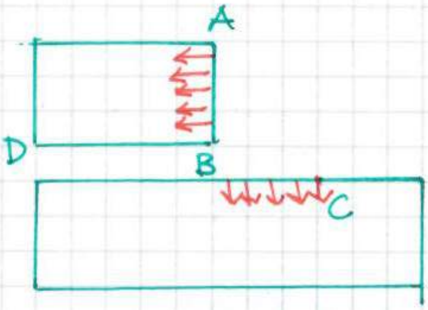
$\underline{F_x = 1800\ N}$

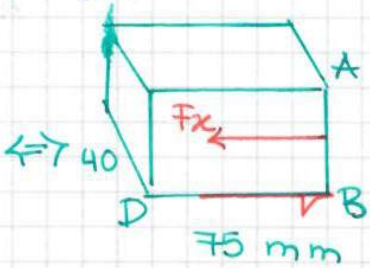
$\uparrow \Sigma F_y = F_y - F\sin\theta = 0$

$\underline{F_y = 2400\ N}$

$$\sigma_{AB} = \frac{F_x}{Area} = \frac{1800\ N}{(25 \times 40)} = \underline{\underline{1,8\ N/mm^2}}$$

$$\sigma_{BC} = \frac{F_y}{Area} = \frac{2400\ N}{(50 \times 40)} = \underline{1,2\ N/mm^2}$$

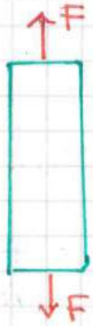




$$\tau_{DB} = \frac{V}{\text{shear area}} = \frac{1800}{75 \times 40}$$

$$= \underline{\underline{0.6\ N/mm^2}}$$

Allowable stress



when F ↑ σ ↑

faliure (load) force : F_{fail}

faliure stress : σ_{fail}

$$\boxed{\sigma_{fail} = \frac{F_{fail}}{Area}}$$

Not to fail (to be safe)

$\sigma \leqslant \sigma_{fail}$ ↳ material property

σ : → due to loading
→ geometry
→ support condition

For safe design

$$\boxed{\text{Factor of safety (FS)} = \frac{\sigma_{fail}}{\sigma_{allowable}}} > 1.$$

$$\boxed{\text{Facture of safety F.S} = \frac{\tau_{fail}}{\tau_{allowable}}} > 1.$$

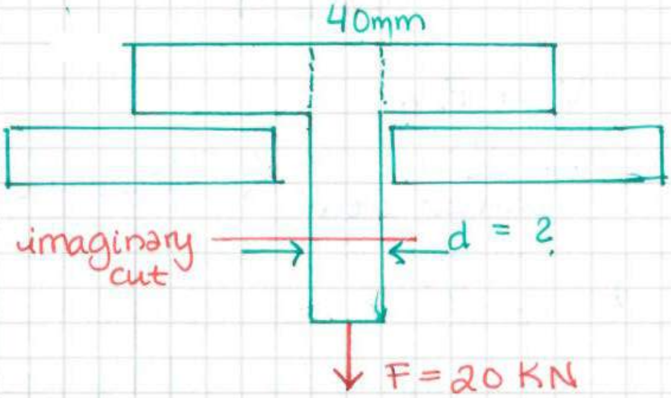
$\sigma \leq \sigma_{allowable}$ (design criteria)

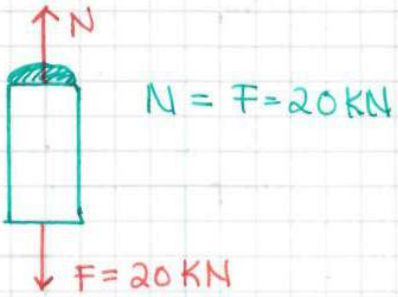
$\tau \leq \tau_{allowable}$ (——"——)

Note:

For a problem either $\sigma_{allowable} / \tau_{allowable}$
or $\sigma_{fail} / \tau_{fail}$

example





Not to fail:

$\sigma \leq \sigma_{allowable}$

$\frac{F}{A} \leq 60$ MPa

$$\frac{20\times10^3}{\left(\frac{\pi d^2}{4}\right)} \leq 60\ N/mm^2 \Rightarrow d \geq 20.6\ mm$$

$\underline{d = 21mm}$

next part : Find mininum thickness of the disk.

Not to tail:

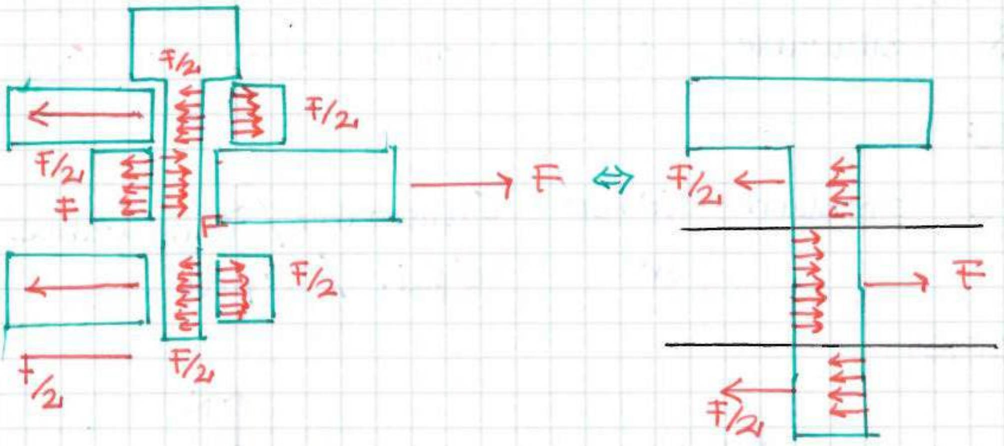
$\tau \leqslant \tau_{allowable}$

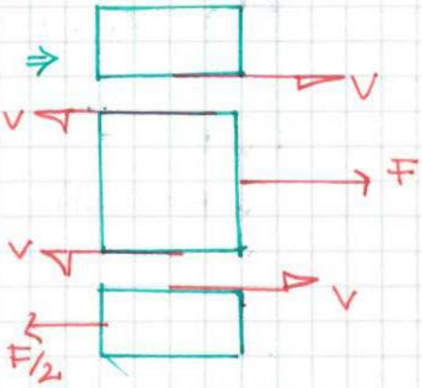
$$\frac{20 \times 10^{3} N}{2\pi \times 20 \times t} \leqslant 35 \, N/mm^{2}$$

$$\underline{\underline{t \geqslant 4,55 \ mm}}$$

example 6:

Pin support:





$V = F/2$

$$\tau = \frac{V}{\text{Shear area}} < \tau_{allowable}$$

* If joint has many bolts force distribute among bolts.

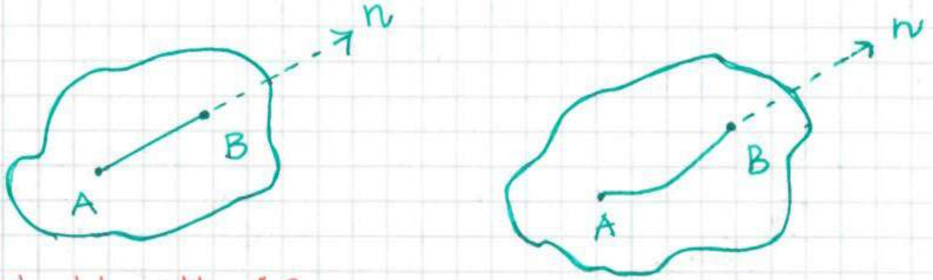
Note:

* If 2 shear planes $V = F/2 \Leftrightarrow \tau = \frac{F/2}{\text{Shear area}}$
* If one shear plane $V = F \Rightarrow \tau = \frac{F}{\text{shear area}}$

chapter 2 : strain

To decribe this deformation in length of line segment and changes in angles ⇒ STRIAN

Normal strain (ε)



original length ΔS

Final length ΔS'

$$\text{strain } (\varepsilon) = \frac{\text{Final length} - \text{original length}}{\text{original length}}$$

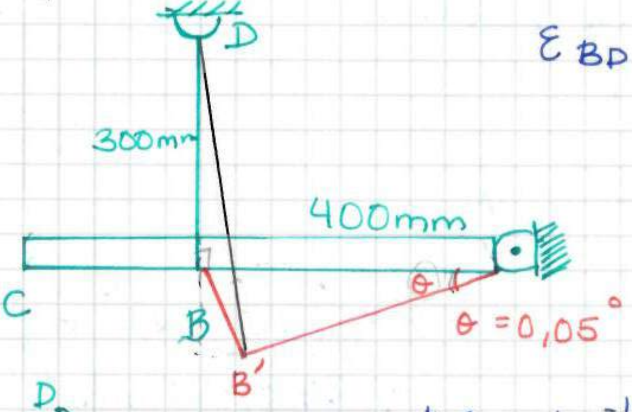
$$\text{strain}(\varepsilon) = \lim_{B \to A} \frac{\Delta S' - \Delta S}{\Delta S} \Rightarrow \Delta S' = (1+\varepsilon) \cdot \Delta S$$

(final length)

Units - no dimension less (mm/mm)

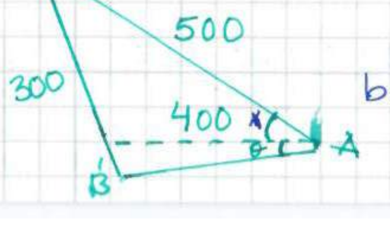
$$\varepsilon \begin{cases} -\text{ve} & \text{contraction} \\ +\text{ve} & \text{elongation} \end{cases}$$

example (1)



$$\varepsilon_{BD} = \frac{DB' - DB}{DB}$$

$DB' = 2$



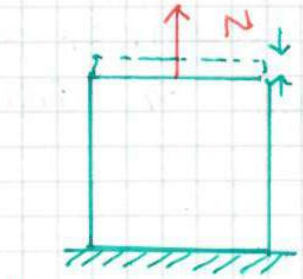
$$* \ \theta = \tan^{-1}\left(\frac{300}{400}\right) = 36{,}87^{\circ}$$

by cosine rule :

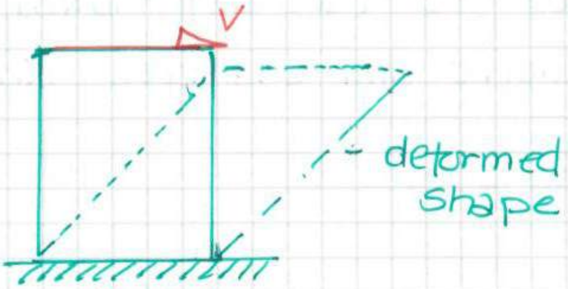
$$DB' = \sqrt{400^2 + 500^2 - (2 \times 400 \times 500 \times \cos 36{,}87)}$$

DB = 300.3491 mm

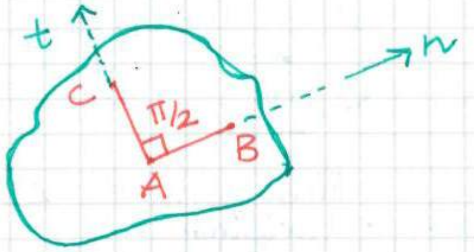
$$\varepsilon_{BD} = \left(\frac{300{,}3491 - 300}{300}\right)\frac{mm}{mm} = \underline{\underline{0{,}00116\ mm/mm}}$$

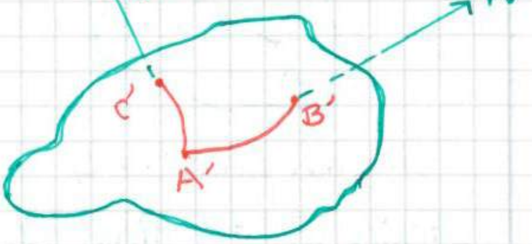


(translate)
Normal strain



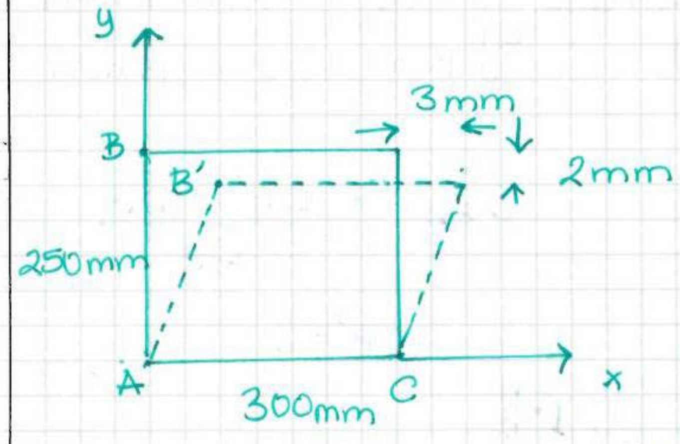
Rotate



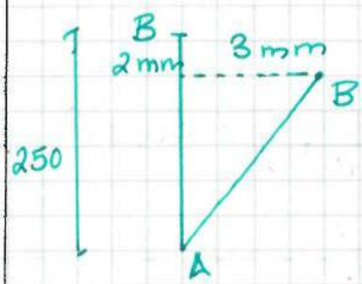


shear γ
strain nt $= \dfrac{\pi}{2} - \underset{\substack{B \to A \text{ along } n \\ C \to A \text{ along } t}}{\lim}\theta'$
relative to nt axes at corner A

example 2



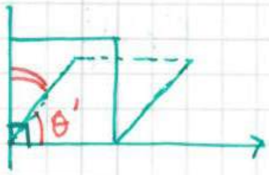
$$\varepsilon_{AB} = \frac{L_{AB'} - L_{AB}}{L_{AB}}$$



$$AB' = \sqrt{(250-2)^2 + 3^2}$$
$$= \underline{248{,}018\,mm}$$

$$\varepsilon_{AB} = \frac{248{,}018 - 250}{250} = \underline{\underline{-7{,}93\times10^{-3}\,mm}}$$

(b) $\gamma_{xy} = \frac{\pi}{2} - \theta'$



$(\pi/2 - \theta')$.

$\gamma_{xy} = \tan^{-1}\left(\frac{3}{250-2}\right) = \underline{0.121 \text{ rad}}$

chapter 3 : mechanical properties of Materials

$\sigma = f(\varepsilon)$

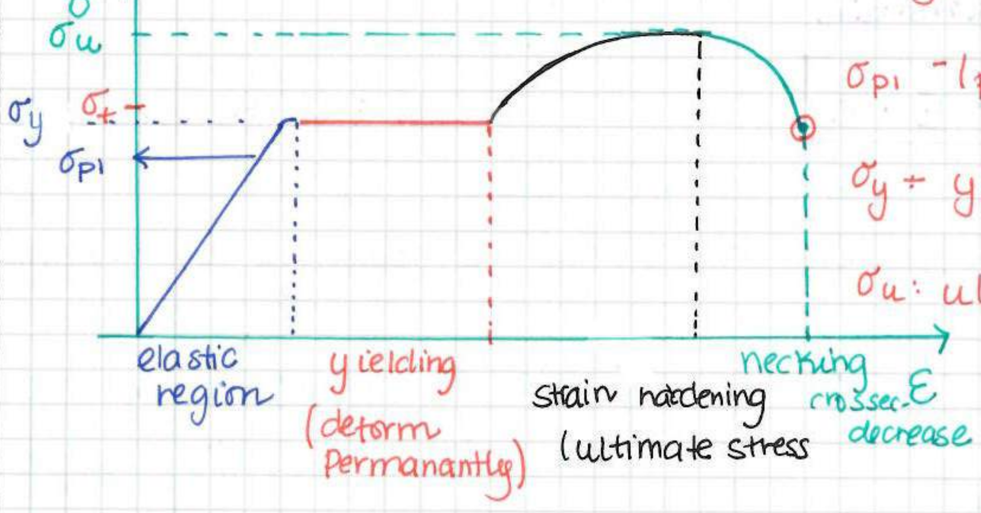
relation between σ and ε.

load (P)	displacement

P/A = Stress (engineering) or nominal stress.
↑ orignal area

δ/L_0 = strain (engineering) also called nominal strain
↑ original length

conventional stress-strain diagram



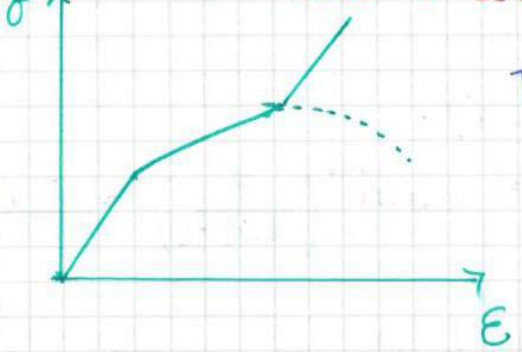
σ_{pl} - (propotional limit)

σ_y : yield stress

σ_u : ultimate stress

σ_f : fracture point

True stress strain digram



True stress = $\sigma_T = \frac{F}{A}$

Behaviours

Ductile materials

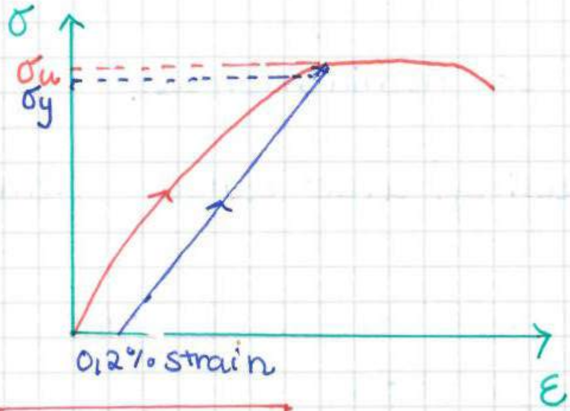
→ yield can be found easily & well defined σ_y

- difficut to σ_y ⇒ offset method

Brittle materials

- breaks without necking, without yielding.

Offset method



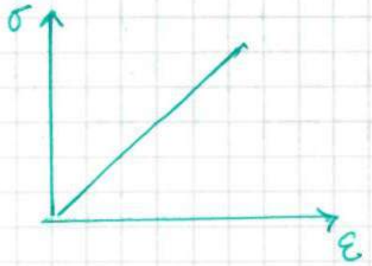
σ_y : yeilding strength

$\boxed{\sigma = E \cdot \varepsilon}$

E : a Konstant [N/m^2 or P

Valid only to the linear portion

E = youngs modulus (modulus of elasticity)



$\tan\theta = E$ (gradiant of thisline)

considering elastic section only

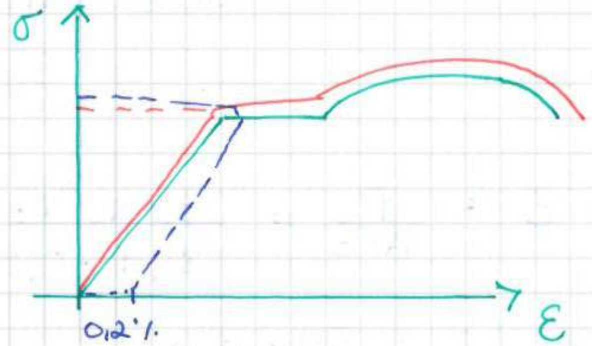
17/02

Summary ch 3

Hooke's law

$\sigma = E \cdot \varepsilon$

↑ young's modulus



Ductile material
necking

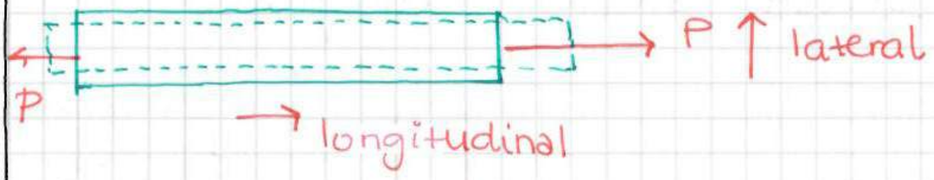
Brittle materials
- easy to break

→ yield stress - well defined

→ If yield not well defined then use offset method (0,2% of strain)

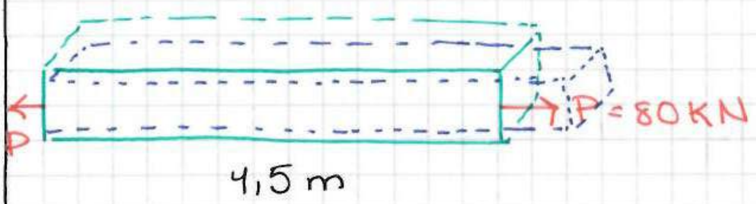
Poisson's ratio

- Ratio of the lateral & longitudinal strains

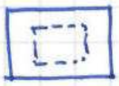


$$\boxed{\nu \text{ - poisson's ratio} = -\frac{\varepsilon_{lateral}}{\varepsilon_{longitudinal}}}$$

Example 2.



$\delta_z = ?$



cross section will reduce

δ_x an δ_y = ?

$$\sigma = E \cdot \varepsilon$$

$$\frac{P}{A} = E \cdot \varepsilon \Rightarrow \varepsilon = \frac{P}{A \cdot E} = \frac{80 \cdot 10^3 N}{0,1 \cdot 0,05} = 16 \cdot 10^6 Pa$$

$$\varepsilon = \frac{L_f - L_o}{L_o} \Rightarrow L_f = \varepsilon \cdot L_o + L_o$$

$$=$$

$$\delta_z = 120 \mu m$$

$$\varepsilon_x = \varepsilon_y = -25,6$$

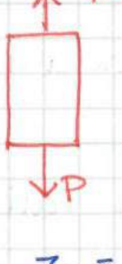
$$\nu_{st} = -\frac{\varepsilon_x}{\varepsilon_z} = +0,32$$

$$\varepsilon_x = \varepsilon_z \cdot 0,32 = 120 \mu m \cdot 0,32 = -25,6$$

$$\delta_x = \varepsilon_x \cdot L_x = -(25,6 \times 10^{-6}) \cdot 0,1 m = \underline{\underline{-2,56 \mu m}}$$

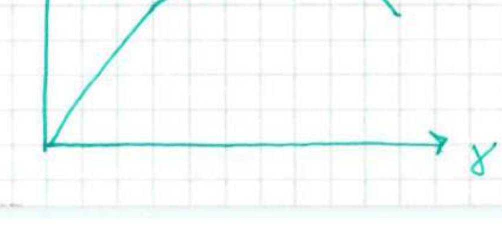
$$\delta_y = \underline{\underline{-1,28 \mu m}}$$

The shear stress- diagram



Hooke's law

$\sigma = E \cdot \varepsilon$



$\tau = G \gamma$

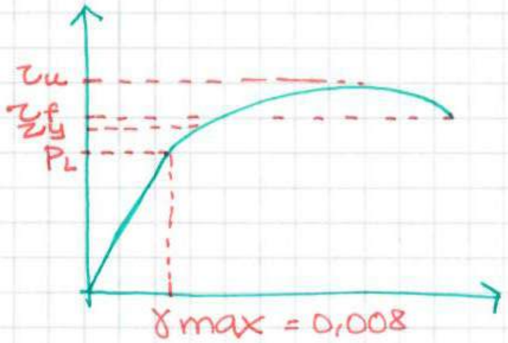
↓

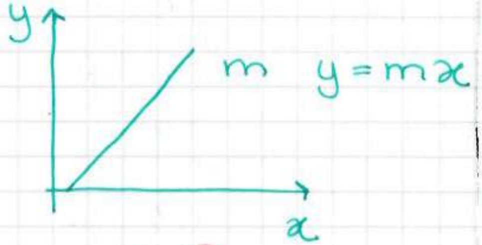
shear modulus of elasticity (modulus of rigidity)

$$\Rightarrow G = \frac{E}{2(1+\nu)}$$

Example 3 :

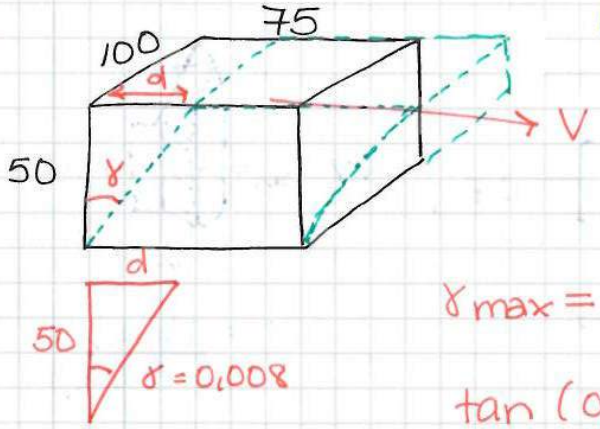
Find the shear modulus. $G = ?$





$\tau = \underline{G} \cdot \gamma$

G is found from the gradient of the propotional curve.



Find max distance d.

What is V necessary to cause displacement.

$\gamma max = \frac{d}{50}$

$\tan(0{,}008) = d/_{50}$

$\Downarrow$

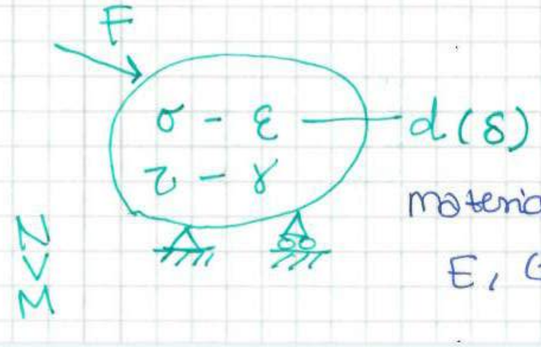
$0{,}008 = d/_{50} \Rightarrow \underline{\underline{d = 0{,}4 m}}$

$\tau = \frac{V}{A}$ (to find dmax we consider $\gamma_{max} \to \tau$)

$\tau_{pl} = 360 = \frac{V}{A}$ $V = 360 \times (75 \times 100)$

$\underline{\underline{V = 2700 kN}}$

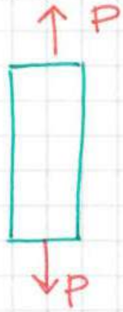
Chapter 4 : Axial load



members which are subjected to tension or compression. axially

material properties

E, G, ν



Hooke's law

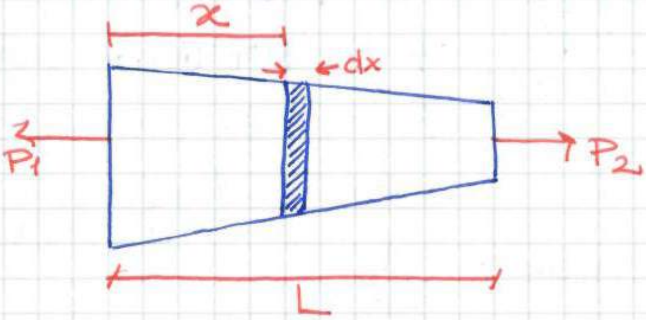
$$\sigma = E \cdot \varepsilon$$

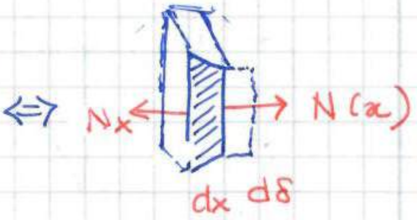
$$P/A = E \frac{\delta}{L} \Rightarrow \boxed{\delta = \frac{PL}{EA}}$$

is valid when

- → A constant
- → P constant
- → E ——"——
- → L ——"——

<u>In general</u> :





$$\sigma = \frac{N(x)}{A(x)}$$

$$\varepsilon = \frac{d\delta}{dx}$$

$$\sigma = E \cdot \varepsilon \Rightarrow \frac{N(x)}{A(x)} = E \cdot \frac{d\delta}{dx}$$

$$d\delta = \frac{N(x)}{E \cdot A(x)} dx$$

$$\boxed{\delta = \int_0^L \frac{N(x)}{E \cdot A(x)} dx}$$

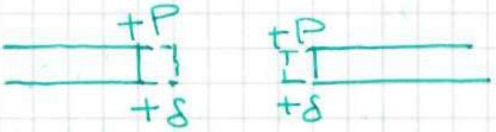
if P and A is constant

$$\delta = \int_0^L \frac{P}{EA} dx = \frac{PL}{EA}$$

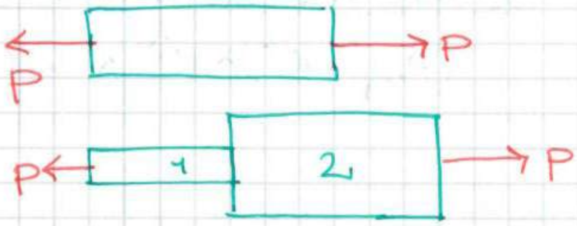
Sign convention

Tensile (+)

elongation (+)



Constant load and cross-sectional area :



$$\delta = \frac{PL}{EA}$$

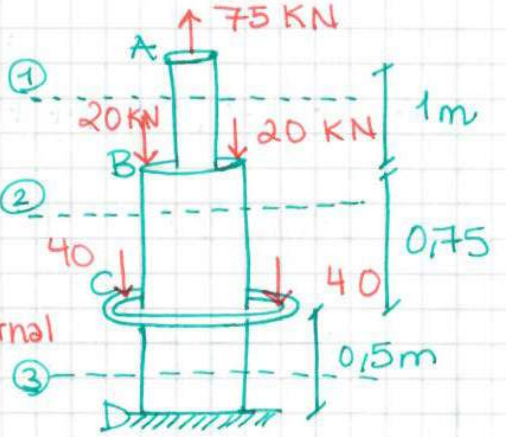
$$\delta = \Sigma \frac{PL}{EA}$$

$$\delta = \frac{P_1 L_1}{E_1 A_1} + \frac{P_2 L_2}{E_2 A_2}$$

example (1)

vertical displacement

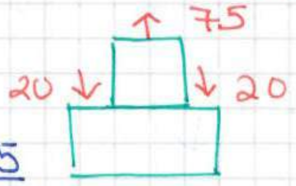
$$\delta_A = \delta_{A/B} + \delta_{B/C} + \delta_{C/D}$$



(delta A relative to B $\delta_{A/B}$)

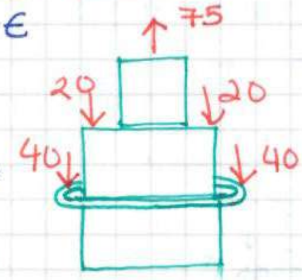
$$\delta_{A/B} = \frac{PL}{AE}$$ (P is "N" an internal force).

$$= \frac{75 \times 1m}{600 \times 10^{-6} \times E}$$



N = 35 KN

$$\delta_{B/C} = \frac{PL}{AE} = \frac{35 \times 0,75}{1200 \times 10^{-6} E}$$



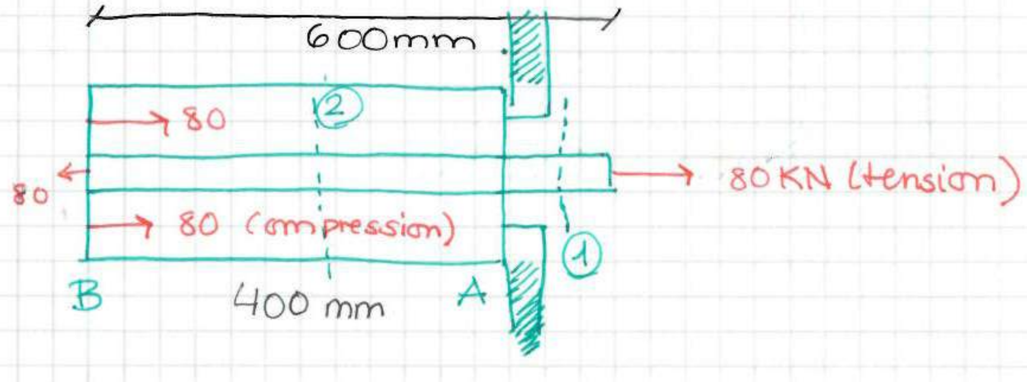
$$\uparrow \quad 75 - 40 - 80 - N = 0$$

N = −45 KN

$$\delta_{C/D} = \frac{PL}{AE} = \frac{-45 \times 0,5}{1200 \times 10^{-6} \cdot E}$$

$$\delta_A = \underline{\underline{0,641 \text{ mm}}}$$

example (2) :



$\delta_C = \delta_{C/B\ (\text{steel bar})} + \delta_{B/A\ (\text{alminum tube})}$

$$\delta_{C/B} = \frac{PL}{A\cdot E_{st}} = \frac{80\times10^{3}\times 6000\,mm\times10^{-3}\,(m)}{(\pi/4\times10^{2})\times10^{-6}\times200\times10^{9}\ N/m^2}$$

$\delta_{C/B} = \underline{0{,}003056\,m}$

Similarly for alminium bar tube : (in compression

$$\overset{\leftarrow}{\delta_{B/A}} = \frac{PL}{A\cdot E_{AT}} = -\frac{80\times10^{3}\times400\times10^{-3}}{(400\times10^{-6})\times70\times10^{9}}$$

$$= -\,0{,}00114\ m$$

$\delta_C = +0{,}00114 + 0{,}003056 =$

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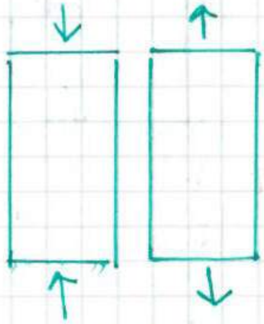
Chapter 4 (continued...)

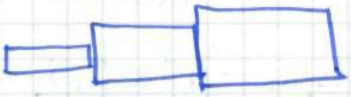


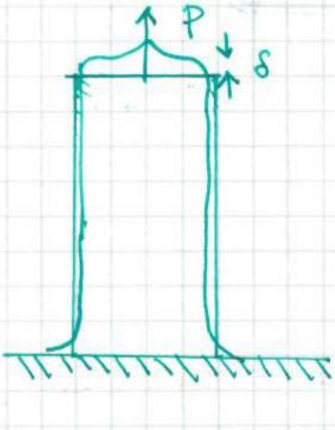
Elongation:

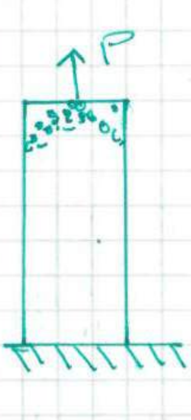
$$\boxed{\delta = \int_0^L \frac{N(x)}{A(x)}\cdot\frac{dx}{E}}$$

For constant load & costant crossection

$$\boxed{\delta = \sum \frac{PL}{AE}}$$

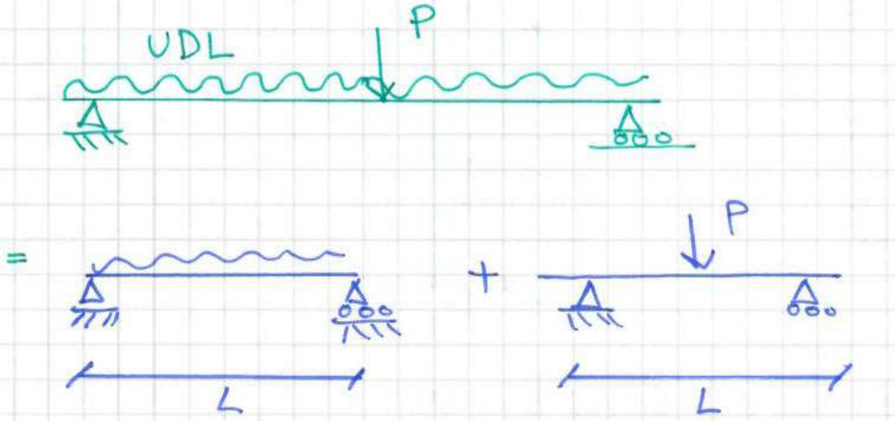




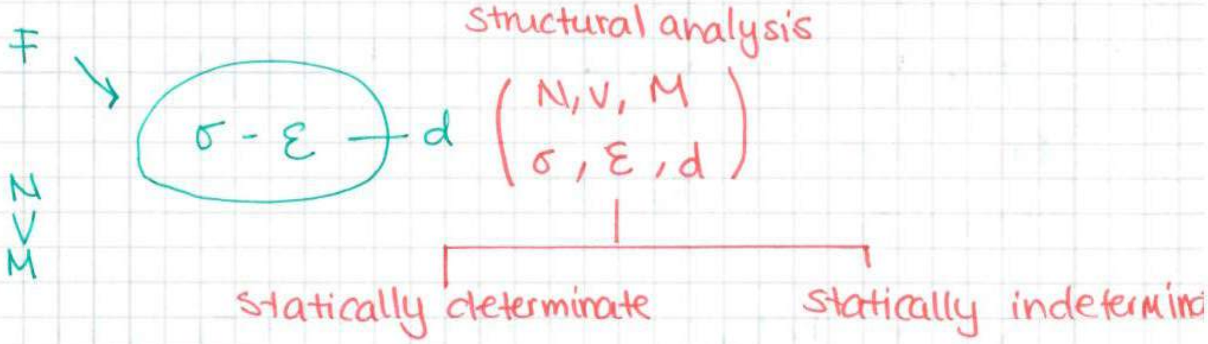


$\delta = \frac{PL}{AE}$

principle of superposition :



4.4 Statically indeterminate Axially loaded member :



Statically determinate conditions :

\# 1. Equilibrium

\# 2. $\sigma - \varepsilon$ $\sigma = E \cdot \varepsilon$ $\tau = G \cdot \gamma$

$F = k \cdot d$

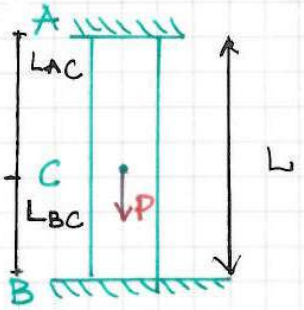
constitutive relations

Statically indeterminate conditions :

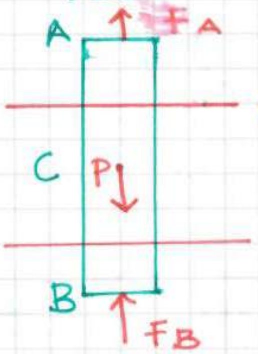
\# 1. Equilibrium

* Compatibility

\# 3 Constitutive relations



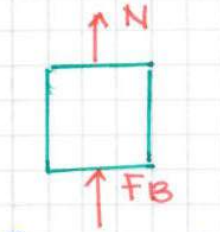
FBD :



1. Equilibrium

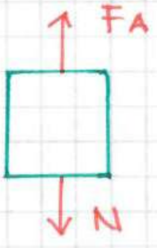
$$\uparrow \; F_A + F_B - P = 0 \qquad ①$$

2. Compatibility (displacement)



$F_B = -N$

$N = -F_B$



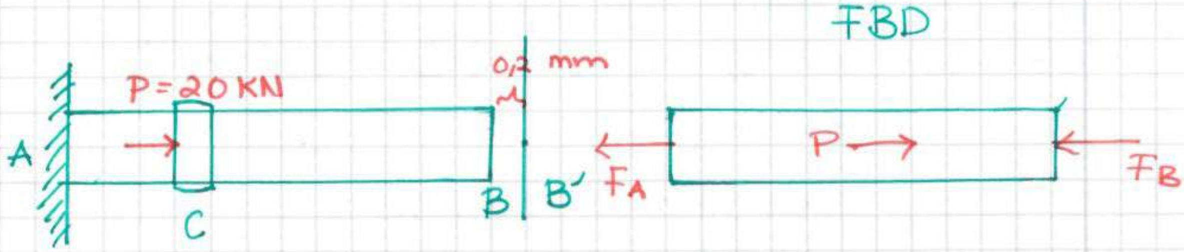
$F_A = N$

$$\delta_{A/B} = 0 = \delta_{A/C} + \delta_{C/B}$$

$$= \frac{F_A L_{AC}}{EA} - \frac{F_B \cdot L_{CB}}{EA} \qquad ②$$

$$\delta_{A/C} = \frac{F_A L_{AC}}{AE} \qquad \delta_{C/B} = -\frac{F_B \cdot L_{CB}}{AE}$$

example 3



Equilibrium :

$F_A + F_B - P = 0 \qquad ①$

$\delta_{B/A} = 0,2 \text{ mm}$

$= \delta_{B/C} + \delta_{C/A} = 0,2 \text{ mm}$

$$= \frac{F_A L_{AC}}{AE} + \frac{(-F_B) \cdot L_{CB}}{AE} = 0,2 \text{ mm}$$

Thermal stress 4.6

Thermal stress : change in temperature

tem ↑ → expansion

tem ↓ → contraction

if material ; homogenous / isotropic

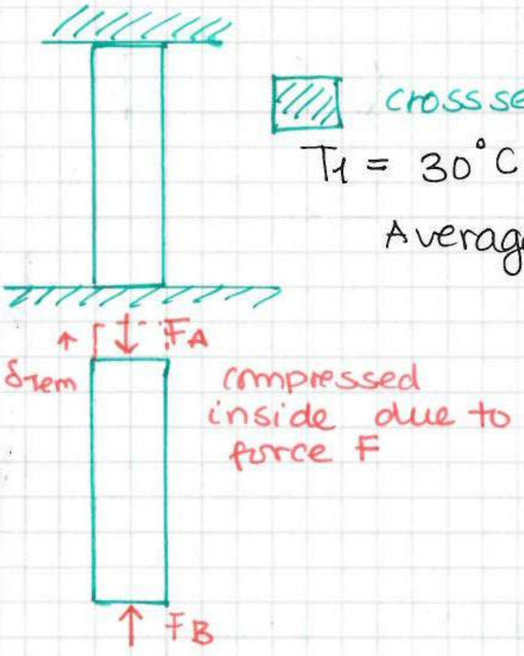
$$\boxed{\delta_T = \alpha \, \Delta T \cdot L}$$

L - original length.

α - linear coeffisient of thermal expansion

ΔT - change in temperature of the member

example 4



▨ cross section 10 mm × 10 mm

$T_1 = 30°C \quad T_2 = 60°$

Average normal stress

$F_A = F_B = F$

compatibility :

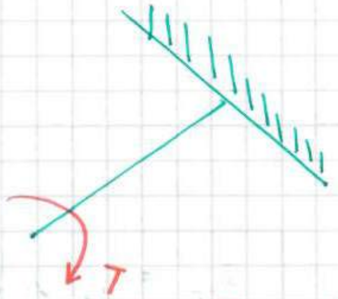
$$\delta_{A/B} = 0 = \delta T - \delta_F = 0$$

$$\alpha \cdot \Delta T \cdot L = \delta_F$$

$$\alpha \cdot \Delta T \cdot L = \frac{FL}{AE}$$

$$\underline{F = 7.2 \text{ KN}}$$

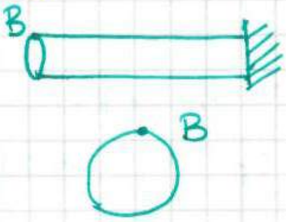
Chapter 5 : Torsion

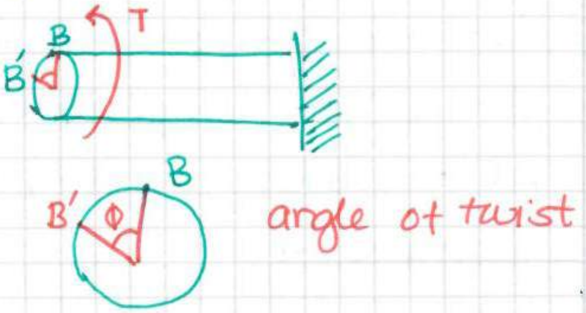


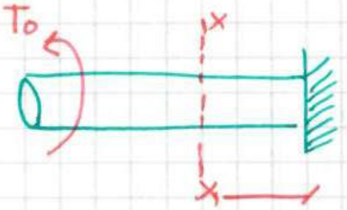
Due to this moment tends to rotate twisting → TORSIO

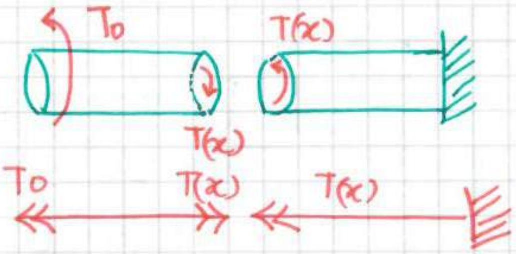
It is a moment - torque

Angle of twist (Φ)







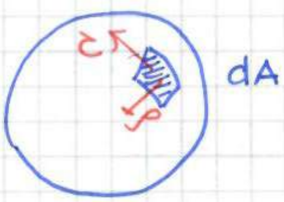


Sign convention :

Use right hand rule

τ τ

1. Equilibrium



$$T(x) = \int_A \tau \cdot dA \cdot \rho$$

τ - shear stress

Assumptions :

(1) Twisting causes circle to remain circle

(2) cross section remains flat.

(3) Radial lines remain straight during deformation

(4) length of the shaft and its radius remain unchanged.

c radius of the bar

$$\frac{\tau_{max}}{c} = \frac{\tau}{\rho}$$

$$\boxed{\tau = \frac{\tau_{max}}{c} \cdot \rho}$$

Then

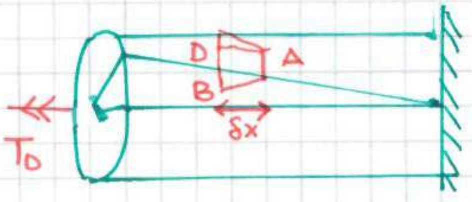
$$T(x) = \int_A \frac{\tau_{max}}{c} \rho^2 \cdot dA$$

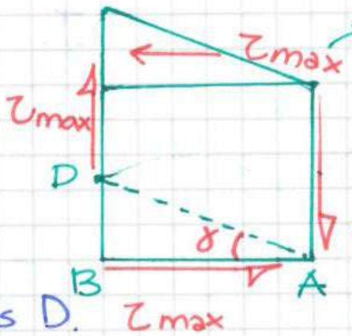
$$T(x) = \frac{\tau_{max}}{c} \underbrace{\int_A \rho^2 \cdot dA}_{J}$$

J - polar moment of Inertia

$$\boxed{\frac{T(x)}{J} = \frac{\tau_{max}}{c} = \frac{\tau}{\rho} \quad - ①}$$

(2) Hooke's law



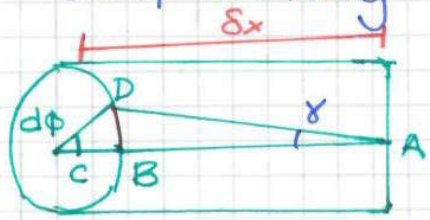


As considered element is located on the top of the surface

due to twisting B moves D.

$$\boxed{\tau_{max} = G\gamma} \quad ②$$

(3) Compatibility



$$\boxed{BD = dx \cdot \gamma = c \cdot d\Phi} \quad ③$$

From equation ② & ③

$$\tau_{max} = G\cdot\gamma = G\cdot c\cdot\frac{d\phi}{dx}$$

$$\frac{\tau_{max}}{c} = \frac{G\,d\phi}{dx}$$

From equation ①

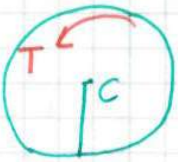
$$\boxed{\frac{T(x)}{J(x)} = \frac{\tau_{max}}{c} = \frac{\tau}{\rho} = G\,\frac{d\phi}{dx}}$$

For constant torque 'T' & constant area throught the length L of the bar material is homogeneous (G-constant). (G = shear modulus of elasticity)

Torsion formula

$$\frac{T}{J} = \frac{\tau_{max}}{c} = \frac{\tau}{\rho} = \frac{G\cdot\phi}{L}$$

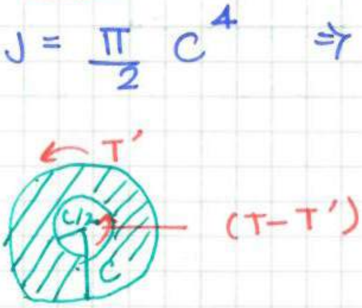
example (1)



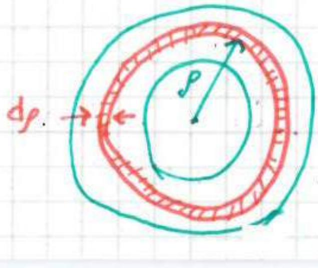
$$\frac{T}{J} = \frac{\tau_{max}}{c}$$

$$T = \frac{\tau_{max}\cdot J}{c}$$

$$J = \frac{\pi}{2}c^4 \Rightarrow T = \frac{\tau_{max}\,\pi/_2\,c^4}{c}$$



lets take a small section from the outer circle.



$$dT' = \rho\cdot\tau\,dA$$

$$T' = \int\rho\cdot\tau\cdot dA\ (2\pi\cdot\rho\cdot d\rho)$$

From torsion formula : $\frac{\tau}{\rho} = \frac{\tau_{max}}{c}$

$$T' = \int_{c/2}^{c} \rho \left(\frac{\tau_{max} \cdot \rho}{c}\right) (2\pi \rho \cdot d\rho)$$

$$= 2\pi \cdot \frac{\tau_{max}}{c} \int_{c/2}^{c} \rho^3 \, d\rho$$

$$T' = \frac{15\pi}{32} \tau_{max} \cdot c^3 \quad ②$$

Combining eq. ① & ②

$$T = \frac{\tau_{max}}{c} \cdot J \quad ① \qquad \tau_{max} = \ldots\ldots$$

$$T' = \frac{15\cancel{\pi}}{3\cancel{2}16} \cdot \frac{T\cancel{c}}{\cancel{\pi/2}_{1}\, \cancel{c^4}} \cdot \cancel{c^3}$$

$$\underline{\underline{T' = 15/16 \, T}}$$

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5,2 Torsion formula

Torsion formula:

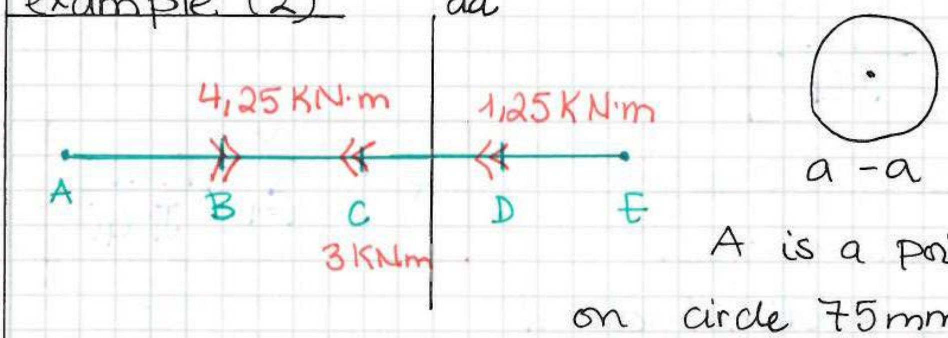
$$\frac{T(x)}{J(x)} = \frac{\tau_{max}}{c} = \frac{\tau}{\rho} = G \cdot \frac{d\varphi}{dx}$$

if torque T - constant
area A - constant } throughout
Length L of the bar

Material is homogenous : (G - constant)

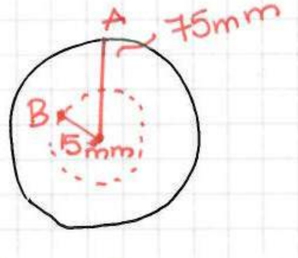
$$\boxed{\frac{T}{J} = \frac{\tau_{max}}{c} = \frac{\tau}{\rho} = \frac{G\varphi}{L}}$$

example (2)

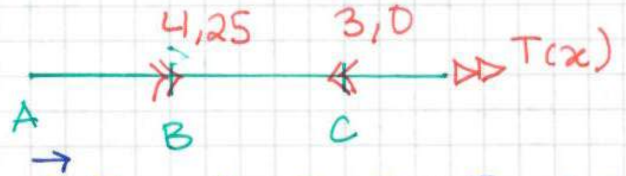


A = C = 75mm

A is a point which lie on circle 75mm radius.



B - is a point on the circle (15mm radius)



$\tau_A = ?$

$\tau_B = ?$

$$\vec{\Sigma F}\ T = 4{,}25 - 3 + T(x) = 0$$

$$\underline{T(x) = -1{,}25\ \text{KNm}} \ \leftarrow\leftarrow$$

Torsion formula:

$$\frac{T(x)}{J} = \frac{\tau}{\rho} \Rightarrow \tau_A = \frac{TC}{J}$$

$$J = \frac{\pi}{2} \times 0{,}075^4\ m^4 = 4{,}97 \times 10^{-5}\ m^4$$

$$= \frac{1{,}25 \times 0{,}075}{4{,}97 \times 10^{-5}}$$

$$\underline{\underline{\tau_A = 1886\ \text{KPa}}}$$

$$\tau_B = \frac{1{,}25 \times 0{,}015}{4{,}97 \times 10^{-5}} = \underline{\underline{377{,}3\ \text{KPa}}}$$

5.4 Angle of twist

From torsion formula

$$G \cdot \frac{d\varphi}{dx} = \frac{T(x)}{J(x)}$$

$$\varphi = \int_0^L \frac{T(x)}{J(x)} \cdot \frac{dx}{G}$$

If T constant

A - constant

G - constant

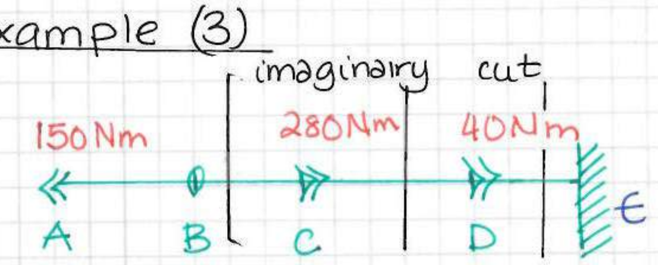
$$\boxed{\varphi = \frac{TL}{JG}}$$

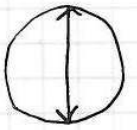
$$\boxed{\varphi = \sum_{i=1}^{n} \frac{T_i L_i}{J_i G_i}}$$

Multiple torques

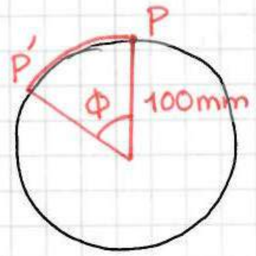
- Several torques
- Cross sectional areal
- G - shear modulus may change.

Example (3)





Circular shaft
14 mm



$PP' = ?$

$PP' = (100\varphi)$

$$\varphi_{A/E} = \varphi_{A/C} + \varphi_{C/D} + \varphi_{D/E}$$

$$= \frac{T_{AC} \cdot L_{AC}}{JG} + \frac{T_{CD} \cdot L_{CD}}{JG} + \frac{T_{DE}\ L_{DE}}{JG} \ *$$

$0 \leqslant x < 0{,}4$

$T(x) = 150\ Nm$

$0{,}4 < x < 0{,}7$

$T(x) + 280 - 150 = 0$

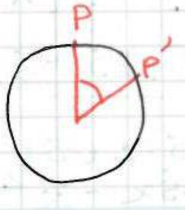
$T(x) = -130\ Nm$

$0{,}7 < x \leqslant 1{,}2$

$T(x) = -170\ Nm \quad (-150 + 280 + 40 + T(x) = 0)$

$$\varphi_{AE} = \frac{150 \times 0{,}4}{JG} + \frac{(-130) \times 0{,}3}{JG} + \frac{(-170) \times 0{,}5}{JG}$$

$\underline{\varphi_{AE} = -0{,}2121 \text{rad}}$



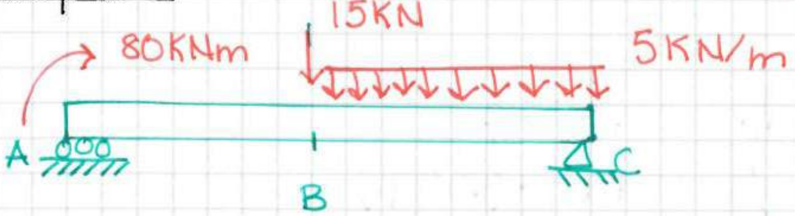
$PP' = 100 \times (0{,}2121)$

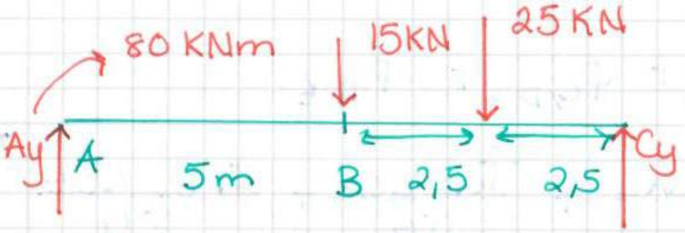
$\underline{PP' = 21{,}2 mm}$

Chapter 6 : Bending

shear and momemen diagram :

example 1





Applying equation of equilibrium :

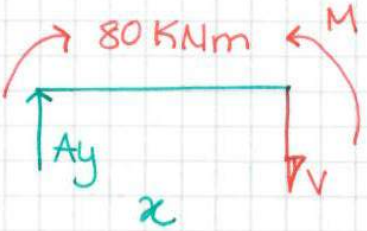
$\rightarrow C_x = 0$

$\uparrow \Sigma = Ay + Cy - 15 - 25 = 0 \Rightarrow Cy = 34.25 \text{ KN}$

$\overset{\curvearrowleft}{C} = 2{,}5 \times 25 + 5 \cdot 15 \text{ KN} - Ay \cdot 10 - 80 \text{ KNm} = 0$

$Ay = 5{,}75 \text{ KN}$

$0 \leq x < 5$



$Ay - V = 0$

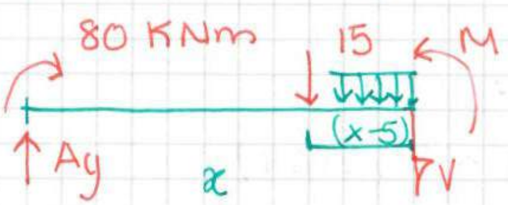
$\underline{V = Ay} = 5{,}75 \text{ KN}$

$\overset{\curvearrowleft}{M} - Ay \cdot x = 80$

$M = 80 + Ay \cdot x$

$M = 5{,}75x + 80$

$5 < x < 10$

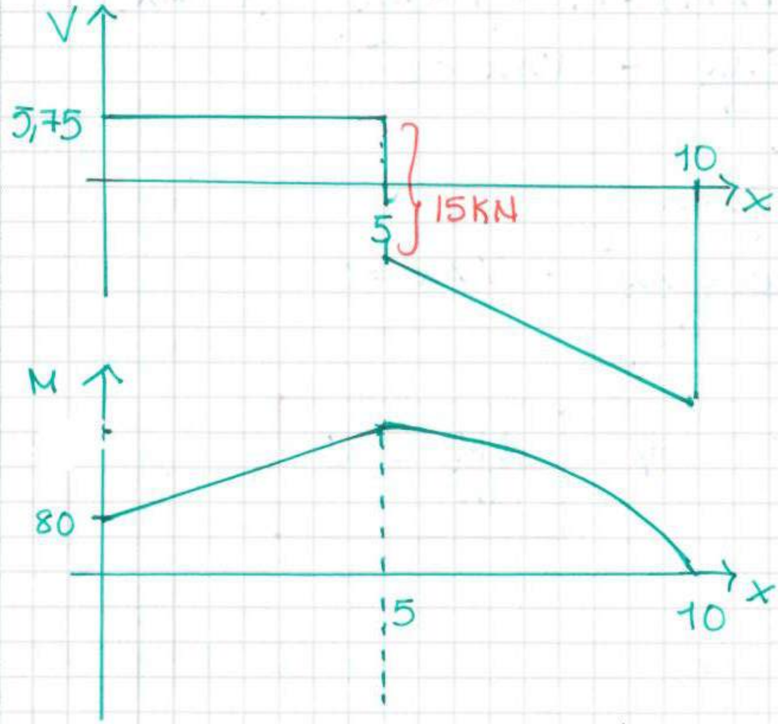


$5{,}75 - 15 - 5(x-5) - V = 0$

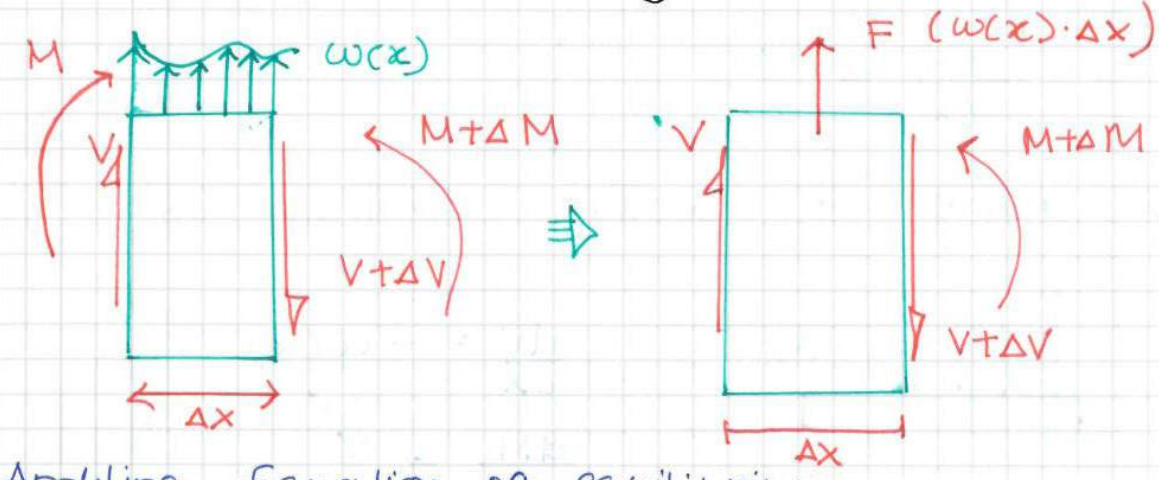
$\underline{V = 15{,}75 - 5x}$

$\overset{\curvearrowleft}{M} + 5(x-5)(x-5)/2 + 15(x-5) - 5{,}75x - 80 = 0$

$M = -2,5x^2 + 15,75x + 92,5$



6,2 Graphical method for constructing shear and moment diagrams



Applying Equation of equilibrium

$\uparrow \; v + F - (V + \Delta V) = 0$

$F - \Delta V = 0$

$w(x)\,\Delta x - \Delta V = 0$

$\Delta V = w(x) \cdot \Delta x$

$\frac{\Delta V}{\Delta x} = w(x)$

$$\boxed{\left(\lim_{\Delta x \to 0}\right) \frac{dv}{dx} = w(x)}$$

Similarly :

$$\overset{\curvearrowleft}{BM} = (M+\Delta M) - M - \left(F \times \frac{\Delta x}{2}\right) - (V \cdot \Delta x) = 0$$

$$\Delta M - \frac{F\Delta x}{2} - V\Delta x = 0$$

$\downarrow$

$\omega(x)\,\Delta x$

$$\Delta M - \underset{\to 0}{\cancel{\frac{\omega(x)\,\Delta x^2}{2}}} - V\Delta x = 0$$

$$\lim_{\Delta x \to 0} \frac{\Delta M}{\Delta x} = V \Rightarrow \boxed{\frac{dM}{dx} = V}$$

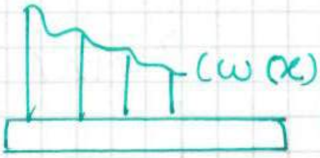
Note :

* when load $\omega(x)$ in downward direction, same expression

$$\boxed{\frac{dV}{dx} = -\omega(x)}$$

$$\boxed{\frac{dM}{dx} = V}$$

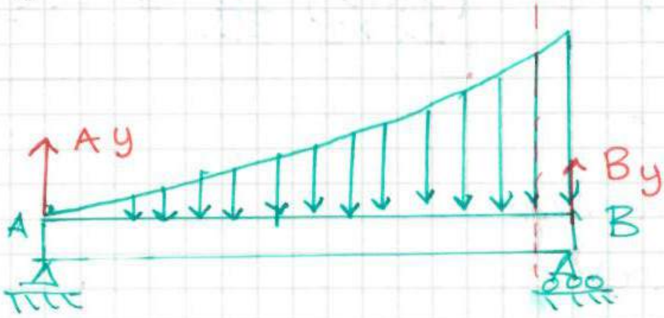
28/02 chapter 6 bending continued...

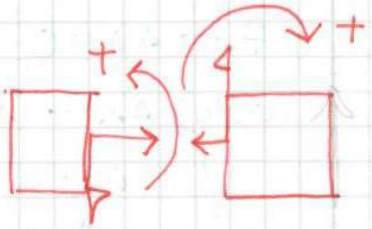


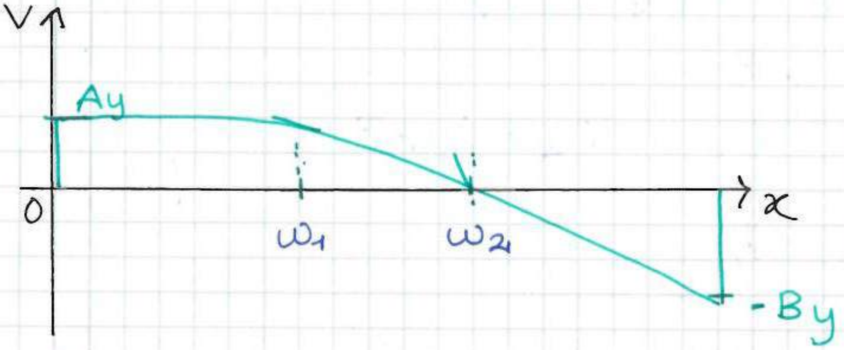
$$\frac{dV}{dx} = -\omega(x)$$

$$\frac{dM}{dx} = V$$

graphical method - to draw BMP & SFD



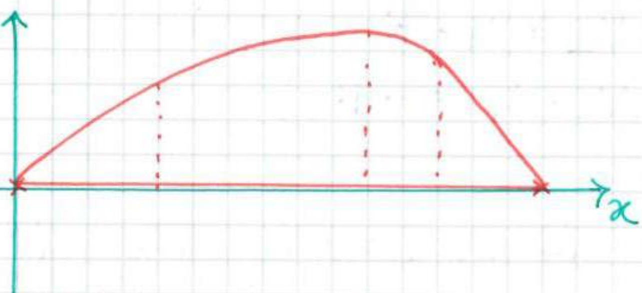




$V_A = A_y$

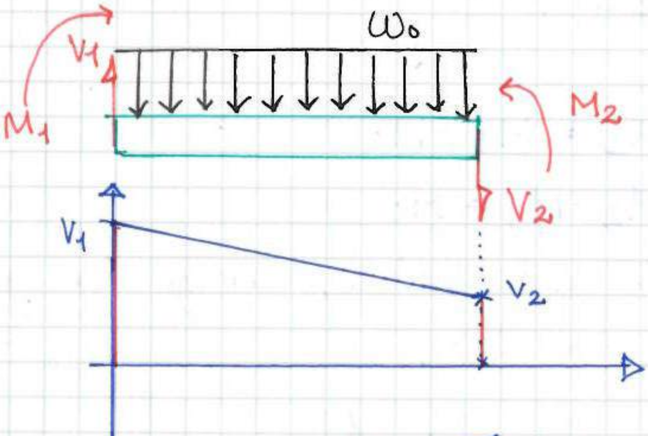
$V_B = -B_y$

Bending moment



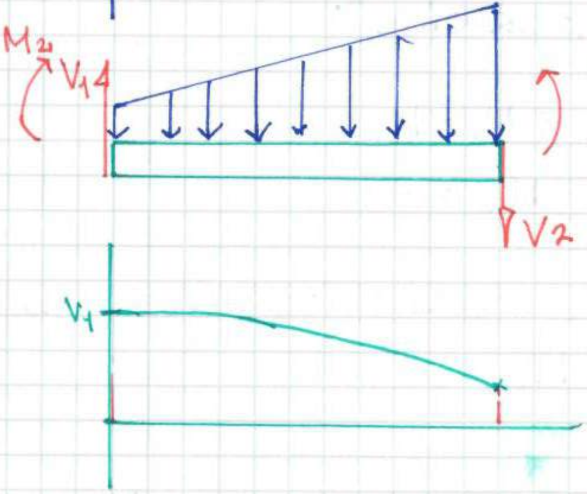
negotive value in shear diagram means slope downwar and vice versa.

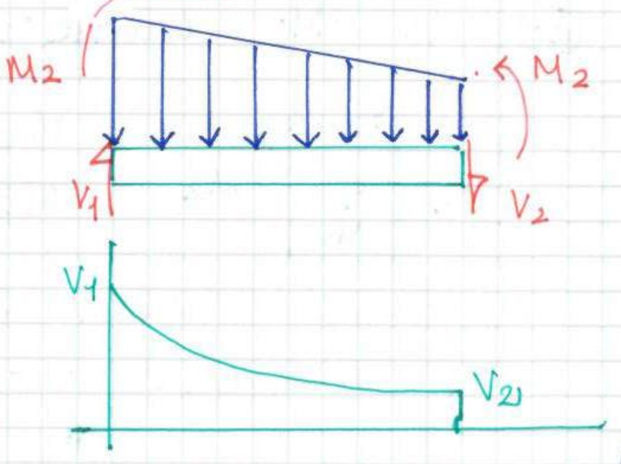
example



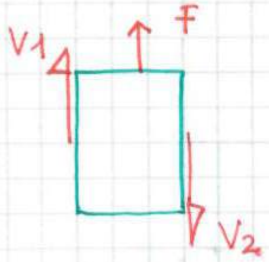
$V_1 > V_2$

$M_2 > M_1$





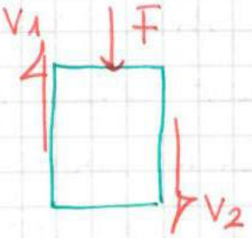
i) If F ↑ (upward)



$V_2 = V_1 + F$

when loading is upward change in shear force $= +F$ $(V_2 - V_1 = F)$ if F is upward.

(ii) If F ↓ (downward)

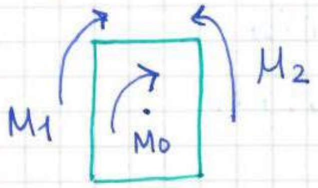


$V_2 = V_1 - F$

$V_2 - V_1 = -F$

when loading is downward change in shear $= -F$

(iii) If M_o clockwise ↷

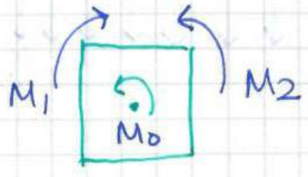


$M_2 = M_1 + M_o$

$M_2 - M_1 = M_o$

when moment is clockwise, change in moment is equal to +-ve M_o.

(iv) If Mo counterclockwise ↶

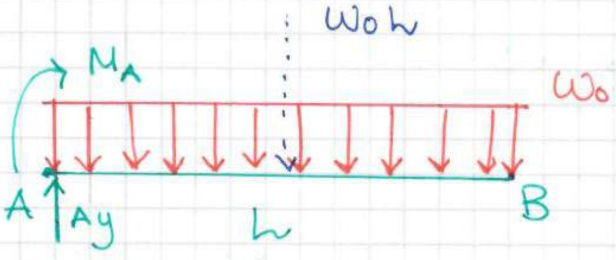


$M_2 = M_1 - M_o$

$M_2 - M_1 = -M_o$

when moment is counterclockwise change in moment $= -M_o$,

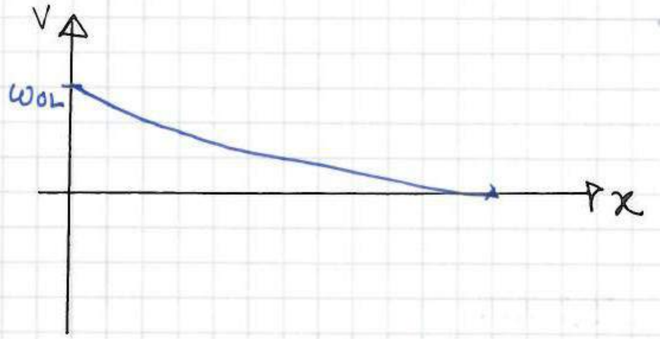
excercise 4

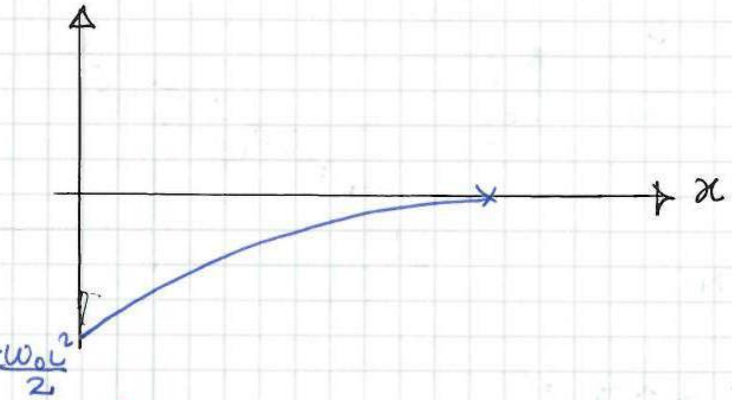


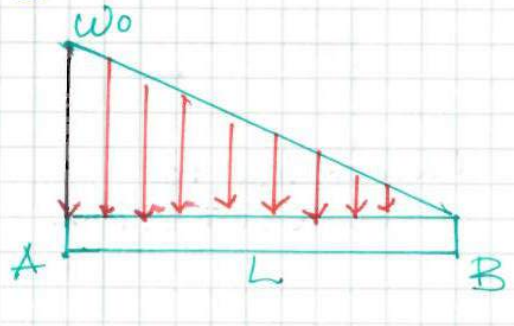
$A_y - w_0 L = 0 \qquad \underline{A_y = w_0 L}$

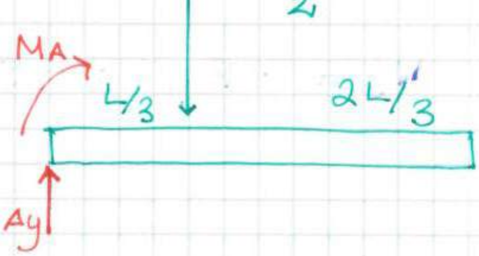
$\curvearrowright M_A + w_0 \cdot L \times L/2 = 0$

$$\underline{M_A = -\frac{w_0 L^2}{2}}$$





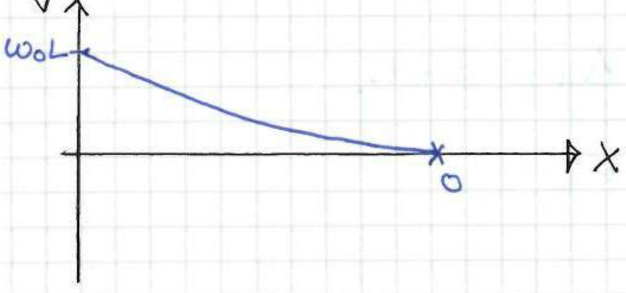




$\uparrow A_y = \frac{\omega_0 L}{2}$ $\qquad V_A = \frac{\omega_0 L}{2}$

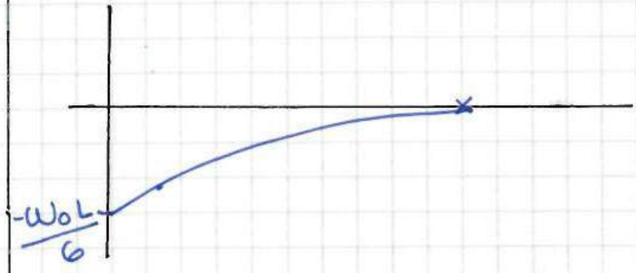
$\curvearrowright_A \quad M_A + \frac{\omega_0 L}{2} \times \frac{L}{3} = 0$

$$M_A = \frac{-\omega_0 L^2}{6}$$



$\frac{dV}{dx} = -\omega(x)$

$\frac{dM}{dx} = V$



example

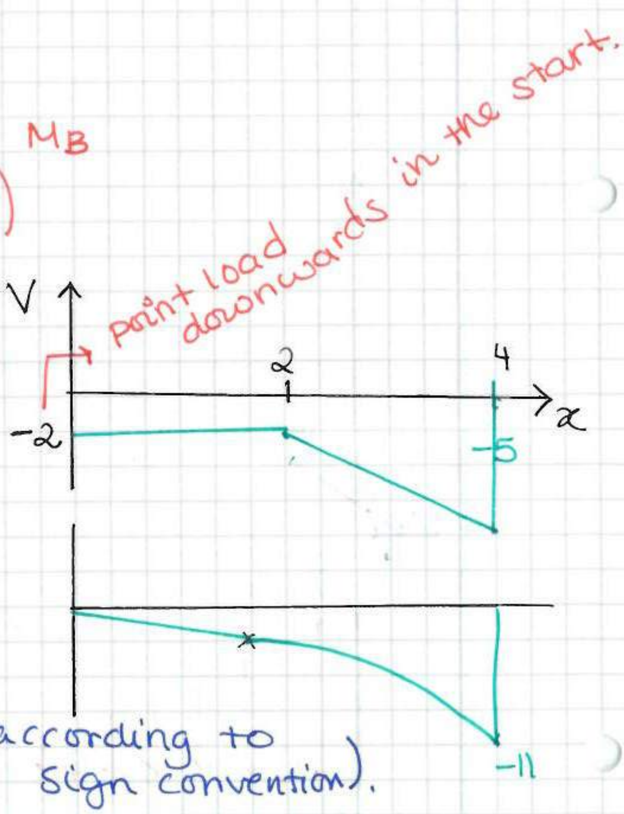
$B_y - 2 - 3 = 0$

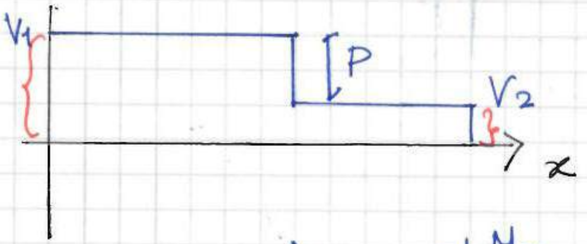
$B_y = 5\ KN$

$\curvearrowleft \; M_B + 3 \times 1 + 2 \times 4 = 0$

$M_B = -11\ KNm$

$V_B = -B_y = -5\ KN$ (according to sign convention).

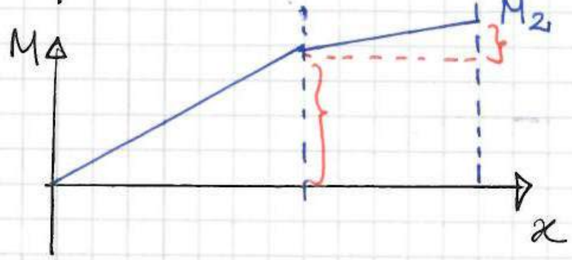




$w(x) = 0$

$\frac{dV}{dx} = 0$

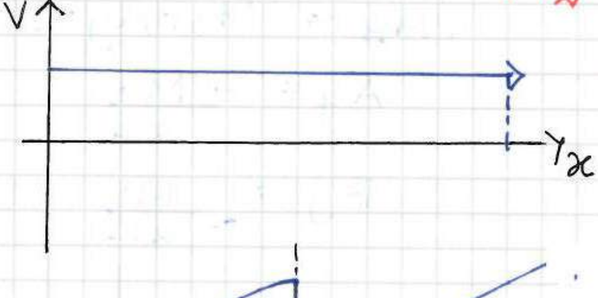
SFD

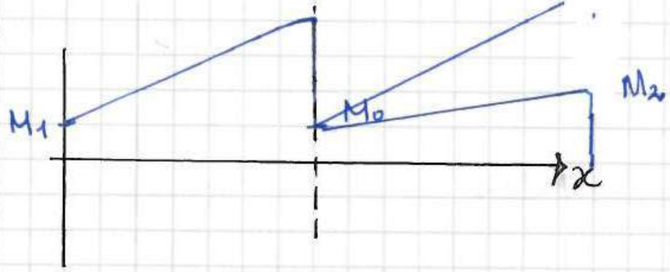


BMD

$V_1 = V_2 = V$

$\frac{dV}{dx} = 0$





$\uparrow A_y = 2P$

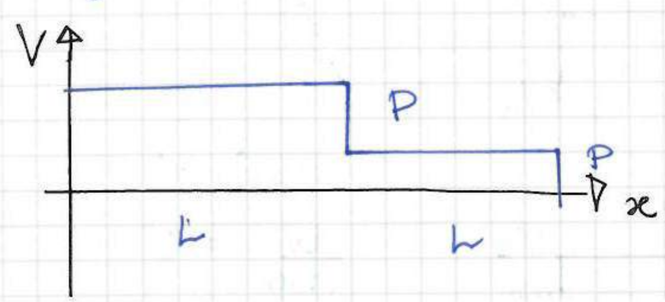
$M_A + PL + P \times 2L = 0$

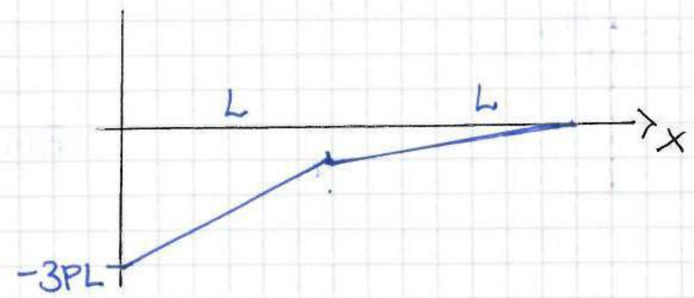
$M_A = -3PL$

$\overset{\curvearrowleft}{-M_A} - PL - 2PL = 0$

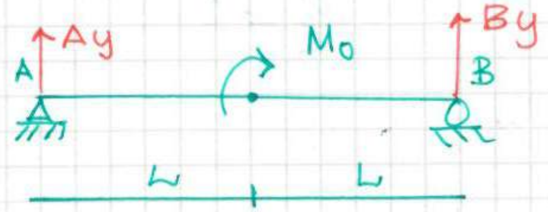
$M_A = -3PL$

$V = A_y \quad = 2P \qquad , \qquad M_A = -3PL$





<u>example</u>

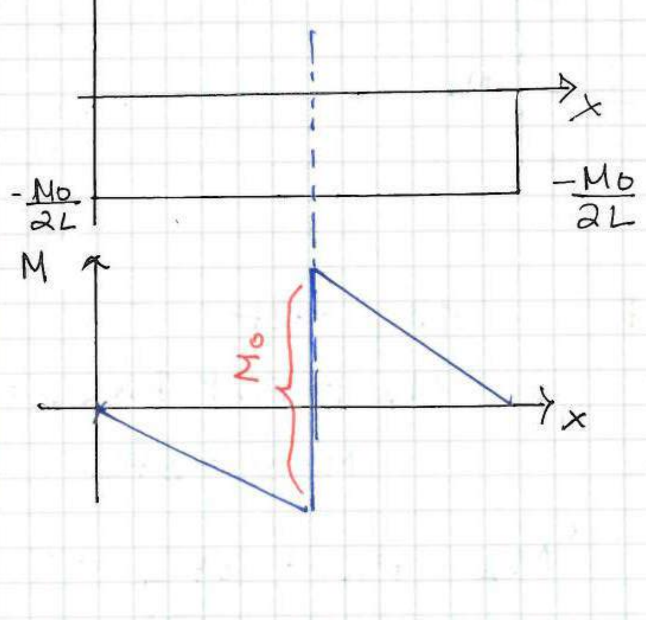


$A_y + B_y = 0$

$\curvearrowright M_B = M_0 + A_y \cdot 2L = 0$

$A_y = -\dfrac{M_0}{2L}$

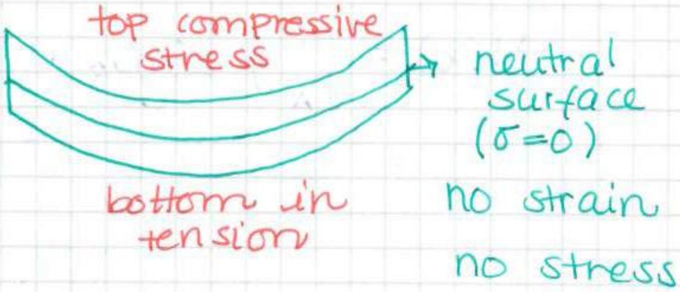
$B_y = \dfrac{M_0}{2L}$

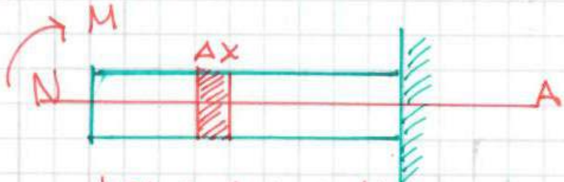


Chapter 6 continued... 6,3

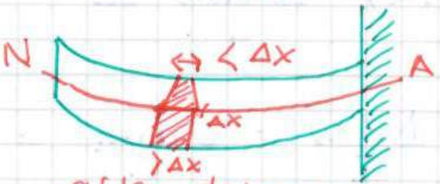
03.03 Bending deformation of a straight member

Before deformation

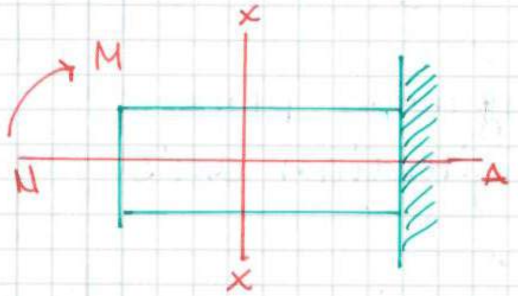


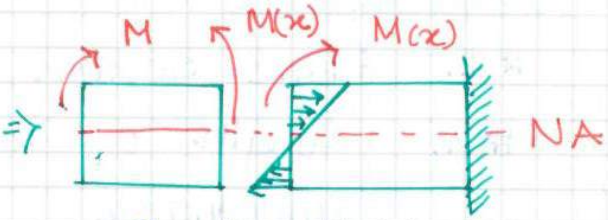


befor deformation

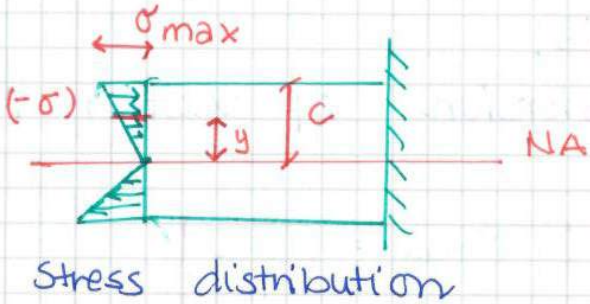


after deformation

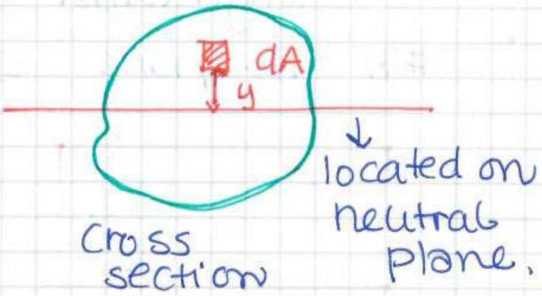




strain distribution



Stress distribution



Cross section

$$M(x) = \int_A y \cdot df$$

$$M(x) = \int_A y \cdot (-\sigma)\, dA \quad ⊛$$

σ - stress dF - force due to σ on dA area

$$\frac{\sigma_{max}}{c} = \frac{-\sigma}{y}$$

$$-\sigma = \frac{\sigma_{max} \cdot y}{c}$$

$$M(x) = \int_A y \, \frac{\sigma_{max}}{c} \cdot y \cdot dA$$

$$M(x) = \frac{\sigma_{max}}{c} \underbrace{\int_A y^2 dA}$$

(I -moment of intertia
Ch 10 - static)

$$M(x) = \frac{\sigma_{max}}{c} I$$

$$\boxed{\frac{M(x)}{I} = \frac{\sigma_{max}}{c} = \frac{-\sigma}{y}}$$ ← Flexture formula.
Bending —ıı—

Assumption

#1. Plane section remains plane

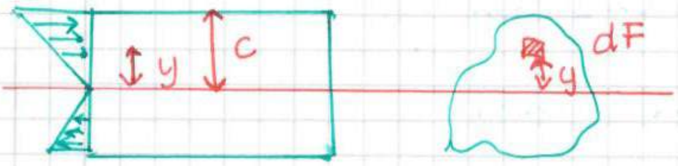
#2. Length of longitudinal axi's remains unchanged.

#3. Plane section remains perpendicular to the longitudinal axis.

#4. In plane distorsion of section is negligib

$$\boxed{\sigma = -\frac{My}{I}} \qquad \boxed{\sigma_{max} = \frac{Mc}{I}}$$

y : perpendicular distance from neutral axis
σ : normal stress at y -distance
I : moment of intertia about the neutral axis.
M : resultant internal moment.



$\Sigma F_x = 0$

$0 = \int_A dF = \int_A \sigma \, dA$

$0 = \underbrace{\frac{-\sigma_{max}}{c}}_{\neq 0} \underbrace{\int_A y \, dA}_{= 0}$

Centriod = $\bar{y}$

$\bar{y} = \frac{\int y \, dA}{\int dA} = 0 \Rightarrow \bar{y} = 0$

$$\boxed{\text{Neutral axis = centroid}}$$

example 7 : Stress distribution is given find M:

(1) $\frac{M}{I} = \frac{\sigma_{max}}{c} = -\frac{\sigma}{y}$ M= 20 MPa

$M = \frac{\sigma_{max}}{c} \cdot I$

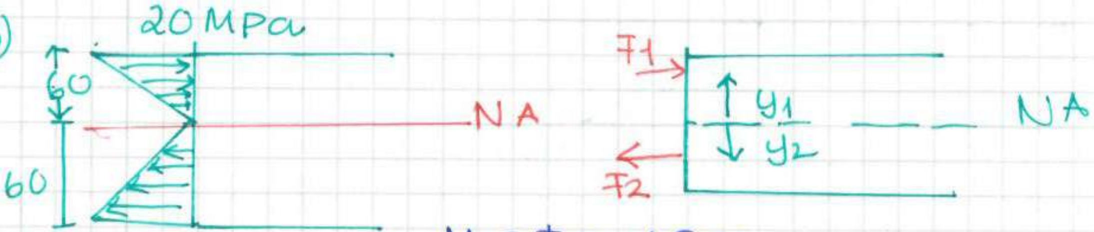
moment of Inertia:

$I = \frac{1}{12} bh^3 = \frac{1}{12} \times 60 \times 60^3 \, mm^4 = 864 \times 10^4 mm^4$

$c = 60mm$

$M = \frac{20 \times 864 \times 10^4 mm^4}{60 mm} = \underline{\underline{2,88 \, KNm}}$

(b)

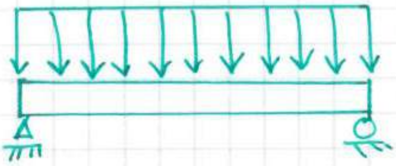


$M = F_1 y_1 + F_2 y_2$

$$F_1 = F_2 = \left(\text{Area} \times \frac{\sigma_{max}}{2} \right)$$

$$y_1 = y_2 = \tfrac{2}{3} \times 60 \text{ mm}$$

example 8



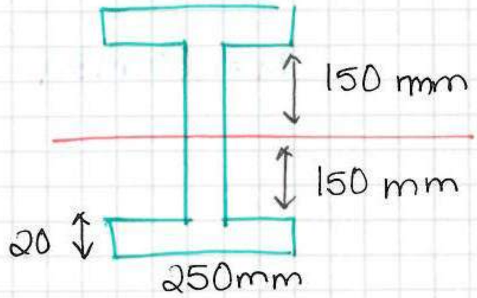
(i) $\sigma_{max} = \frac{M(x)}{I} \cdot y_{max}$

maximum stress of considered cross section where BM is M(x)

ii) $\sigma_{max} = \frac{M_{max} \, y_{max}}{I}$

maximum stress of the beam

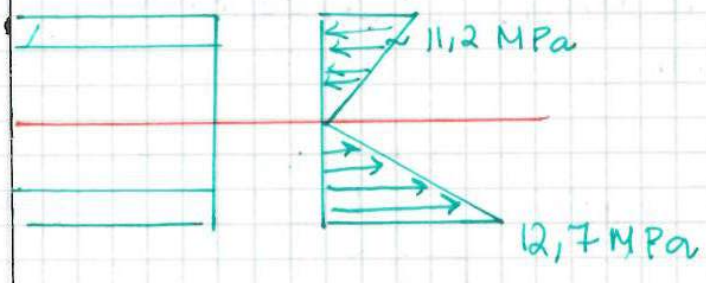
* Every cross-section has it's own maximum.



$$I = \frac{1}{12} \times 250 \times 340^3 - 2 \times \frac{1}{12} \times 115 \times 300^3 = 301;3 \times 10^{-6}$$

$$\sigma_{max} = \frac{22,5 \times 10^3 \times 170 \times 10^{-3}}{301,3 \times 10^{-6}}$$

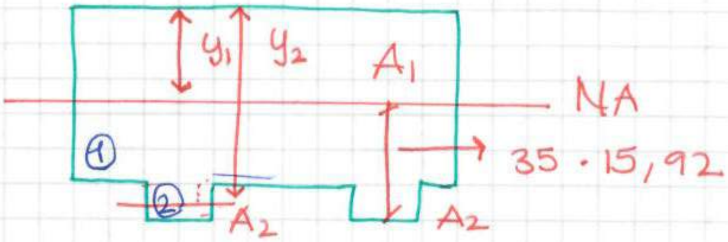
$$\underline{\underline{\sigma_{max} = 12,7 \text{ MPa}}}$$



example 9

with ribs.

$$\frac{\sigma_{max}}{c} = \frac{M}{I}$$



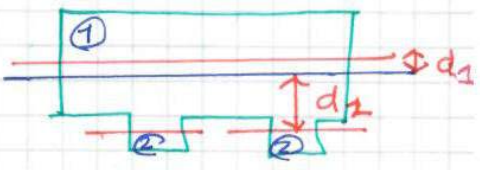
To find NA:

As this is not symmetric center is not the centroid

$$\bar{y} = \frac{\sum \bar{y} A}{\sum A}$$

$$= \frac{y_1 A_1 + y_2 \cdot A_2 \cdot \times 2}{A} -$$

$y_1 = \frac{30}{2}$ $\quad A_1 = 60\times30$



$y_2 = (30 + 5/2)$ $\quad A_2 = (10\times5)$

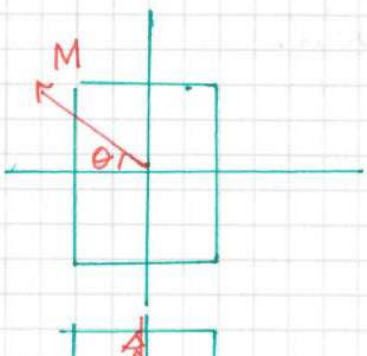
$\bar{y} = 15,92$ mm or $0,01592$ m

$I = (I_1 + A_1 d_1^2) + 2(I_2 + A_2 d_2^2)$

$I = 0,164\times10^{-6}\ m^4$

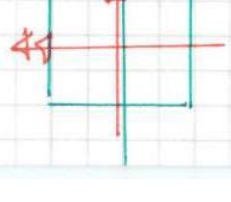
$$\sigma_{max} = \frac{40 \times 0,01908}{0,1642\times10^{-6}} = 4,65\ MPa$$

6,5 Usymmetric Bending



θ: angle with prinsiple z axis

θ = +ve when it is from +2 axis to + y axis



$M_z = M\cos\theta$

$M_y = M\cdot\sin\theta \Rightarrow \boxed{\frac{M_y}{M_z} = \tan\theta}$

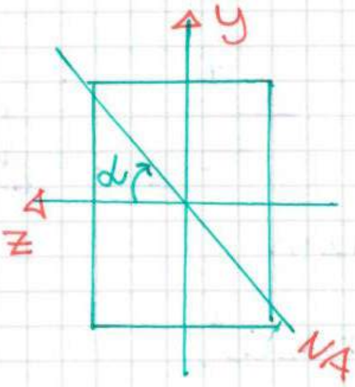
Resultant normal stress

$$\sigma = \frac{-M_z y}{I_z} + \frac{M_y z}{I_y}$$

↑ stress due to M_z is compressive in +-ve y direction

↑ stress due to M_y is tensile in positive z

Orientation of NA



As no normal stress acts on the neutral axis

$$\sigma = 0$$

$$= -\frac{M_z}{I_z} + \frac{M_y z}{I_y} = 0$$

$$\frac{M_z y}{I_z} = \frac{M_y \cdot z}{I_y} \Rightarrow \frac{y}{z} = \frac{M_y}{M_z} \cdot \frac{I_z}{I_y}$$

$$\tan \alpha = y/z = \tan\theta \cdot \frac{I_z}{I_y}$$

$$\tan \alpha = \frac{I_z}{I_y} \tan\theta$$ orientation of the neutral axis.

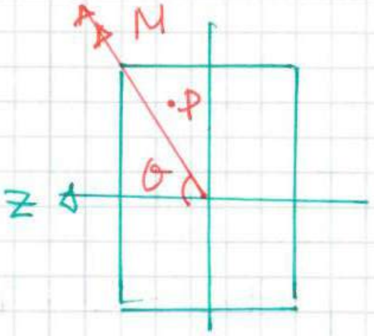
7/03

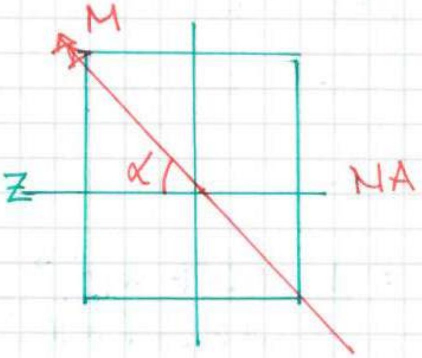
Summary ch. 6

$\sigma = \frac{-My}{I}$ flexture formula.

↑ tensile

* Location of neutral axis coincide withe the centriod of crosssection.

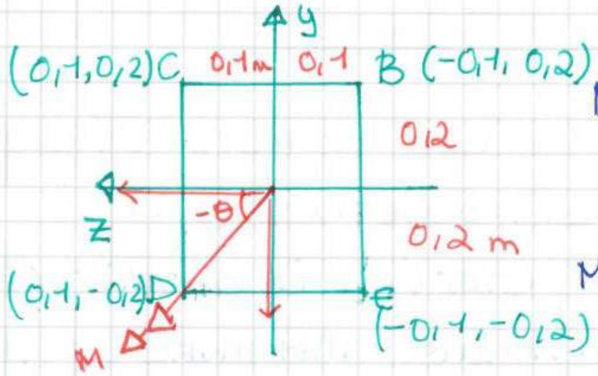




Stress at point P =

example

Find $\sigma_B, \sigma_C, \sigma_D, \sigma_E$ at each corners, orientation α of NA :



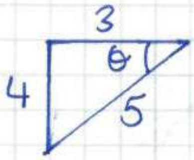
M = 12 kNm (given)

$M_z = M \cdot \cos\theta$

$= 12 \cdot \cos\theta = 12\ (3/12)$

$M_y = M \cdot \sin\theta$

$= -12\ (4/5) = -9,6\,kNm$



$$\sigma = \frac{-M_z y}{I_z} + \frac{M_y z}{I_y}$$

Moment of Inertia :

$I_z = 1/12 \times 0,4 \times 0,2^3\ m^4 = 0,2667 \times 10^{-3}\ m^4$

$I_y = \frac{1}{12} \times 0{,}2 \times 0{,}4^3 \text{ m}^4 = 1{,}067 \times 10^{-3} \text{ m}^4$

$\sigma_B = -\frac{(7{,}2 \times 10^3) \times 0{,}2}{0{,}2667 \times 10^{-3}} + -\frac{(9{,}6 \times 10^3)(-0{,}1)}{1{,}067 \cdot \times 10^{-3}}$

$\underline{\sigma_B = 2{,}25 \text{ MPa}}$ (feil)

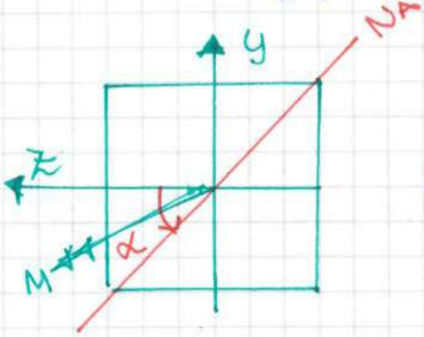
$\sigma_C = 2{,}25 \text{ Mpa}$

$\sigma_D = -4{,}95 \text{ Mpa}$

$\sigma_E = 4{,}95 \text{ MPa}$

orientation $\tan \alpha = \frac{I_z}{I_y} \tan \theta$

$\tan \alpha = \frac{1{,}067 \times 10^{-3}}{0{,}2667 \times 10^{-3}} \left(-4/3\right) = -79{,}4°$ [Check

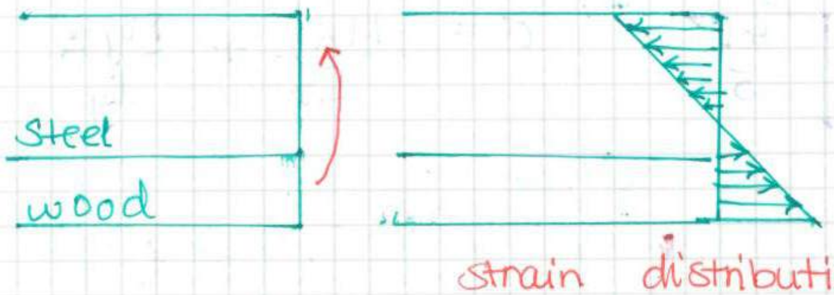


[Check]

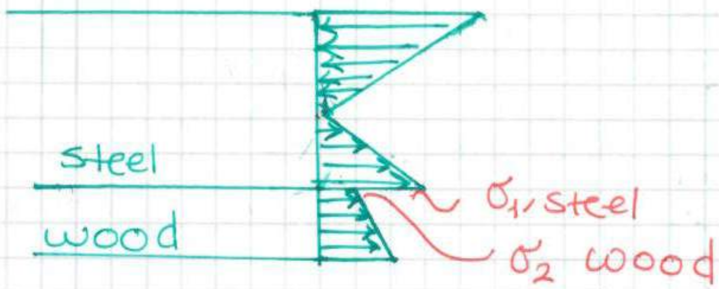
6,6 Composite Beams

Beams contructed of two or more different materials are refered to as composite beams.

The transformasjon factor (n) is a ratio of the moduli of the different materials that make up the beam.



strain distribution



Bending stress distribution

$\sigma_{1\,steel} = E_s\,\varepsilon$ ①

σ_{2} wood $= E_w \cdot \varepsilon$ ②

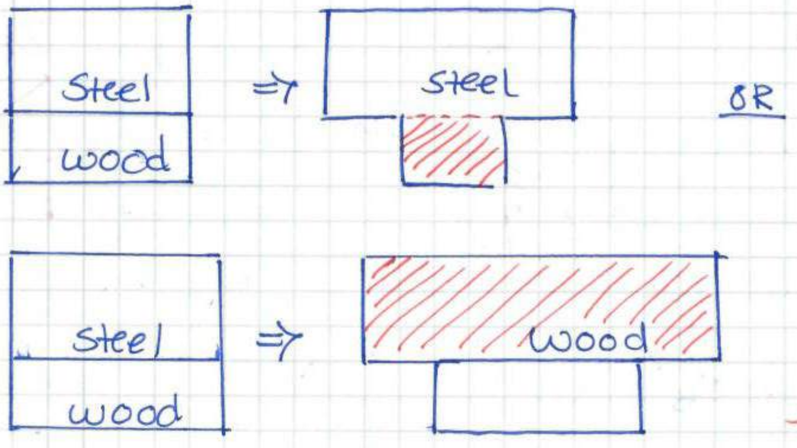
$$\boxed{\frac{\sigma_{1,\,steel}}{\sigma_{2}\ wood} = \frac{E_s}{E_w}}$$ modulus ratio

stress distribution is not linear flexture formula cannot be applied. Therefore we transform the cross section to a same material (one material

$\sigma_s = \frac{dF}{A_s} = E_s\,\varepsilon$ — ③

$\sigma_w = \frac{dF}{A_w} = E_s\,\varepsilon$ — ④

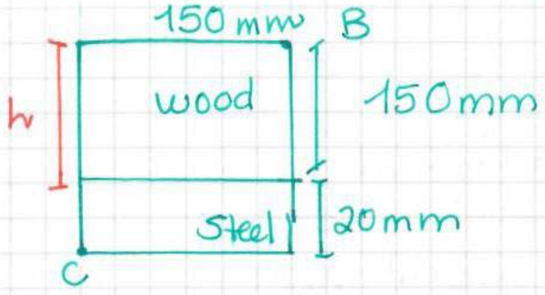
$$\frac{3}{4} : \boxed{\frac{A_w}{A_s} = \frac{E_s}{E_w}}$$



$A_s = \frac{E_w \cdot A_w}{E_s}$

$A_w = \frac{E_s \cdot A_s}{E_w}$

example (11)



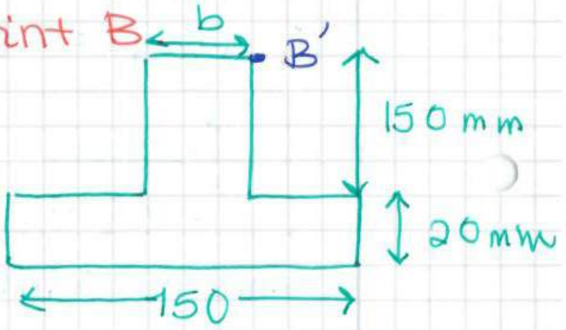
$\sigma_B = ?$

$\sigma_C = ?$

given $M = 2\,KNm$

h cannot change due to

We need to find stress at point B



Transformed section

$$A_{steel} = A_w \times \frac{E_w}{E_s}$$

$$\underset{mm}{b} \times 150 = \underset{mm^2}{(150 \times 150)} \times \left(\frac{12}{200}\right)$$

$$\underline{\underline{b = 9\,mm}}$$

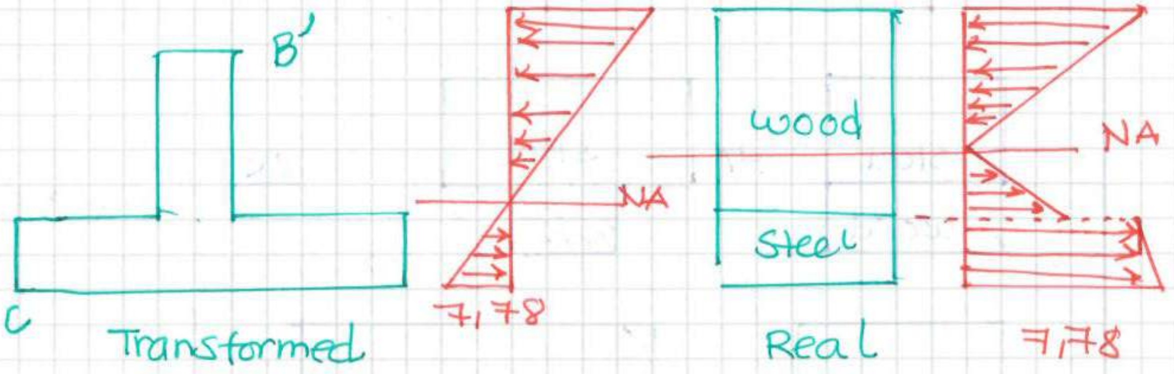
$$\frac{M}{I} = \frac{-\sigma}{y} = \frac{\sigma_{max}}{c}$$

$$I = 9{,}358 \times 10^{-6}\ mm^4$$

$$\sigma_B' = -\frac{My}{I} = -\frac{2 \times 10^3 \times (170 - 36{,}38) \times 10^{-3}}{9{,}358 \times 10^{-6}}$$

$$\sigma_B' = -28{,}6\ MPa$$

$$\sigma_C = 7{,}78\ MPa$$



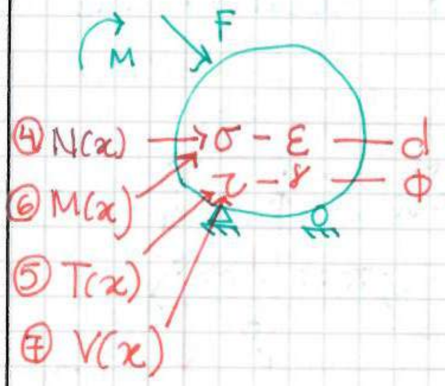
Transformed

Real

$$\sigma_B = \frac{E_w}{E_s}\,\sigma_B'$$

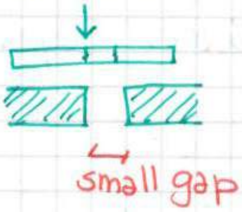
$$1{,}71 \Leftarrow \left(\frac{12}{200}\right)(-28{,}6)$$

chapter 7 - Transverse shear



Types of shear stress :

#1.



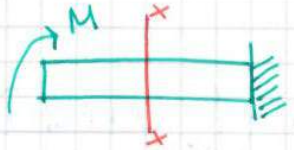
Shearing ⇐ ch①
tearing off

#2.

shear stress
due to torsion

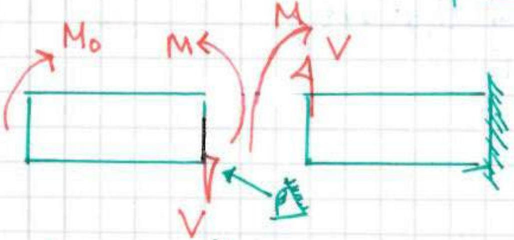
$\tau = \frac{T\rho}{J}$ ch.⑤

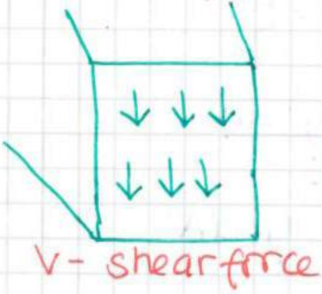
#3.



(continued...)

⇒

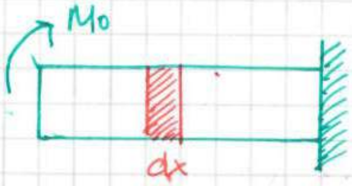


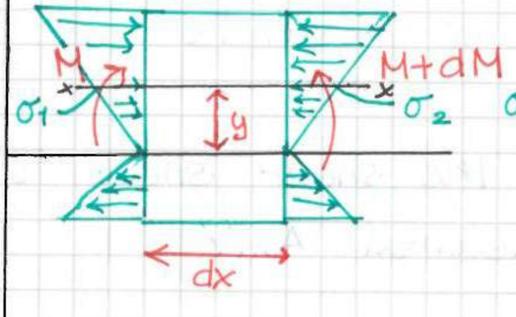


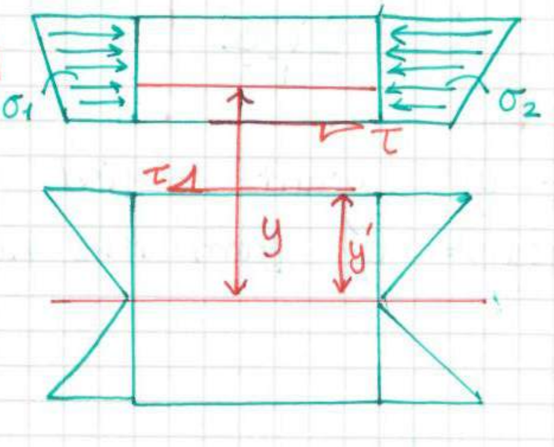
V - shear force

$\tau_{avg} = \frac{V}{\text{shear area}}$?

(uniformly distributed
this formula is ok)

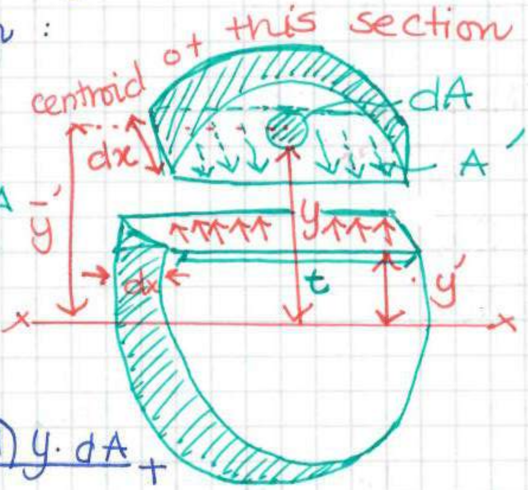






$\sigma_1 = \frac{My}{I}$ $\sigma_2 = \frac{(M+dM)y}{I}$

considering the upper right section and applying equilibrium equation :



$\rightarrow \Sigma F_x = 0$

$$\Rightarrow \int_{A'} \sigma_1 \, dA - \int_{A'} \sigma_2 \, dA + (\tau \times dx \times t) = 0$$

$$\int_{A'} \frac{My}{I} dA - \int_{A'} \frac{(M+dM)y \cdot dA}{I} + \tau t dx = 0$$

$$-\int_{A'} \frac{dMy}{I} dA + \tau t dx = 0$$

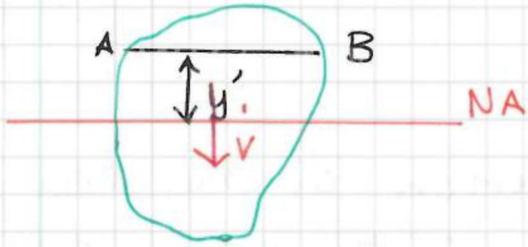
$$\tau t dx = \int_{A'} \frac{dMy \cdot dA}{I}$$

$$\tau = \underbrace{\frac{dM}{dx}}_{V} \cdot \frac{1}{It} \int_{A'} y \, dA$$

$$\tau = \frac{V}{It} \underbrace{\int_{A'} y \, dA}_{Q = \bar{y}' \cdot A'}$$

$$\boxed{\tau = \frac{VQ}{It}}$$ Shear-formula

If you want to find the shear stress τ at hight y' from the neutral Axis

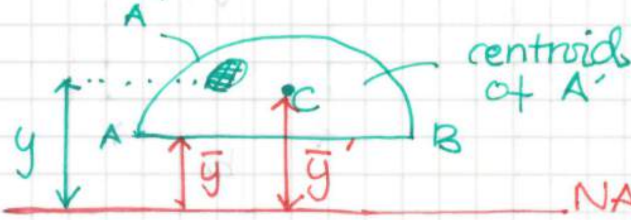


Step 1

Make an imaginary cut at height y' from NA [along AB line]

Step 2 :

t - length of AB

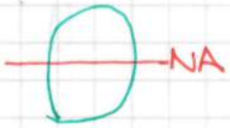


Step 3 :

$$Q = \int_A y \, dA = \bar{y}' A'$$

Step 4

V- Internal shear force across the crosssection (given or from shear force diagram)



Step 5

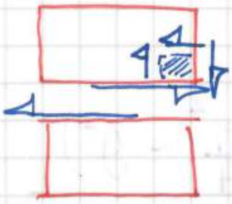
I_{NA} - 2^{nd} moment of area of the entire crossection about the neutral axis

Step 6

$$\tau = \frac{VQ}{It}$$

Note (i) : Q & t are new

(ii) :

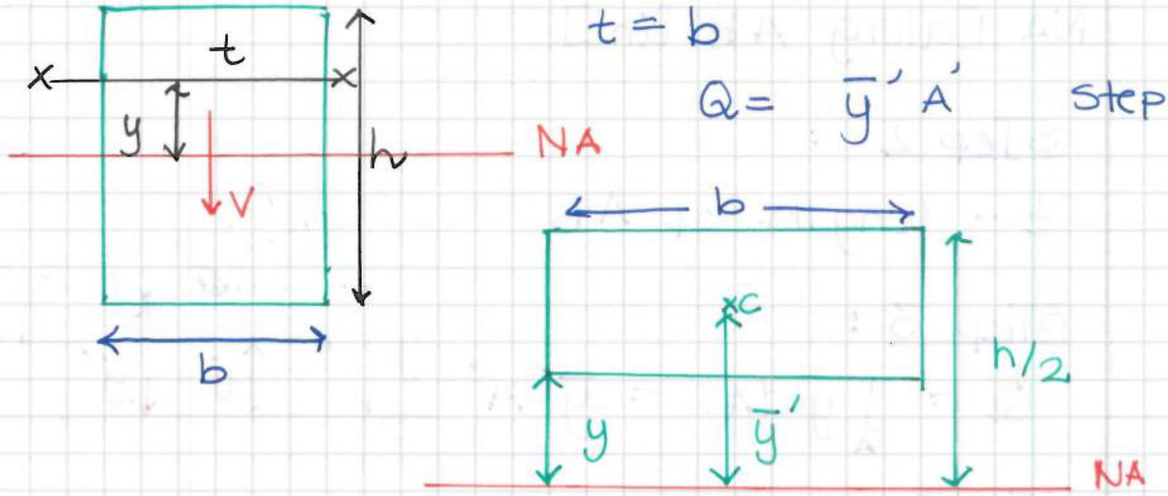


Complimentary property of shear

7.2

example 1

Determine the distribution of the shear stress over the cross section of the beam shown in Figure.



$t = b$

$Q = \bar{y}' A'$ Step ③

$A' = b\,(h/2 - y)$

$\bar{y}' = \dfrac{y + (h/2 - y)}{2} = \left(\frac{h}{2} + y\right) \cdot \frac{1}{2}$

$Q = \frac{1}{2}(h/2 + y) \times b\left(\frac{h}{2} - y\right) = \left(\frac{h^2}{4} - y^2\right)\frac{b}{2}$

Step ④

$V = V$

Step ⑤

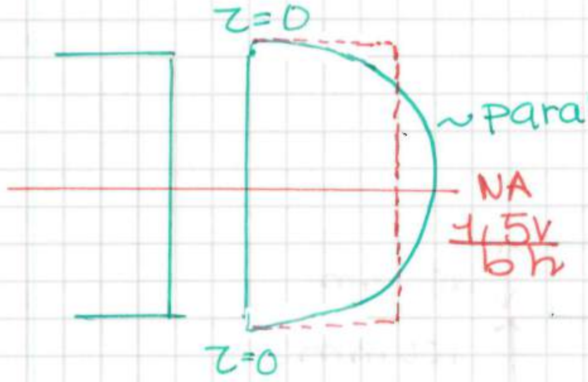
$I = 1/12\, bh^3$

Step ⑥

$$\tau = \frac{VQ}{It} = \frac{V \cdot \frac{\cancel{b}}{2}\left(\frac{h^2}{4} - y^2\right)}{\cancel{b} \cdot \frac{1}{12} bh^3} = \frac{6V\left(\frac{h^2}{4} - y^2\right)}{bh^3}$$

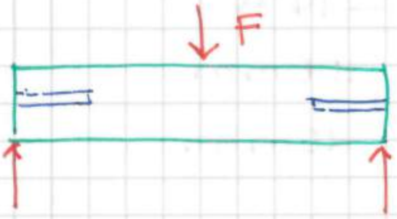
$y = 0 \quad \tau = \frac{3V}{2bh}$

$y = h/2 \quad \tau = 0$



Cracks in the neutral axis

$\tau_{avg} = \frac{V}{bh}$

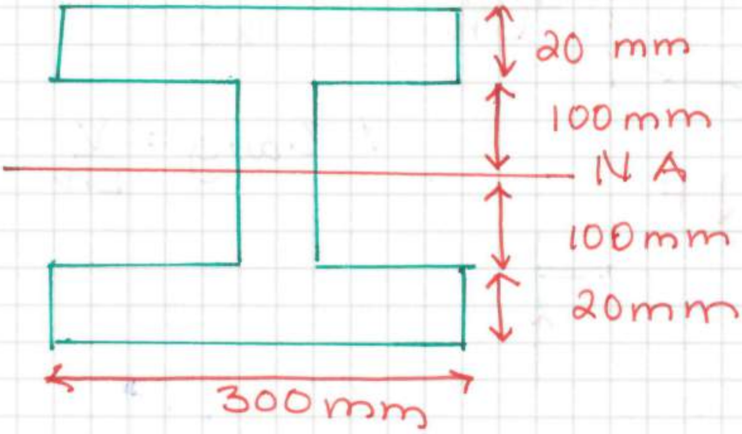


10/03 **Bending induced shear stress**

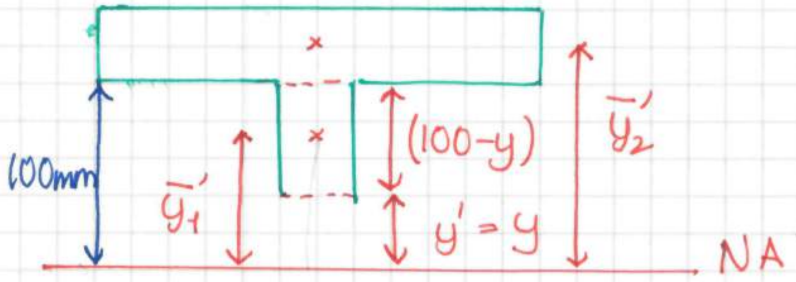
$$\tau = \frac{V \cdot Q}{I \cdot t}$$

ex. 2

$V = 80\ KN$



Step 1 :



if y is between 0 and 100.

$t = 15\ mm$

$Q = \bar{y}_1' A_1 + y_2' A_2 \qquad Q = (100^2 - y^2)\frac{15}{2} + (300 \times 20 \times 110)$

$\bar{y}_1' = y + \frac{(100-y)}{2} = \frac{y + 100}{2}$

$\bar{y}_2' = 110\ mm$

$A_1 = (100 - y) \cdot 15\ mm^2$

$A_2 = 20 \times 300\ mm^2$

$V = 80 \times 10^3\ N$

$I = \frac{1}{12} \times 15 \times 200^3 + 2 \times \left(\frac{1}{12} \times 300 \times 20^3\right)$
$+ (300 \times 20 \times 110^2) = 1,556 \times 10^8\ mm^4$

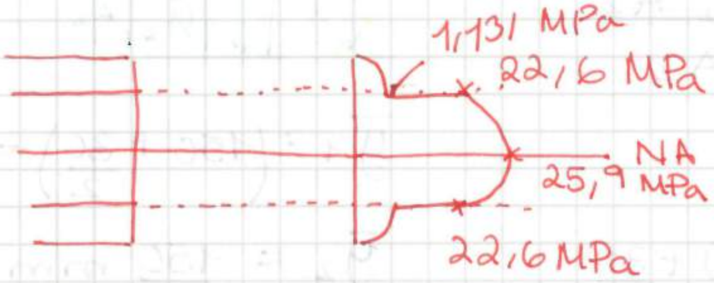
$\tau = \frac{VQ}{It} = \frac{15}{2}(100^2 - y^2) + \dots$

$$\tau = \frac{80 \times 10^3 \left[(100^2 - y^2)\frac{15}{2} + 300 \times 20 \times 110\right]}{1,556 \times 10^8 \times 15}$$

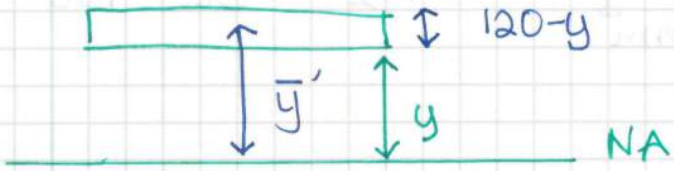
$\tau = \left[\frac{15}{2}(100^2 - y^2) + (6,60 \times 10^5)\right] 3,428 \times 10^{-5}$

when $y = 0$ $\underline{\underline{\tau = 25,19\ MPa}}$

when $y = 100$ $\underline{\underline{\tau = 22,6\ MPa}}$



If $100 < y \leq 120$ $t = 300\ mm$



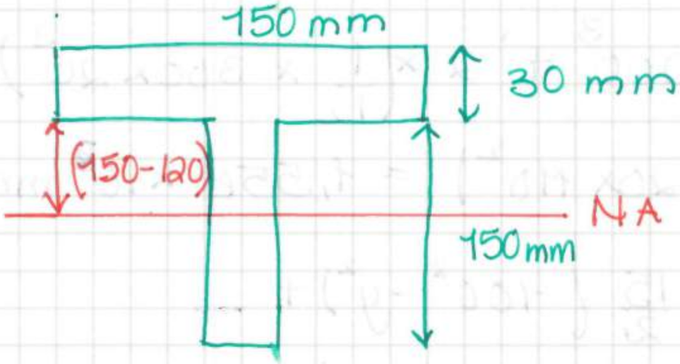
$Q = \bar{y}' A'$ $\bar{y}' = y + \frac{120 - y}{2} = \frac{y + 120}{2}$

$Q = (120^2 - y^2) \times 150$ $A' = (120 - y) \times 300$

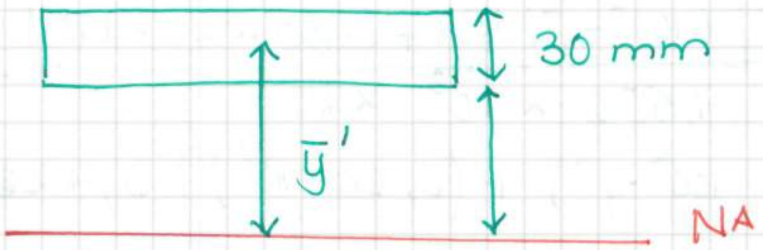
$\tau = \frac{VQ}{It} = \frac{80 \times 10^3 \times (120^2 - y^2)}{1,556 \times 10^6 \times 300} =$

$y = 100$ $\tau = 1,131\ MPa$ $y = 120$ $\tau = 0$

example (3)



step 1
uimaginary cut



$t = 30$ mm

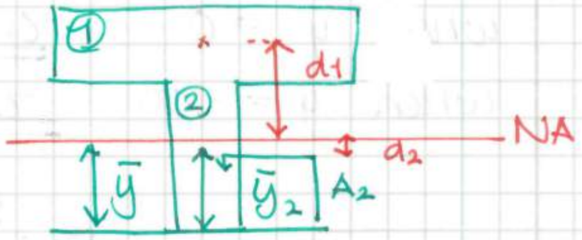
$Q = \bar{y}'A'$

$A' = 30 \times 150$

$\bar{y}' = ?$

$$\bar{y} = \frac{\Sigma \bar{y}_i A_i}{\Sigma A_i}$$

$$= \frac{\bar{y}_1 A_1 + \bar{y}_2 \cdot A_2}{A_1 + A_2}$$



$\bar{y} = 120$ mm

$\bar{y}' = (150 - 120) + \frac{30}{2}$

$= 45$mm

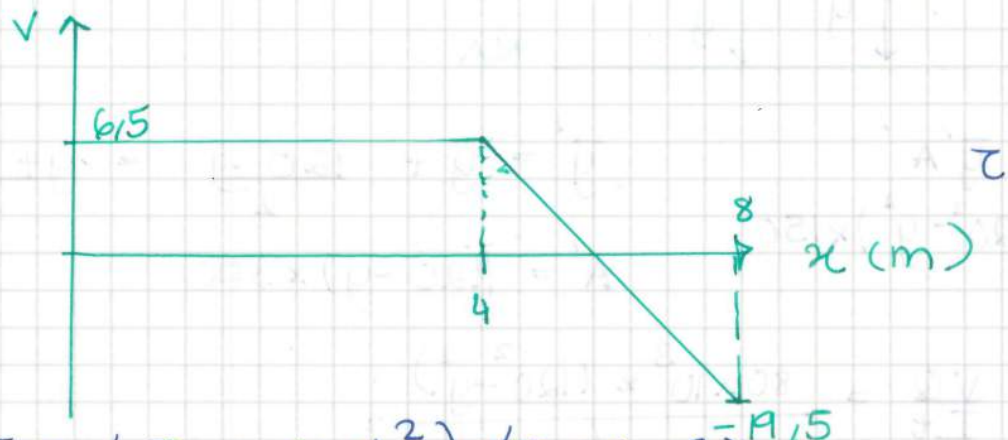
$Q = 30 \times 150 \times 45$ mm^3

$\bar{y}_1 = \left(150 + \frac{30}{2}\right) = 165$mm

$\bar{y}_2 = \frac{150}{2}$ mm

$A_1 = 30 \times 150$

$A_2 = 30 \times 150$



$\tau_{max} \rightarrow V_{max}$

$V_{max} = 19,5$KN

$I = (I_1 + A_1 d_1^2) + (I_2 + A_2 d_2^2)$

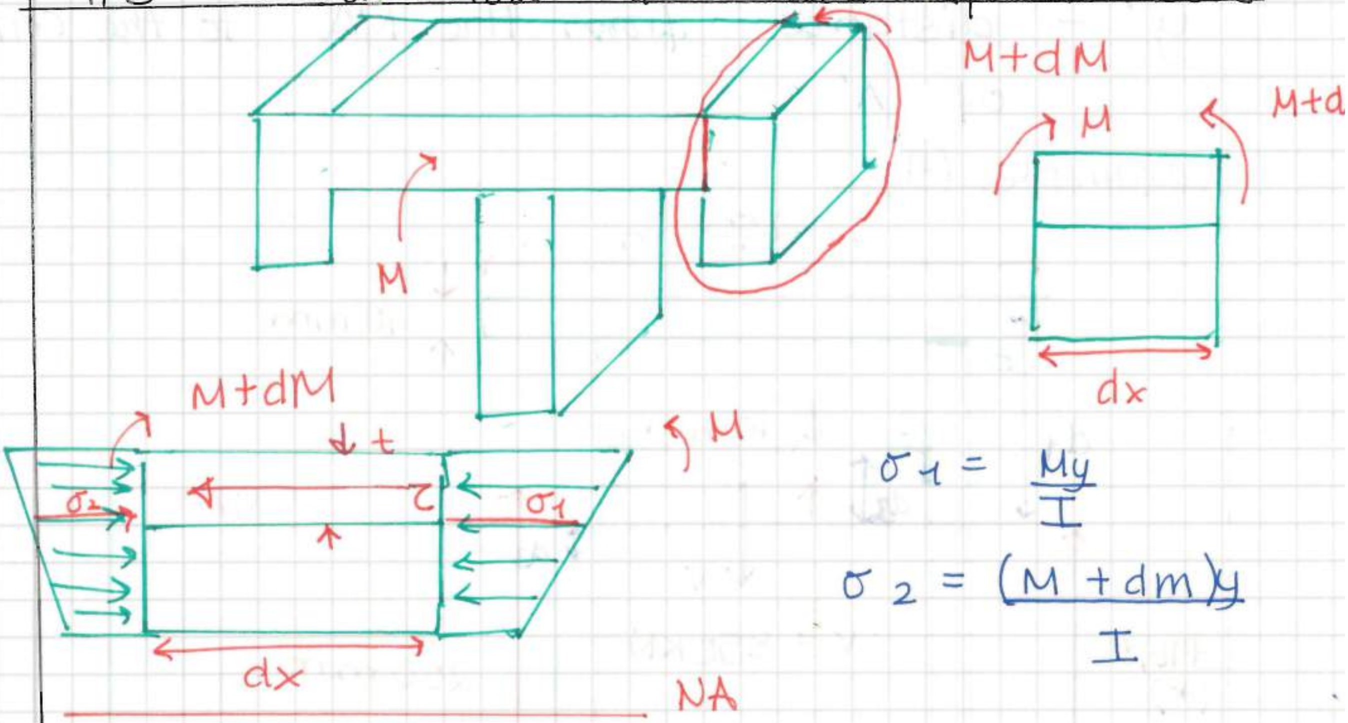
$d_1 = 165 - 120 =$

$d_2 = 120 - 75$, $A_2 = 30 \times 150$ $A_1 = 30 \times 150$

$I = 2{,}7 \times 10^{7}\ mm^{4}$

$\tau_{max} = \frac{VQ}{It} = \underline{\underline{48{,}75\ MPa}}$

7,3 Shear Flow in build-up members



$\sigma_1 = \frac{My}{I}$

$\sigma_2 = \frac{(M+dm)y}{I}$

Applying equation of equilibrium

$$-\int_{A'} \sigma_2\, dA + \int_{A'} \sigma_1\, dA + \tau\, dx\, t = 0$$

$$\int \frac{dMy}{I}\, dA = \tau\, t\, dx$$

$$\frac{dM}{dx}\, \frac{1}{I} \int_{A'} y\, dA = \underbrace{\tau t}_{q} = \text{shear flow}$$

Shear flow :

shear force per unit length of the glue along the beam.

$\int_A y\, dA = Q$

$Q = \bar{y}' A'$ where

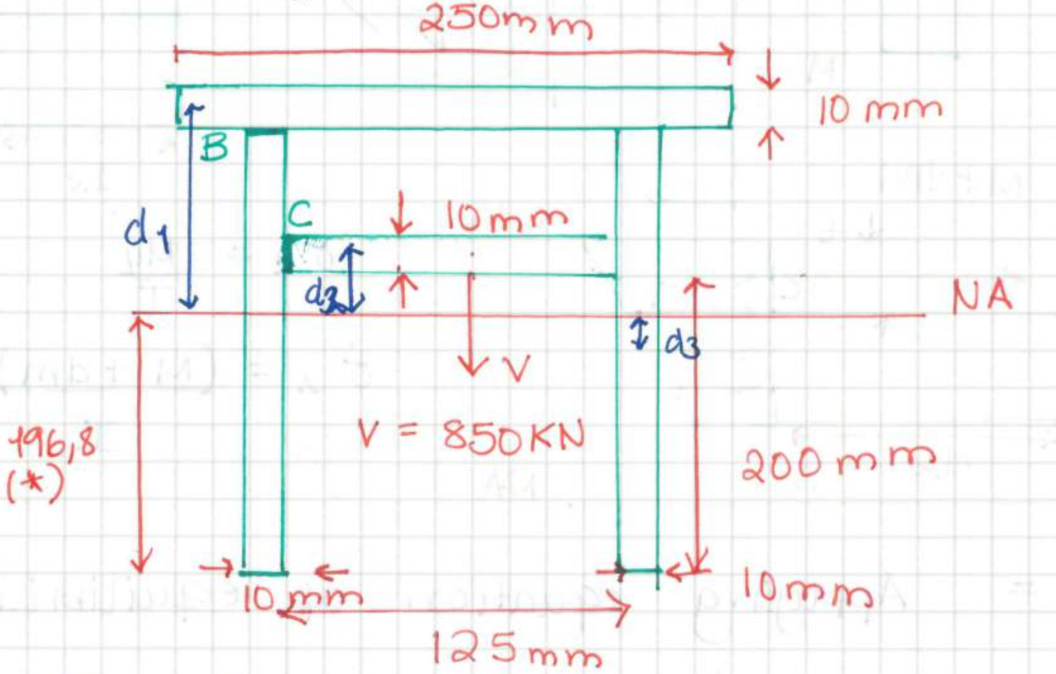
A' = crosssectional area of the segment.

$$\boxed{\frac{VQ}{I} = q}$$

A' - c/s area of the segment that is connected to the beam at the juncture whear shear flow is to be calculated

$\bar{y}'$ - distance from the NA to the centroid of A'

example (4)

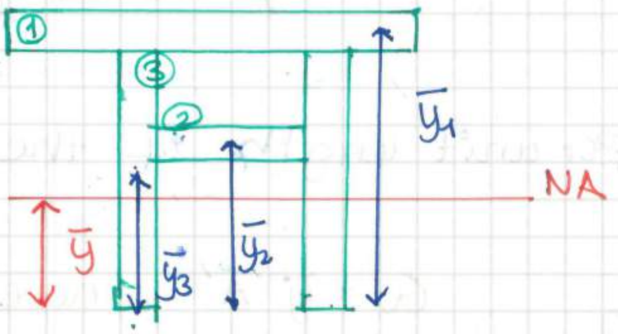


Shear flow

$q = \frac{VQ}{I}$ $\qquad$ $V = 850\,KN$

$\qquad\qquad\qquad I = ?$

I = we need neutral axis distance.



$$\bar{y} = \frac{\Sigma \bar{y}_i A_i}{\Sigma A_i} = \frac{\bar{y}_1 A_1 + \bar{y}_2 A_2 + 2 \cdot \bar{y}_3 A_3}{A_1 + A_2 + A_3}$$

$A_1 = (250 \times 10) = mm^2$

$A_2 = (10 \times 125)\ mm^2$

$A_3 = (300 \times 10)\ mm^2$

$\underline{\bar{y} = 196,8\ mm}$ (*)

$$I = (I_1 + A_1 d_1^2) + (I_2 + A_2 d_2^2) + (I_3 + A_3 d_3^2) \times 2$$

$d_1 = 305 - 196,8$ $A_1 = 250 \times 10$

$d_2 = 205 - 196,8$ $A_2 = 10 \times 125$

$d_3 = 196,8 - 150$ $A_3 = 300 \times 10$

$I = 87,52 \times 10^6\ mm^4$

$q = \frac{VQ}{I} \Rightarrow$ $Q = ?$

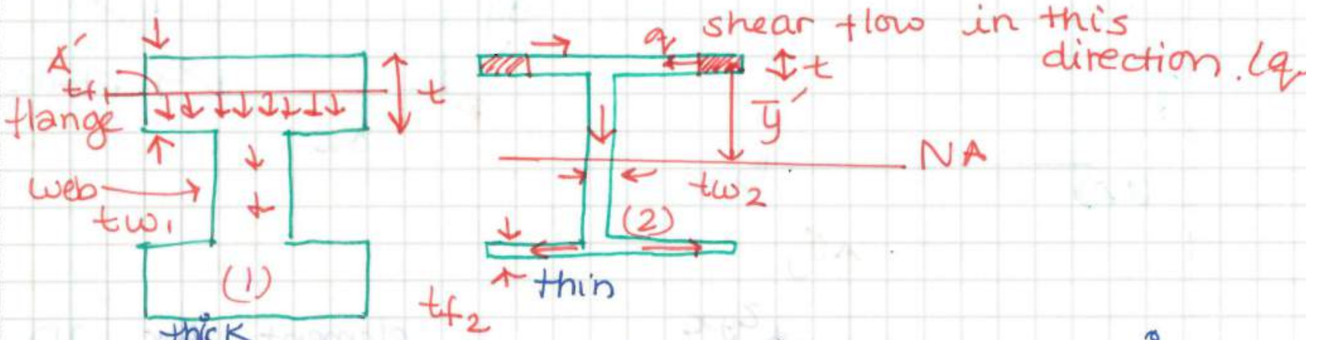
$\overline{y_B}' = 305-$
$196,8$

$Q = (250 \times 10)(305 - 196,8) = 270500\ mm$

$q = \frac{VQ}{I} = 2630,205\ N/mm$

$q_B = \frac{1}{2} q = \frac{2630,205}{2}$ ...

7,4 Shear Flow in thin wall members



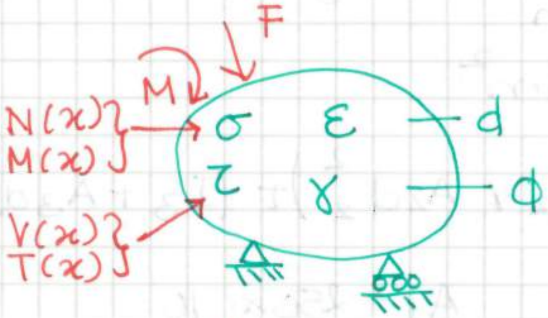
Thickness <<< compared to other dimensions (height/width)

We use shear flow instead of shear stress.

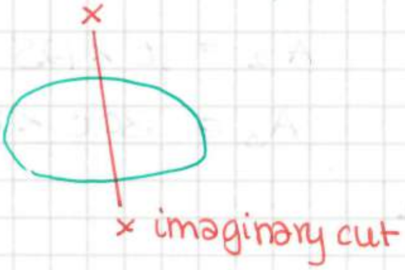
$q = VQ/I$

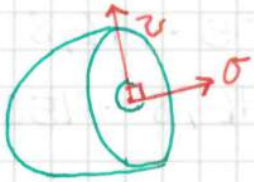
14.03

Chapter 9 : Stress transformation

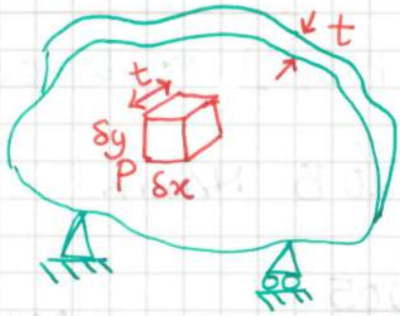


Stress transformation :
(stress at a point).



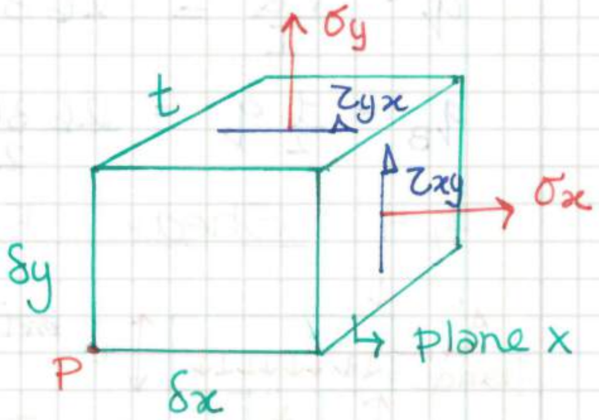


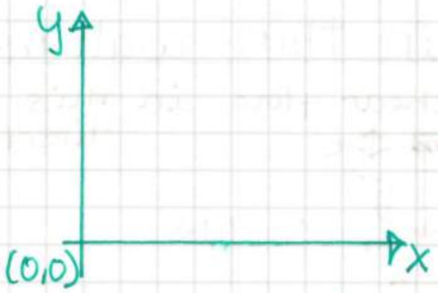
Considering thin plate :

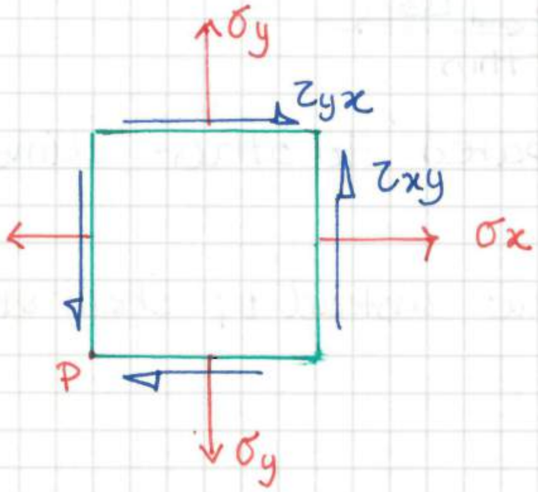


$t \ll$ other dimensions

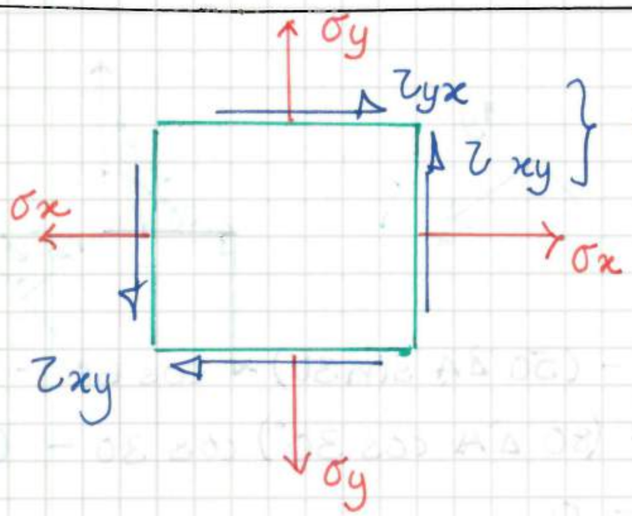
2D plane stress problem







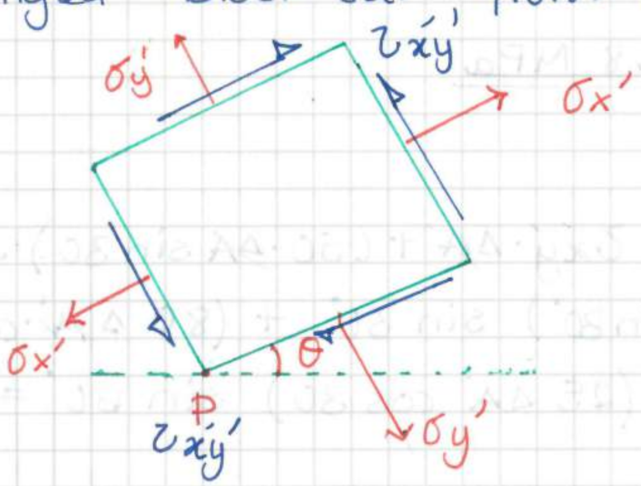
element for 2D plane stress problem



same ∴ τ_{xy}

If there is no body force.

Angled block out from the plate with angle θ.

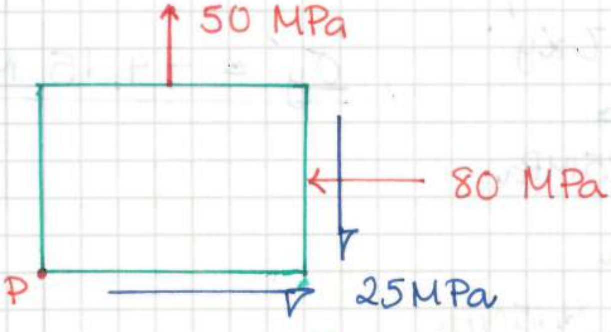


$\sigma_x \neq \sigma_x'$

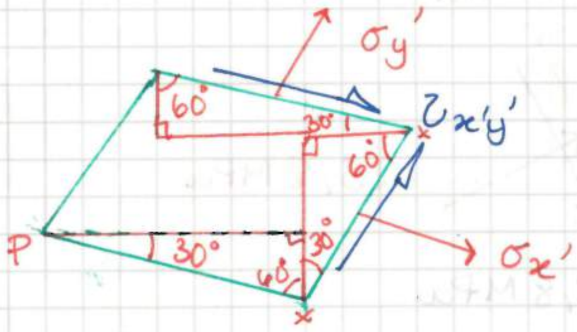
$\tau_{xy} \neq \tau_{x'y'}$

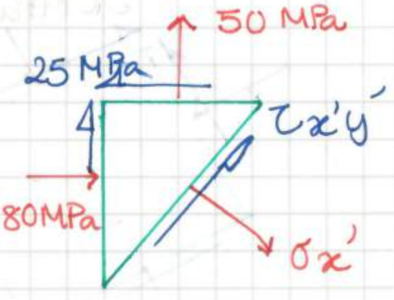
If we know σ_x, σ_y, τ_{xy}, Objective
Find σ_x', σ_y', $\tau_{x'y'}$

example (1)

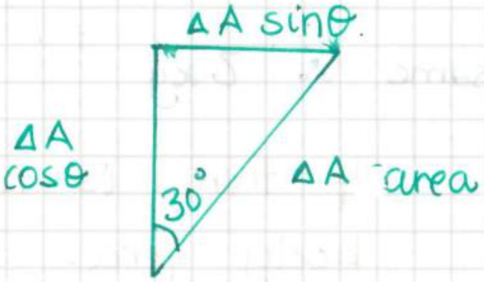


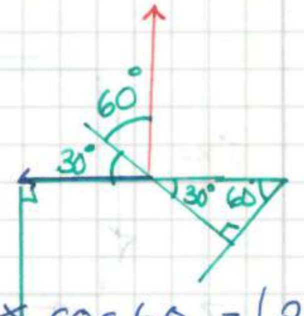
$\sigma_x = -80$ MPa
$\sigma_y = 50$ MPa
$\tau_{xy} = -25$ MPa





considering equilibrium of this element



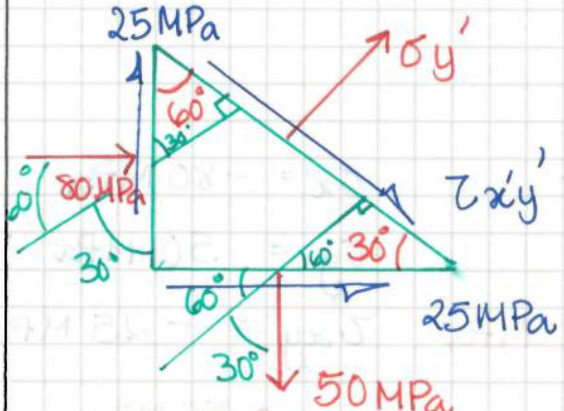


$$\Sigma F = \sigma_{x}' \cdot \Delta \cancel{A} - (50\ \Delta\cancel{A}\ \sin 30) \times \cos 60 - (25\ \Delta\cancel{A} \sin 30^\circ)$$
$$\cos 30^\circ + (80\ \Delta\cancel{A}\ \cos 30^\circ) \cos 30 - (25\ \Delta\cancel{A} \cos 30^\circ)$$
$$\cos 60^\circ = 0$$

$$\underline{\sigma_{x}' = -25{,}8\ \text{MPa}}$$

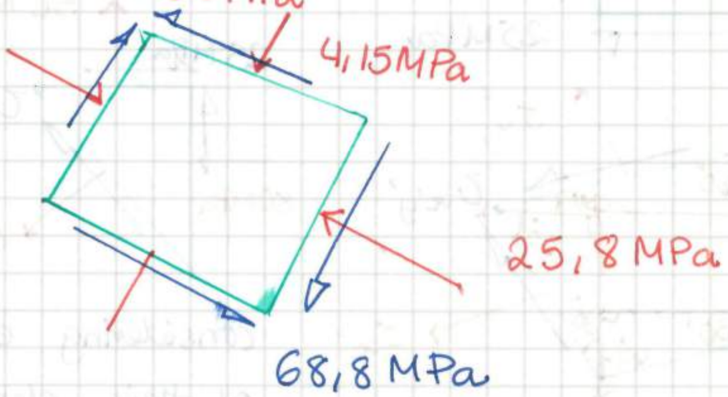
$$\Sigma F = \tau_{x'y'} \cdot \Delta\cancel{A} + (50 \cdot \Delta\cancel{A} \sin 30) \cdot \sin 60 - (25\ \Delta\cancel{A}$$
$$\sin 30^\circ)\ \sin 30^\circ + (80\ \Delta\cancel{A} \cdot \cos 30)\ \sin 30^\circ$$
$$+ (25\ \Delta\cancel{A}\ \cos 30)\ \sin 60^\circ = 0$$

$$\underline{\tau_{x'y'}' = -68{,}8\ \text{MPa}}$$



Applying equation of equilibrium

$$\underline{\sigma_{y}' = -4{,}15\ \text{MPa}}$$



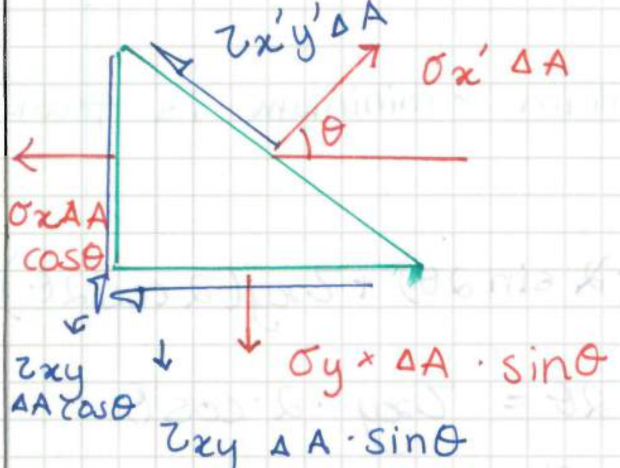
9.2 General eq. of Plane-stress transformation:

θ = counter clockwise

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

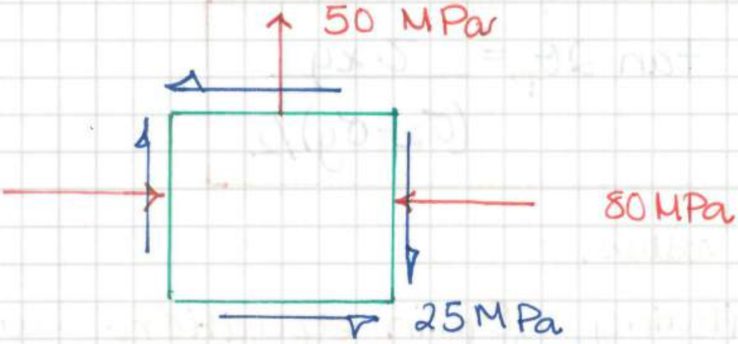
$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$



[if σx, σy, τxy is known we can use this formula directly]

example (2)



$\sigma_x = -80$ MPa

$\sigma_y = 50$ MPa

$\tau_{xy} = -25$ MPa

$\theta = -30°$

$\sigma_{x'} = -25,85$ MPa

$\sigma_{y'} = -4,15$ MPa

$\tau_{x'y'} = -68,8$ MPa

9,3 Principal stress and Maximum In Plane Stress

At a point, there are infinite number of σ_x', σ_y', τ_{xy}'

out of those → there is

$$\left.\begin{array}{l}\sigma_x' \text{ maximum}\\ \sigma_x' \text{ mininum}\end{array}\right\}\text{Principal stress at Point P}$$

$\tau_{x'y'}$ maximum

$$\sigma_x' = \frac{\sigma_x+\sigma_y}{2} + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_x' = f(\theta)$$

$\frac{d\sigma_x'}{d\theta} = 0$ maximum & minimum is found

$$\frac{d\sigma_x'}{d\theta} = \left(\frac{\sigma_x-\sigma_y}{\cancel{2}}\right)(-\cancel{2}\sin 2\theta) + \tau_{xy}(2\cos 2\theta) = 0$$

$$(\sigma_x-\sigma_y)\cdot\sin 2\theta = \tau_{xy}\cdot 2\cdot\cos\theta$$

$$\frac{\sin 2\theta}{\cos 2\theta} = \frac{\tau_{xy}}{(\sigma_x-\sigma_y)/2}$$

$$\Rightarrow \boxed{\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x-\sigma_y)/2}}$$

θ_p has 2 vallues.

By substituting θ_p in equation we can find principle values for their stresses.

$$\boxed{\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

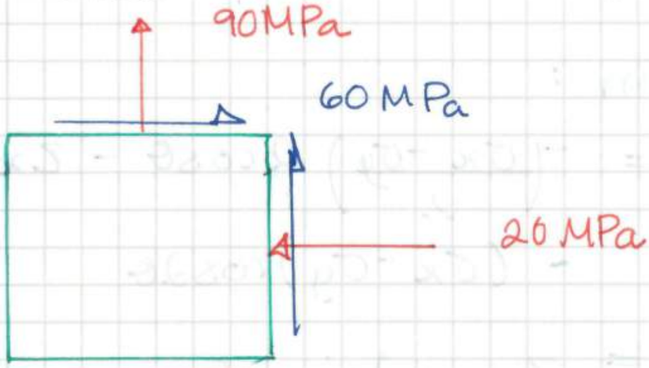
σ_1 - maximum normal stress

σ_2 - minimum normal stress.

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

by substituting θ_p $\boxed{\tau_{x'y'} = 0}$

example (3)



$\sigma_x = -20$ MPa

$\sigma_y = 90$ MPa

$\tau_{xy} = 60$ MPa

$\sigma_{1,2}$

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{60}{(-20-90)/2}$$

$\tan 2\theta_p = -1{,}09$

$2\theta_p = \tan^{-1}(-1.09) = -47{,}49°$ or $180 + (-47{,}49°)$

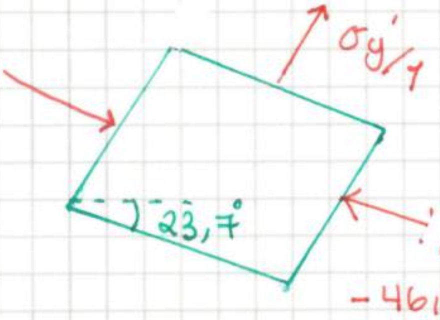
$\theta_p = -23{,}7°$ or $66{,}3°$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \left(\frac{-20+90}{2}\right) \pm \sqrt{\left(\frac{-20-90}{2}\right)^2 + 60^2}$$

$\sigma_1 = 116$ MPa (maximum)

$\sigma_2 = -46{,}4$ MPa (mininum)



-116 MPa $\sigma_{x'}$ = equilibrium eq.

using $\theta_p = -23{,}7°$

to find the plane in which each normal stresses acts

$-46{,}4$ MPa

$\sigma_{x'} = -46{,}4$ MPa

Maximum In-plane shear stress

$$\tau_{x'y'} = -\left(\frac{xx - \sigma_y}{2}\right) \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\tau_{x'y'} = f(\theta)$$

For maximum :

$$\frac{d\tau_{x'y'}}{d\theta} = -\left(\frac{\sigma_x - \sigma_y}{2}\right) 2\cos\theta - \tau_{xy} \cdot 2 \cdot \sin 2\theta = 0$$

$$-(\sigma_x - \sigma_y)\cos 2\theta$$

$$= \tau_{xy} \cdot 2\sin 2\theta$$

for maximum shear stress $\theta = \theta_s$

$$\boxed{\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}}}$$

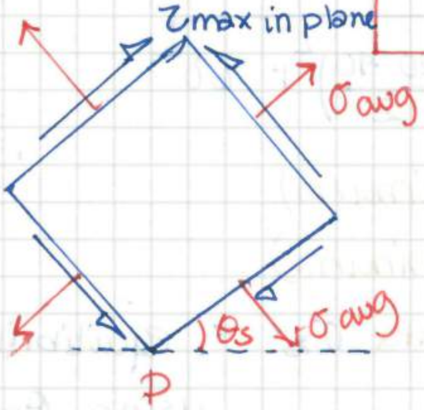
θ_s has 2 values

By substituting θ_s in the τ_{xy} equation.

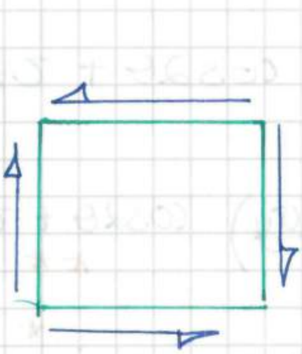
$$\boxed{\tau_{\max \text{ in plane}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

when $\tau_{\max}$

$$\boxed{\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}}$$



example (4)



Given

$\sigma_x = 0$

$\sigma_y = 0$

$\tau_{xy} = -\tau$

(a) $\tau_{\max \text{ in plane}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \pm\tau$

$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = 0$

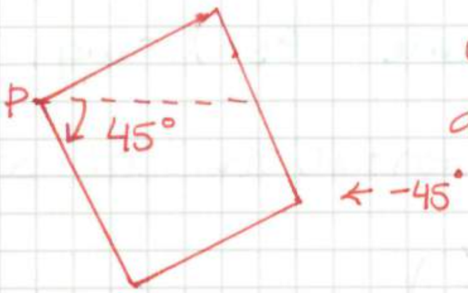
(b) $\tan 2\theta_P = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-\tau}{0} = \infty$

$2\theta_P = \tan^{-1}(\infty) = -90^\circ$ or $180 + (-90^\circ) = 90^\circ$

$\underline{\theta_P = +45^\circ}$

$\sigma_{1,2} = \pm\sqrt{0 + \tau^2} = \pm\tau$

$\underline{\sigma_1 = \tau}$ $\underline{\sigma_2 = -\tau}$



To find the plane in which each normal stress act $\sigma_x' = -\tau$

9.4 Mohr's Circle - plane stress

σ_x σ_y τ_{xy} known

σ_x' σ_y' $\tau_{x'y'}$ unknown (variable)

Graphical representation of stress at a point

$$\sigma_{x'} = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\left[\sigma_{x'} - \left(\frac{\sigma_x+\sigma_y}{2}\right)\right]^2 = \left[\left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta \underset{**}{+} \tau_{xy}\sin 2\theta\right]^2 \quad ①$$

$$\left[\tau_{x'y'}\right]^2 = \left[-\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos \overset{*}{2}\theta\right]^2 \quad ②$$

①+②

$$\left[\sigma_{x'} - \left(\frac{\sigma_x+\sigma_y}{2}\right)\right]^2 + (\tau_{x'y'})^2 = \underbrace{[\,*\,]^2 + [\,**\,]^2}_{R^2 \text{ = konstant value}}$$

$$(x-c)^2 + y^2 = R^2$$

(Equation of a circle)

7.03 Summary

If we know $\sigma_x, \sigma_y, \tau_{xy}$

- $\sigma_{x'}, \sigma_{y'}, \tau_{xy'}$ - state of stress at a point
- $\sigma_{1,2}$, θ_p - principle stresses
- τ_{max} inplane θ_s, σ_{avg}.

Graphical representation of stress at a point

$$\underset{\text{known}}{\begin{pmatrix}\sigma_x \\ \sigma_y \\ \tau_{xy}\end{pmatrix}} \qquad \underset{\text{unknown}}{\begin{pmatrix}\sigma_{x'} \\ \sigma_{y'} \\ \tau_{x'y'}\end{pmatrix}}$$

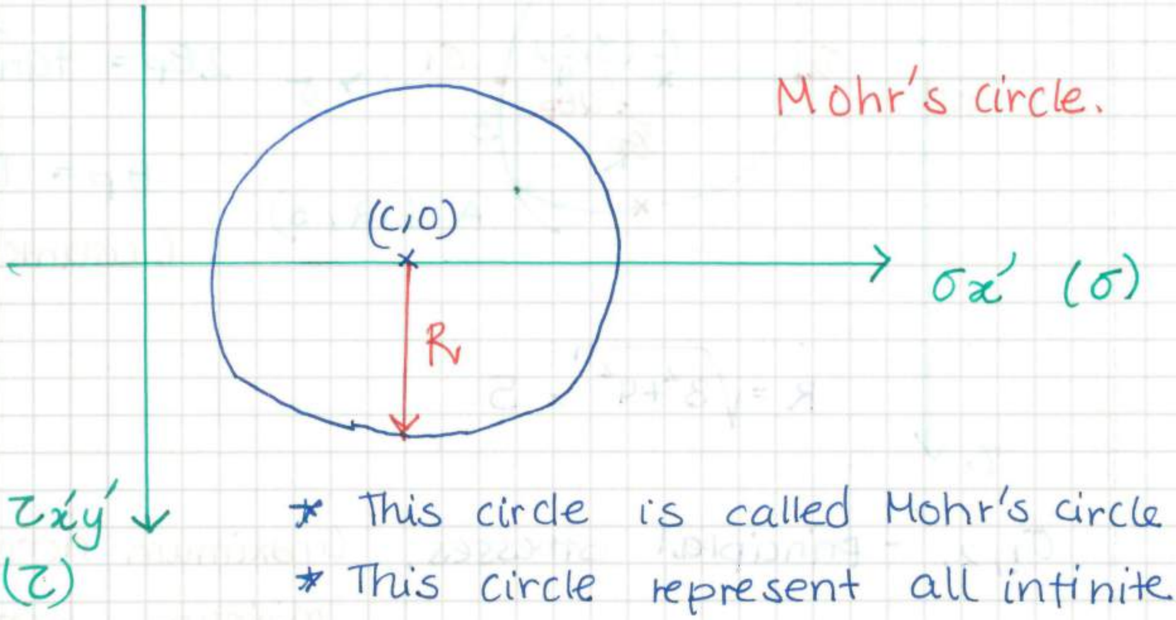
$$\left[\sigma_{x'} - \left(\frac{\sigma_x+\sigma_y}{2}\right)\right]^2 + \tau_{x'y'}^2 = \left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2$$

$$(\sigma_{x'} - c)^2 + \tau_{x'y'}^2 = R^2$$

$$C = \frac{\sigma_x + \sigma_y}{2}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

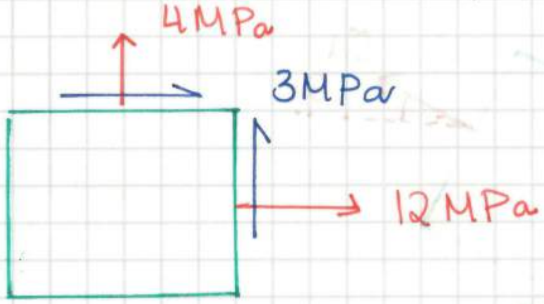
R : Radius C : coordinate of the center



* This circle is called Mohr's circle

* This circle represent all infinite number of stress at point.

<u>example</u>



σ_x = 12 MPa

σ_y = 4 MPa

τ_{xy} = 3 MPa

Step 1 - Find the center of the circle.

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{12+4}{2} = 8 \text{ MPa}.$$

Step 2 - Establish σ, axis (positive right) & τ axis (positive down wards.

Step 3 - plot the center of circle using positive sign convention, for σ_x, σ_y τ_{xy}

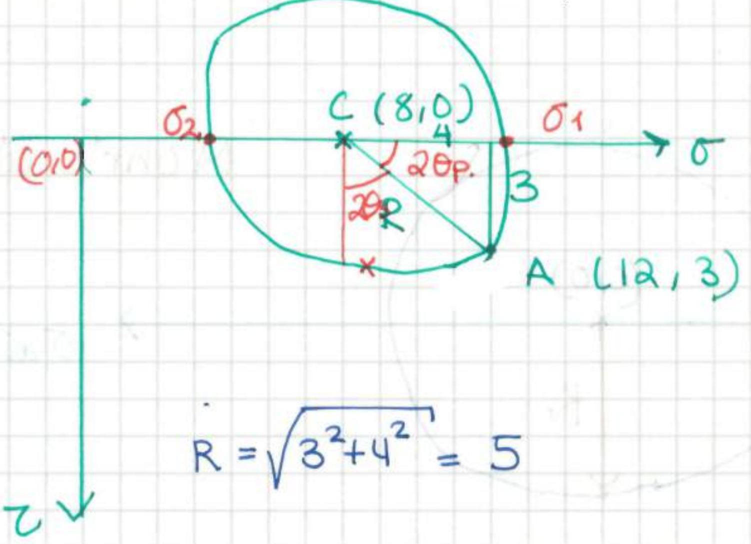
Step 4 - plot point A (σ_x, τ_{xy}) (12,3)

Step 5 - connect C & A (CA line)

$$R = CA = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

CA is the radius of the circle

Step 6 - As R is known, sketch the circle.



$2\theta_p = \tan^{-1}(3/4)$

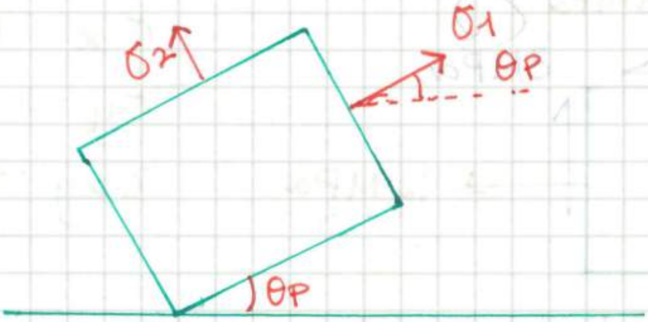
$\theta_p = 18{,}43°$ (counterclockwise)

$R = \sqrt{3^2+4^2} = 5$

$\sigma_{1,2}$, - principal stresses, (maximum normal stress minimum normal stress)

$\sigma_1 = (8+5) = 13$ MPa

$\sigma_2 = 8-5 = 3$ MPa

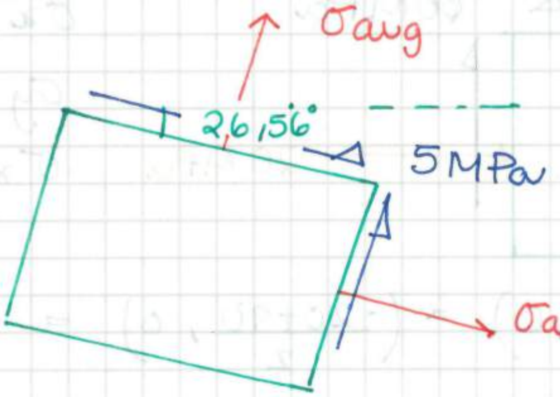


Note : ½ (Mohr's circle orientation)

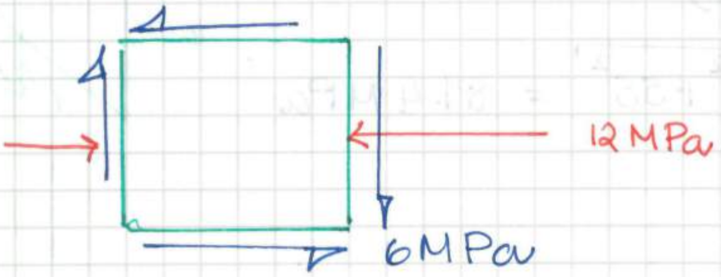
= orientation of stress element.

τ_{max} in plane = 5MPa

$2\theta_s = \tan^{-1}(4/3)$ $\theta_s = 26{,}56°$ (clockwise)



example.



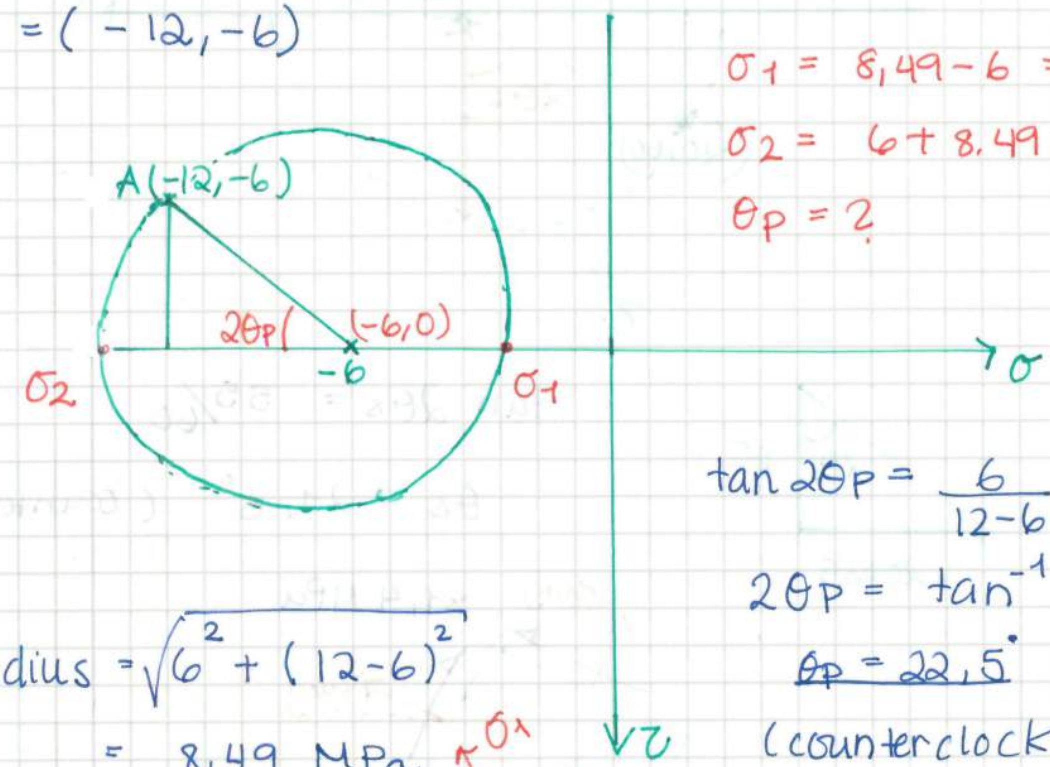
σ_x = -12 MPa

σ_y = 0

τ_{xy} = -6 MPa

$$C = \left(\frac{\sigma_x + \sigma_y}{2}, 0\right) = \left(\frac{-12}{2}, 0\right) = (-6, 0)$$

A = (-12, -6)



σ_1 = 8,49 - 6 = 2,49

σ_2 = 6 + 8.49 = 14,5

θ_p = ?

$$\tan 2\theta_P = \frac{6}{12-6}$$

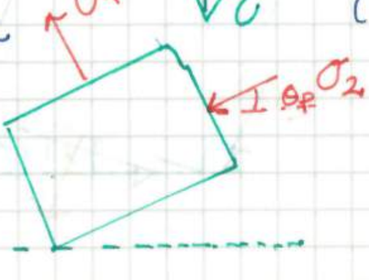
$2\theta_P = \tan^{-1}(1) = 4$

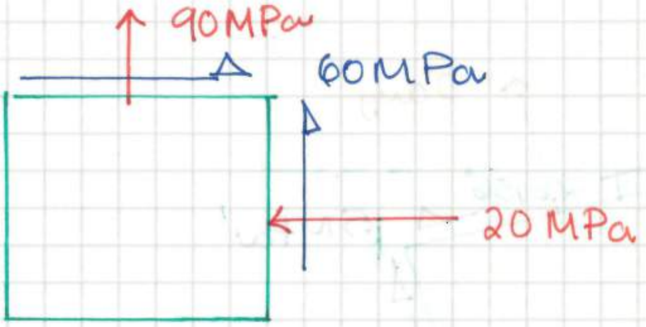
$\theta_P = 22,5°$

(counterclockwise)

$$\text{radius} = \sqrt{6^2 + (12-6)^2}$$

$$= 8,49 \text{ MPa}$$





$\sigma x = -20 MPa$

$\sigma y = 90 MPa$

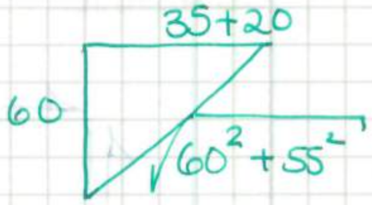
$\sigma_{xy} = 60 MPa$

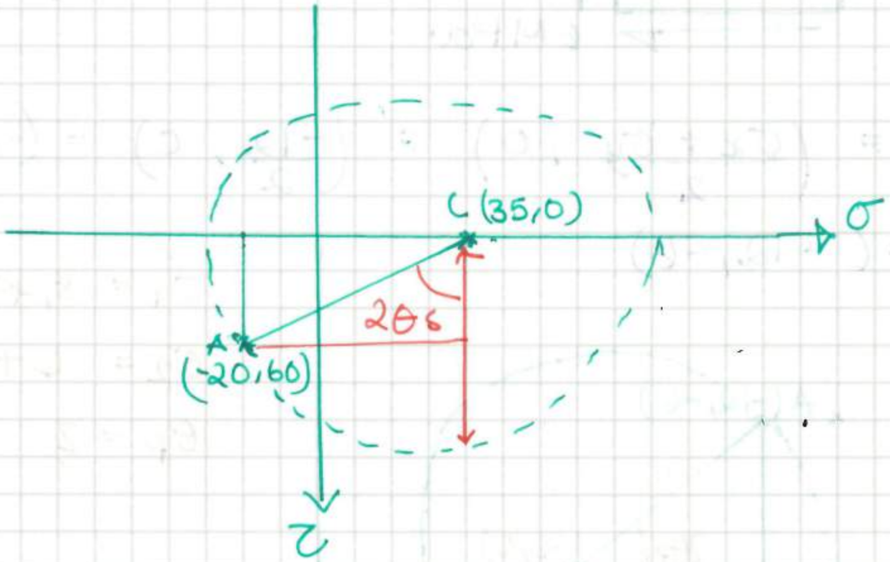
$C = \left(\frac{\sigma x + \sigma y}{2}, 0\right) = \left(\frac{-20+90}{2}, 0\right) = (35, 0)$

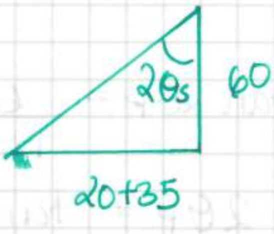
Referance

$A = (-20, 60)$

Radius $\sqrt{60^2 + 55^2} = 81.4 MPa$

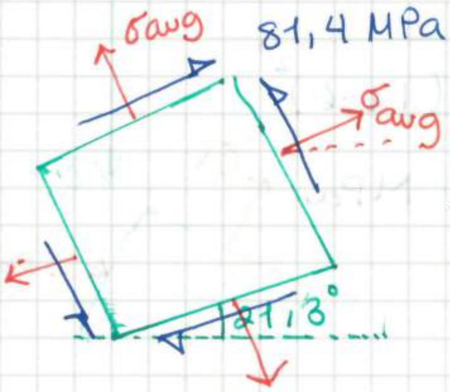




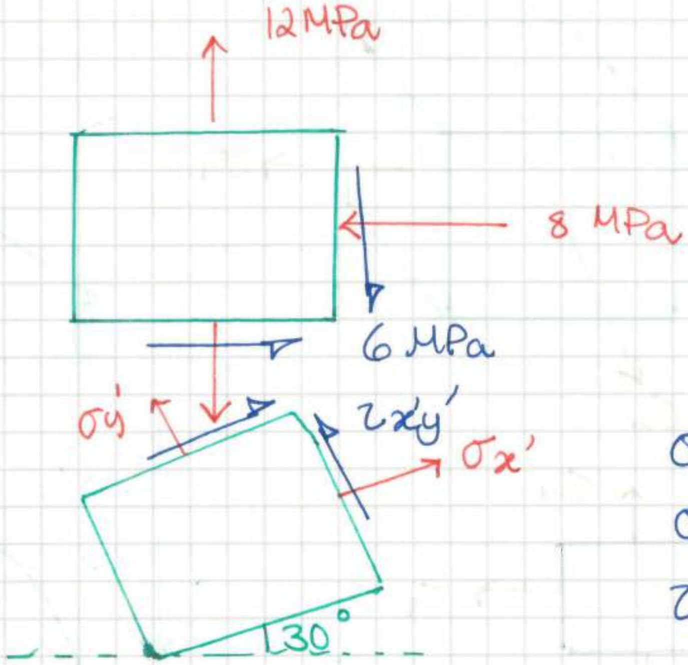


$\tan 2\theta s = 55/60$

$\theta s = 21,3°$ (counter clockwise)



example (7)



$\sigma x = -8 MPa$

$\sigma y = 12 MPa$

$\tau xy = -6 MPa$

$\sigma x' = ?$

$\sigma y' = ?$

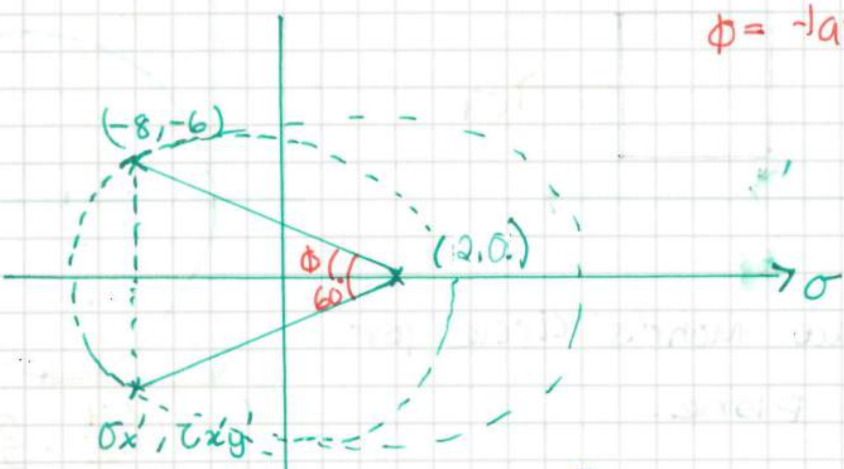
$\tau x'y' = ?$

center in $\left(\frac{\sigma x + \sigma y}{2}, 0\right) = (2, 0)$

$A = (-8 ; -6)$

Radius $\sqrt{(8+2)^2 + 6^2} = 11,66$

$\phi = \tan^{-1}\left(\frac{6}{10}\right) = 30,96°$



$\sigma y' = 2 + 11,66 \cos 29,04°$

$= 12,2 MPa.$

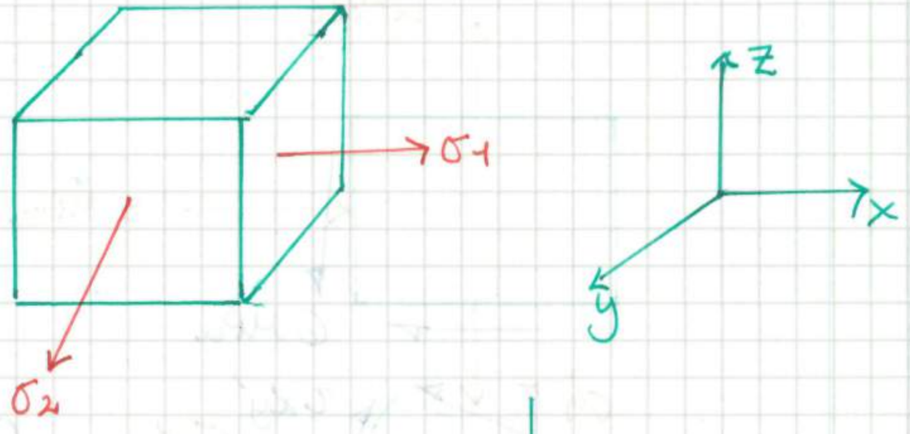
$\sigma x' = -\left[(11,66 \times \cos 29,04°) - 2 \right]$

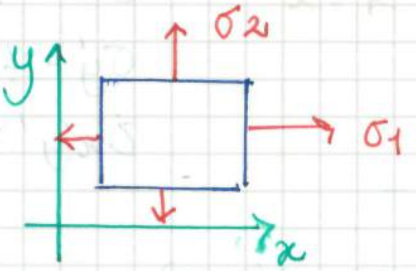
$\sigma x' = -8,20 MPa$

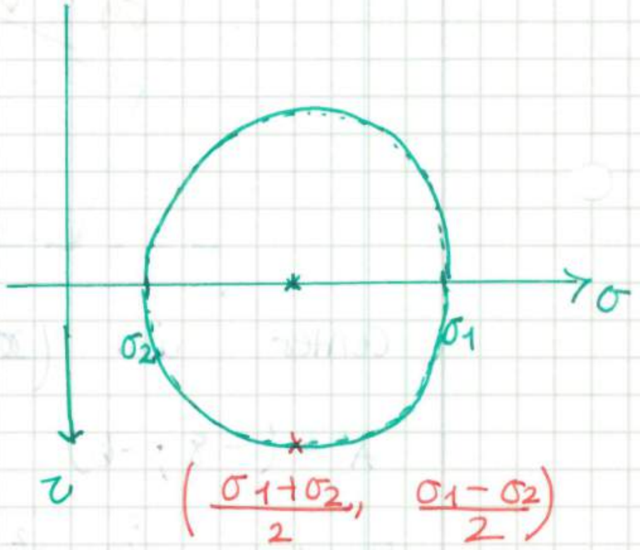
$\tau xy = 11,66 \sin 29,04°$

$= 5,66 MPa$

9.5 Absolute maximum shear stress ($\tau_{abs\,max}$)

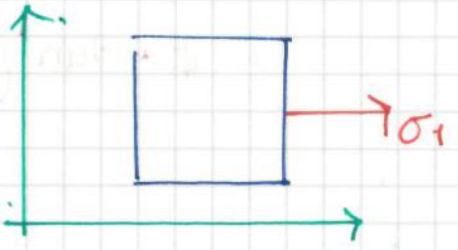


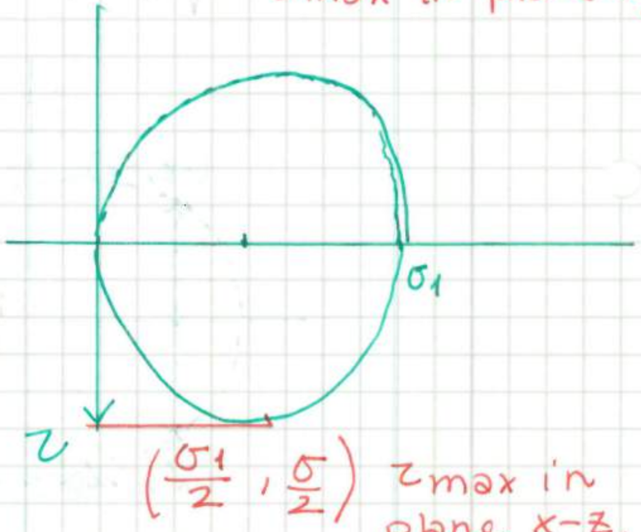




τmax in plane x-y

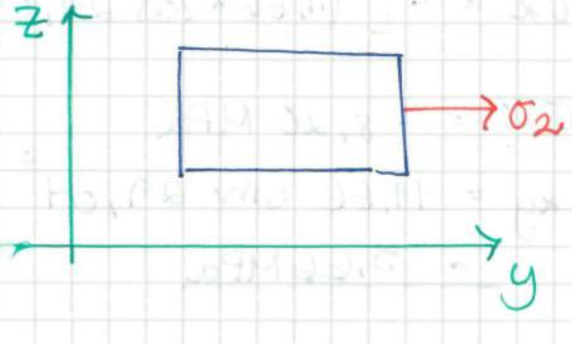
(2) Draw mohr's circle for x-z plane

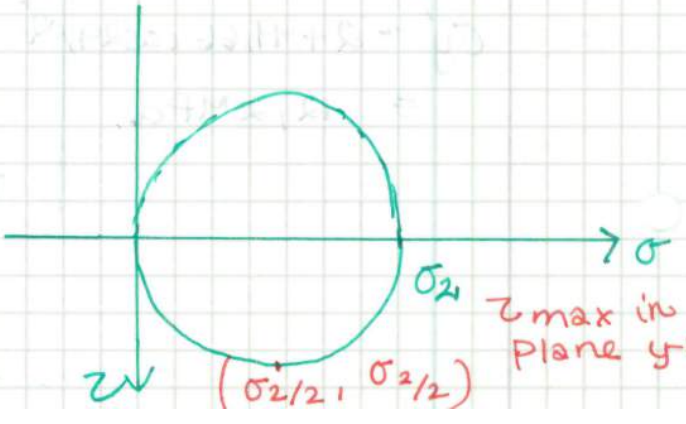




τmax in plane x-z.

(3) Draw Mohr's circle for y-z plane.

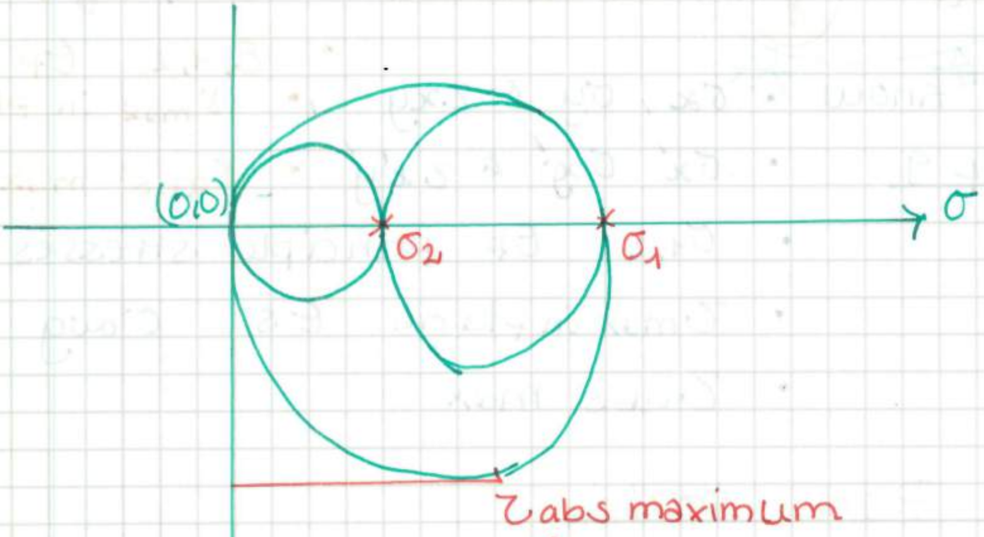


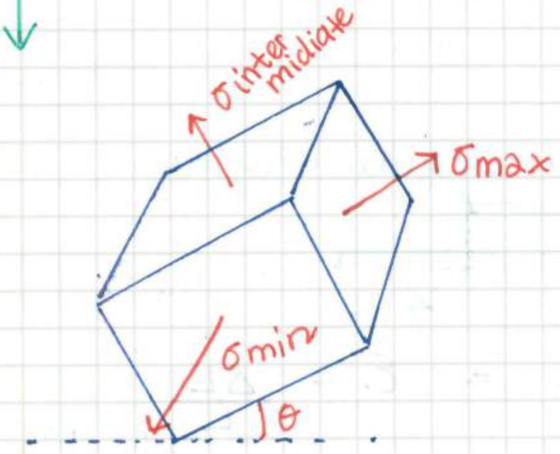


τmax in Plane y-z

Note

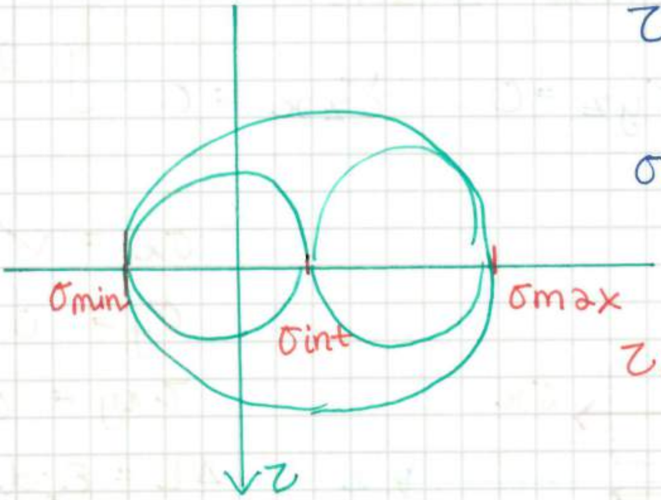
We can draw 3 Mohr's circles in one coordinate system (combine 3 mohr's circles)





$\sigma_{max} < \sigma_{int} < \sigma_{mi}$

We can draw mohr's circle,



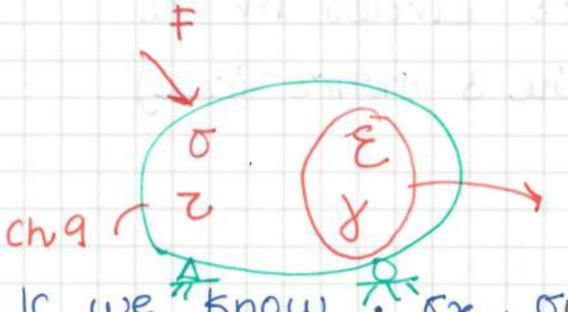
$\tau_{abs\,max} = \left(\frac{\sigma_{max}-\sigma_{min}}{2}\right.$

$\sigma_{avg} = \left(\frac{\sigma_{max}+\sigma_{min}}{2}\right.$

$\tau_{abs\,max} = \frac{32-0}{2} = 16$

21/03

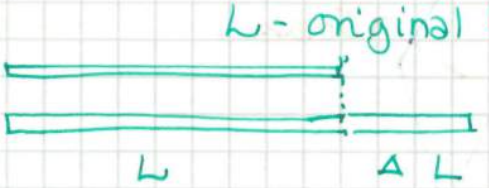
Chapter 10: Strain Transformation



ch 10
- $\varepsilon_{x'}, \varepsilon_{y'}, \gamma_{x'y'}$
- $\varepsilon_{1,2}$ θ_p
- γ_{max} in plane θ_s & ε_{avg}
- $\gamma_{abs\ max}$

If we know
Chap 9
- σ_x, σ_y & τ_{xy}
- $\sigma_{x'}$ $\sigma_{y'}$ & $\tau_{x'y'}$
- $\sigma_{1,2}$ θ_p principle stresses
- τ_{max} in plane θ_s σ_{avg}
- $\tau_{abs\ max}$

10.1 Plane - strain

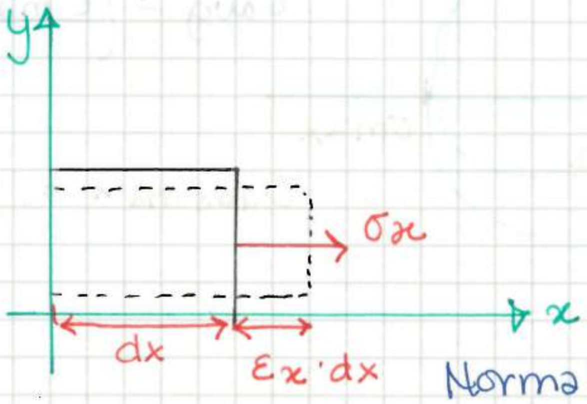


Linear strain = $\varepsilon = \frac{\Delta L}{L}$

In 3D: ε_x, ε_y, ε_z, γ_{xy}, γ_{yz}, γ_{zx}

But, for plane strain

$\varepsilon_z = 0$, $\gamma_{yz} = 0$ $\gamma_{zx} = 0$ } 2D



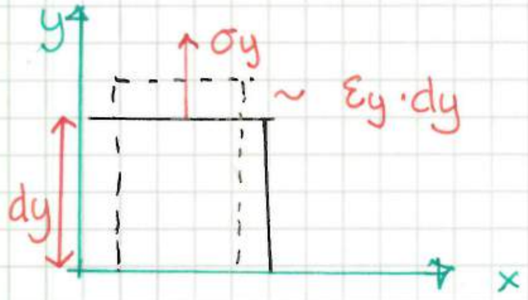
σ_x = ✓

$\sigma_y = 0$

$\tau_{xy} = 0$

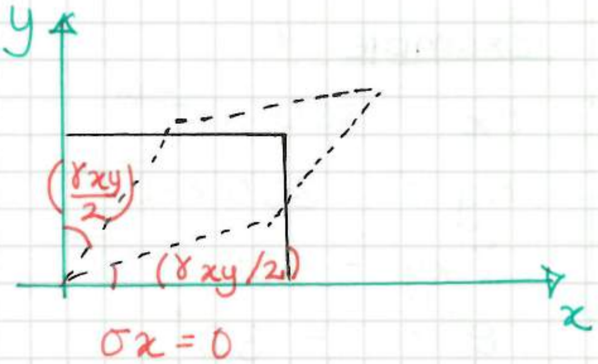
$\Delta L = \varepsilon_x \cdot dx$

Normal strain x-direction ε_x



$\sigma x = 0$
$\sigma y = V$
$\tau xy = 0$

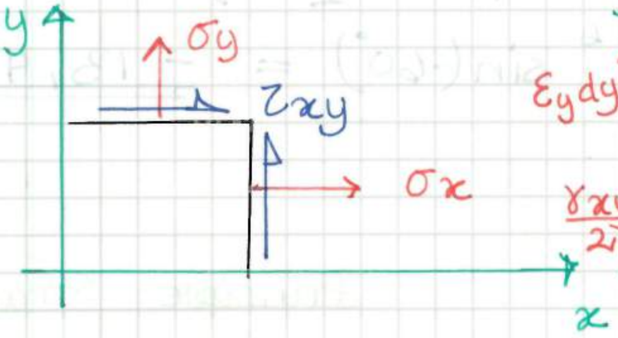
Normal strain in y-direction ε_y

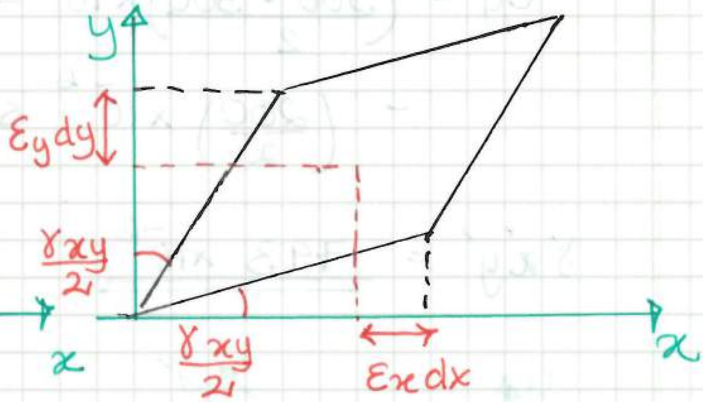


$\sigma x = 0$
$\sigma y = 0$
$\tau xy = V$

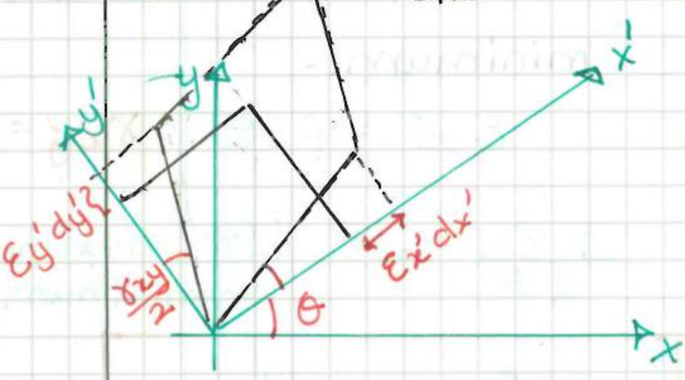
Shear strain γ_{xy}

If the element is subjected to $\sigma_x, \sigma_y, \tau_{xy}$ then deformed shape ?





For a $\theta°$ oriented element:



θ = counterclockwise

If we know $\varepsilon_x, \varepsilon_y, \gamma_{xy}$

$$\boxed{\varepsilon_{x'} = \left(\frac{\varepsilon_x+\varepsilon_y}{2}\right) + \left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta}$$

$$\boxed{\varepsilon_{y'} = \frac{\varepsilon_x+\varepsilon_y}{2} - \frac{\varepsilon_x-\varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta}$$

$$\boxed{\frac{\gamma_{x'y'}}{2} = -\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta}$$

Example 1

$\varepsilon_x = 500 \times 10^{-6}$

$\varepsilon_y = -300 \times 10^{-6}$

$\gamma_{xy} = 200 \times 10^{-6}$

$\theta = -30^\circ$

$$\hookrightarrow \varepsilon_{x'} = \left(\frac{\varepsilon_x + \varepsilon_y}{2}\right) + \left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

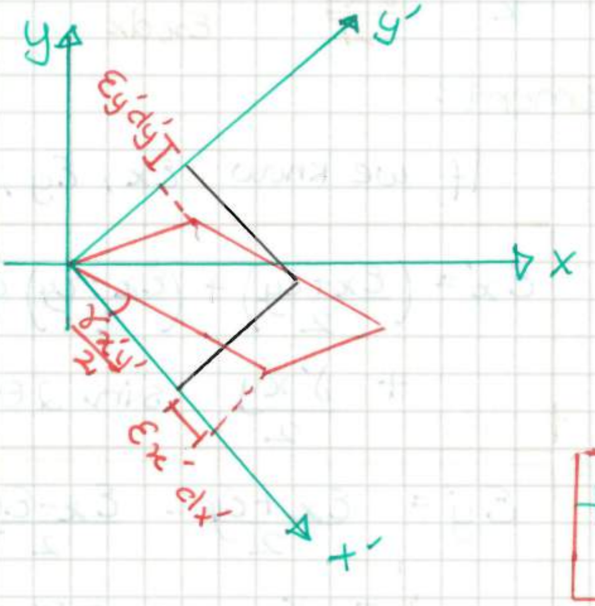
$$= \left(\frac{500-300}{2}\right)\times 10^{-6} + \left(\frac{500+300}{2}\right)\times 10^{-6} \times \cos(-60^\circ)$$

$$+ \frac{200}{2}\times 10^{-6} \sin(-60^\circ) = \underline{\underline{213 \times 10^{-6}}}$$

$$\varepsilon_{y'} = \left(\frac{500-300}{2}\right)\times 10^{-6} - \left(\frac{500+300}{2}\right)\times 10^{-6} \cos(-60^\circ)$$

$$- \left(\frac{200}{2}\right)\times 10^{-6} \sin(-60^\circ) = \underline{\underline{-13,4 \times 10^{-6}}}$$

$$\gamma_{x'y'} = \underline{\underline{793 \times 10^{-6}}}$$



Principle Strains

maximum normal strain/ minimum.

$\varepsilon_{x'} = f(\theta)$, $\gamma_{xy} = 0$

$\frac{d\varepsilon_{x'}}{d\theta} = 0$ for maximum or minimum

$$\boxed{\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}}$$

By substituting θ_p in equation $\varepsilon_{x'}$

$$\boxed{\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}}$$

Example (2)

$\varepsilon_x = -350 \times 10^{-6}$

$\varepsilon_y = 200 \times 10^{-6}$

$\gamma_{xy} = 80 \times 10^{-6}$

Find principle strain $\varepsilon_{1,2}$?

Orientation θ_p - ?

↳ $\tan 2\theta_p = \dfrac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \dfrac{80}{(-350-200)}$

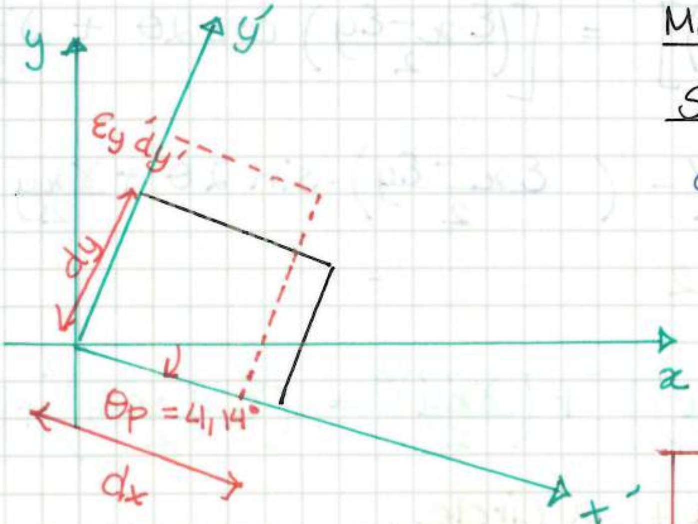
$\theta_p = \underline{\underline{-4,44° \ \& \ 85,9°}}$

$$\varepsilon_{1,2} = \left(\frac{-350+200}{2}\right) \times 10^{-6} \pm \sqrt{\left(\frac{-350+200}{2}\right)^2 + \left(\frac{80}{2}\right)^2}$$

$\underline{\underline{\varepsilon_1 = 203 \times 10^{-6}}}$ $\underline{\underline{\varepsilon_2 = -353 \times 10^{-6}}}$

To find which of these values of strains deforms the element in the x' direction use eq $\varepsilon_{x'}$ and $\theta_p = -4,14°$

$\varepsilon_{x'} = -353 \times 10^{-6}$



Max in plane shear Strain

as $\gamma_{x'y'} = f(\theta)$

$$\frac{d\left(\frac{\gamma_{x'y'}}{2}\right)}{d\theta} = 0$$

for maximum

$$\boxed{\tan 2\theta_s = -\left(\frac{\varepsilon_x - \varepsilon_y}{\gamma_{xy}}\right)}$$

By substituting θ_s in $\frac{\gamma_{x'y'}}{2}$ eq.

$$\boxed{\frac{\gamma_{\text{max in plane}}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}}$$

Example (3)

Given $\varepsilon_x = -350 \times 10^{-6}$ Find

$\varepsilon_y = 200 \times 10^{-6}$ γ_{max} in plane

$\gamma_{xy} = 80 \times 10^{-6}$ θ_s - orientation

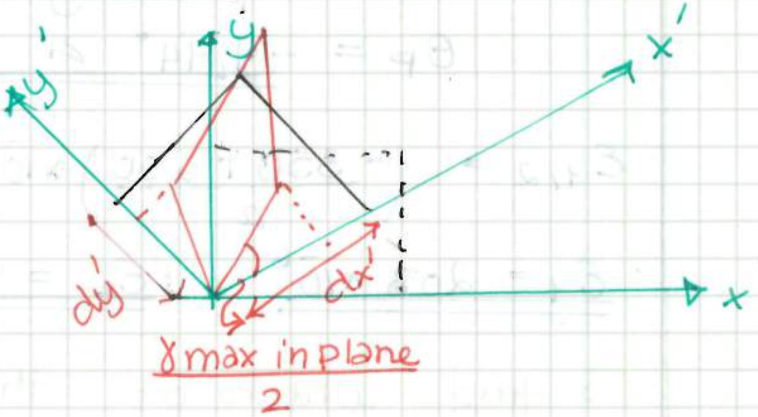
$$\tan 2\theta_s = -\left(\frac{-350-200}{80}\right) \times 10^{-6}$$

$$\theta_s = 40{,}9^\circ \quad \text{or} \quad 130{,}9^\circ$$

$$\frac{\gamma_{max \text{ in plane}}}{2} = \underline{\underline{556 \times 10^{-6}}}$$

$$\gamma_{x'y'} = 556 \times 10^{-6}$$

$$\varepsilon_{avg} = -75 \times 10^{-6}$$



Mohr's circle - Plane strain

$$\varepsilon_{x'} = \left(\frac{\varepsilon_x + \varepsilon_y}{2}\right) + \left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\left[\varepsilon_{x'} - \left(\frac{\varepsilon_x + \varepsilon_y}{2}\right)\right]^2 = \left[\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta\right]^2 \quad -①$$

$$\left(\frac{\gamma_{xy}}{2}\right)^2 = \left[\left(-\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right) \cdot \sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta\right]\right]^2 \quad -②$$

Equation 1 + 2

$$\left[\varepsilon_{x'} - \left(\frac{\varepsilon_x + \varepsilon_y}{2}\right)\right]^2 + \left(\frac{\gamma_{x'y'}}{2}\right)^2 = \left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2$$

equation of a circle

$$(\varepsilon_{x'} - C)^2 + \left(\frac{\gamma_{x'y'}}{2}\right)^2 = R^2 \qquad \text{center } (C, 0)$$

Radius = R

#1. Sign convention of axis

ε axis – positive to the right

$\frac{\gamma}{2}$ axis - positive downwards

#2. Reference point $A = \left(\varepsilon_x, \frac{\gamma_{xy}}{2}\right)$ – (Given values)

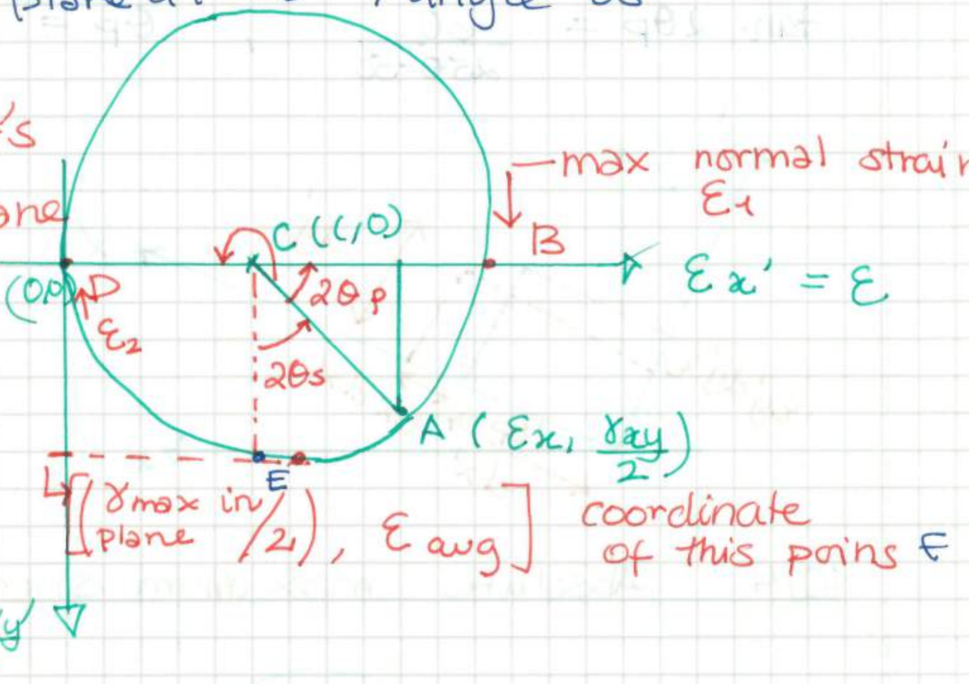
#3. Center of the circle $(C, 0) = \left(\frac{\varepsilon_x + \varepsilon_y}{2}, 0\right)$

#4 Radius of the circle = CA

#5. ε_1 and ε_2 are principle strain (at B&D θ_p

#6. $\frac{\gamma}{2}$max in plane at E, angle θ_s

This is called Mohr's circle for plane strain



Example (4)

Given $\varepsilon_x = 250 \times 10^{-6}$, $\varepsilon_y = -150 \times 10^{-6}$

$\gamma_{xy} = 120 \times 10^{-6}$

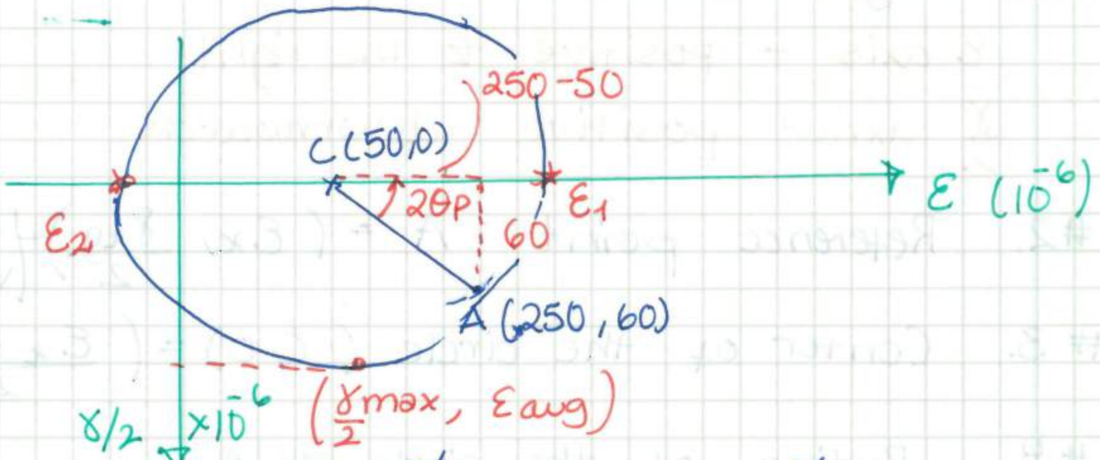
Reference $A = \left(\varepsilon_x, \frac{\gamma_{xy}}{2}\right) = (250 \times 10^{-6}, 60 \times 10^{-6})$

center $= (C, 0) = \left(\frac{\varepsilon_x + \varepsilon_y}{2}, 0\right) = \left(\left(\frac{250 - 150}{2}\right) \times 10^{-6}, 0\right)$

$= (50 \times 10^{-6}, 0)$

Radius $= \sqrt{60^2 + (250-50)^2}$

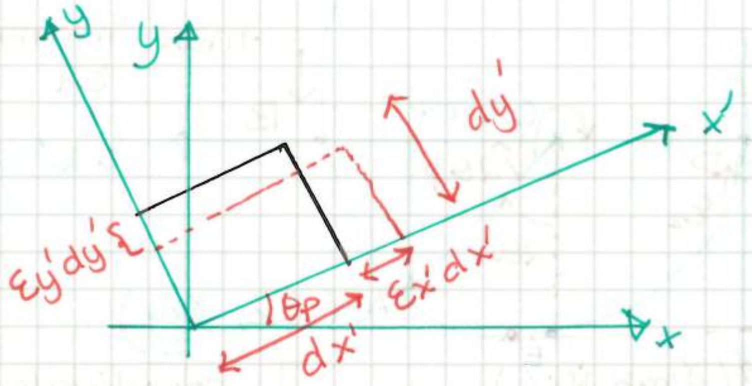
$= 208{,}8 \times 10^{-6}$



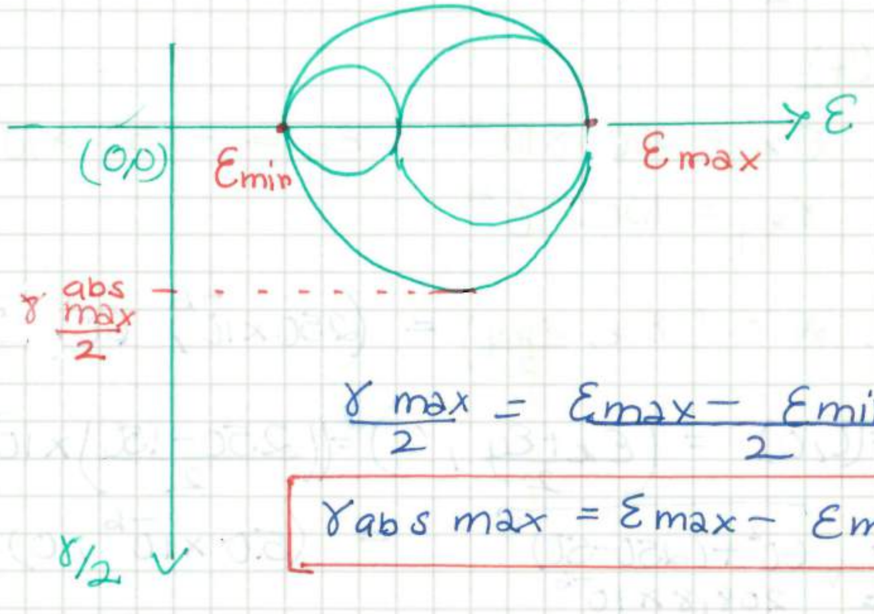
$$\varepsilon_1 = (208,8+50)\times10^{-6} = \underline{\underline{258,8\times10^{-6}}}$$

$$\varepsilon_2 = (208,8-50)\times10^{-6} = -159\times10^{-6}$$

$\tan 2\theta_p = \dfrac{60}{250-50}$, $\theta_p = 8,35^\circ$ counterclockwise



10,4 Absolute maximum shear strain



$$\frac{\gamma_{max}}{2} = \frac{\varepsilon_{max} - \varepsilon_{min}}{2}$$

$$\boxed{\gamma_{abs\,max} = \varepsilon_{max} - \varepsilon_{min}}$$

$$\varepsilon_{avg} = \frac{\varepsilon_{max} + \varepsilon_{min}}{2}$$

Summary

strain at a point for a given θ orientation

- $\varepsilon x'$, $\varepsilon y'$, $\frac{\gamma x'y'}{2}$
- Principle strains $\varepsilon_1, \varepsilon_2$
- Max in plane shear strain
  strain γ_{max} in plane
- Absolute Max $\gamma_{abs\,max}$.

example (5)

Given : $\varepsilon x = -400 \times 10^{-6}$

$\varepsilon y = 200 \times 10^{-6}$

$\gamma xy = 150 \times 10^{-6}$

maximum in plane shear strain

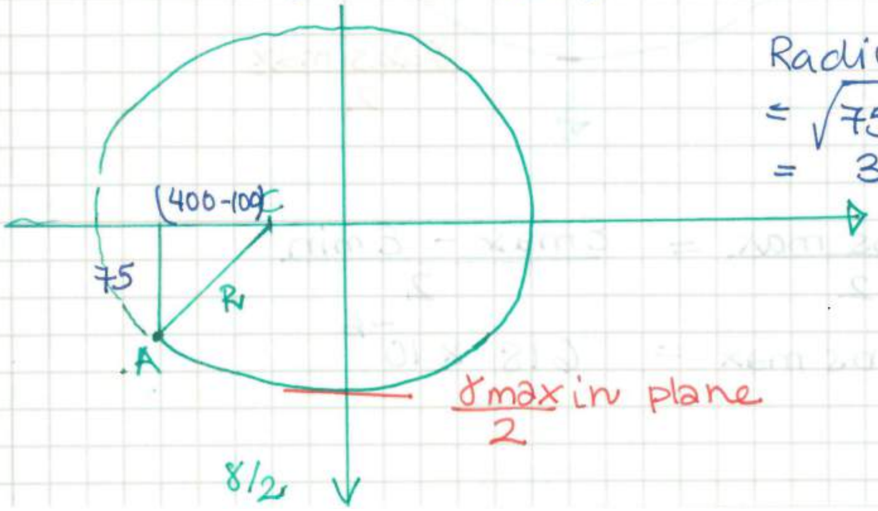
γ_{max} in plane = ?

$$\frac{\gamma_{max\ in\ plane}}{2} = \left(\varepsilon_{\substack{max\\ in\ x-y\ plane}} - \varepsilon_{\substack{min\\ xy\ plane}}\right)$$

Lets draw Mohr's circle

center $\left(\frac{\varepsilon x + \varepsilon y}{2}, 0\right) (C, 0) = (-100 \times 10^{-6}, 0)$

Reference $A\left(\varepsilon x, \frac{\gamma xy}{2}\right) = (-400 \times 10^{-6}, 75 \times 10^{-6})$



Radius CA

$= \sqrt{75^2 + (400-100)^2}$

$= 309 \times 10^{-6}$

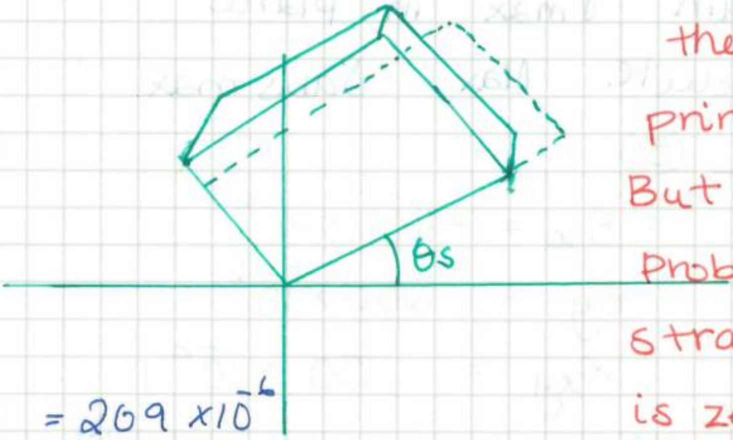
$$\frac{\gamma_{max\ in\ plane}}{2} = 309 \times 10^{-6}$$

$$\gamma_{max\ in\ plane} = \underline{\underline{618 \times 10^{-6}}}$$

$$\gamma_{abs\,max} = \gamma_{max\ in\ plane}$$

xy , yz , zx

If I draw the strain element;

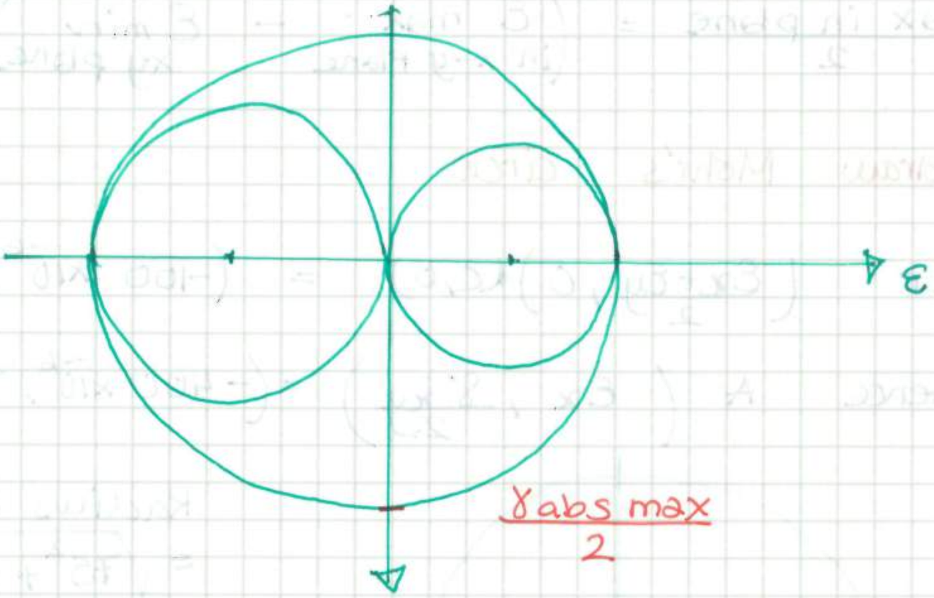


there are 3 principle strains But in this problem principle strain in z-directio is zero (ε_z=0)

- $\varepsilon_{max} = 209 \times 10^{-6}$

$\varepsilon_{int} = 0$

$\varepsilon_{min} = -409, \times 10^{-6}$



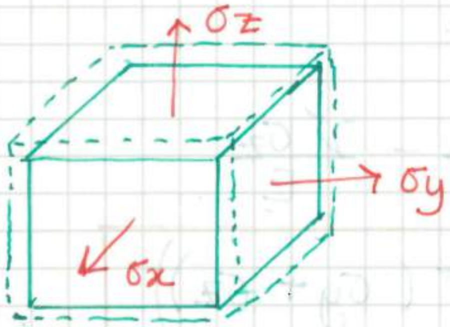
$$\frac{\gamma_{abs\,max}}{2} = \frac{\varepsilon_{max} - \varepsilon_{min}}{2}$$

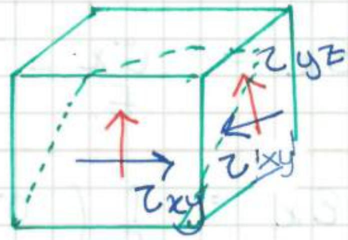
$$\gamma_{abs\,max} = 618 \times 10^{-6}$$

10.6 Material - property (Stress strain relation)

Objective ; to find ε & σ relationship.

If the material at a point is subjected to a state of triaxial stress $\sigma_x, \sigma_y, \sigma_z$
$\tau_{xy}, \tau_{yz}, \tau_{zx}$





These stresses can be related to strains

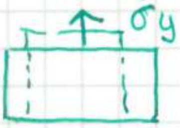
- * Hooke's law ($\sigma = E \cdot \varepsilon$, $\tau = G \cdot \gamma$)
- * Poission's ratio ($\nu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}}$)
- * Principle of superposition

Let's consider only x-direction :

Case 1 when σ_x is applied, element elongates in x- direction.

From Hook's law $\varepsilon_x' = \frac{\sigma_x}{E}$ ①

Case 2 when σ_y is applied

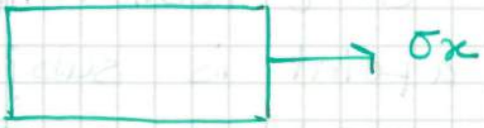


strain due to σ_y

$= \varepsilon_x'' = -\nu \frac{\sigma_y}{E}$ ②

$$\nu = -\frac{\text{lateral strain}}{\text{longitudinal strain}} = -\frac{\varepsilon_x}{\varepsilon_y}$$

case III when σ_x & σ_y are applied



$$\varepsilon_x = \frac{\sigma_x}{E} + \left(-\frac{\nu\sigma_y}{E}\right)$$

when σ_y, σ_x, σ_z

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\boxed{\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu(\sigma_y + \sigma_z)\right)}$$

Similarly

$$\boxed{\varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu(\sigma_x + \sigma_z)\right]}$$

$$\boxed{\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]}$$

Hook's law for shear stress & strain,

$$\boxed{\gamma_{xy} = \frac{1}{G}\tau_{xy}} \qquad \boxed{\gamma_{yz} = \frac{1}{G}\tau_{yz}}$$

$$\boxed{\gamma_{zx} = \frac{1}{G}\tau_{zx}}$$

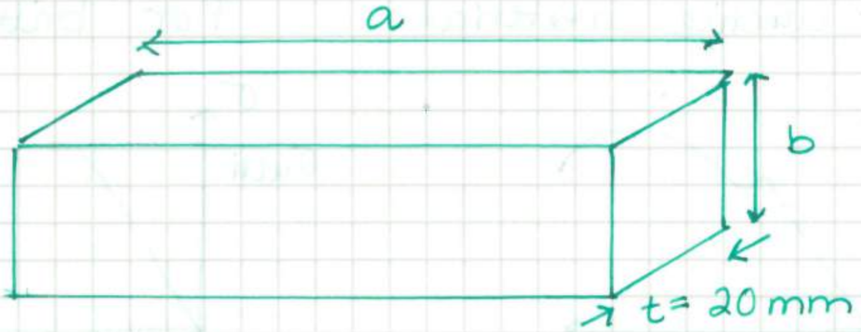
* Relation between Elastic modulus (E) shear modulus (G) & Poisson's ratio

$$\boxed{G = \frac{E}{2(1+\nu)}}$$

Example (6)

The copper bar is subjected to a uniform loading along its edges as shown in fig. If a = 300 m b = 50 mm and t = 20 mm before the load is applied, find its new length, width and thickness after application of the load. E = 120 GPa

$\nu = 0{,}34$



$\sigma_x = 800$ MPa $\qquad \sigma_z = 0$

$\sigma_y = -500$ MPa

$\tau_{xy} = 0$

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\,(\sigma_y + \sigma_z)\right)$$

$$= \frac{1}{120\times10^3}\left[800 - (0{,}34)(-500+0)\right] = 0{,}00808$$

$$\varepsilon_y = -0{,}00643$$

$$\varepsilon_z = \frac{1}{E}\left(\underset{0}{\cancel{\sigma_z}} - \nu\,(\sigma_y + \sigma_z)\right] = -0{,}00085$$

New length:

$$a' = (a + \varepsilon_x \cdot a)$$

$$= 300\,(1 + 0{,}00808) = 302{,}8\text{ mm}$$

New width

$$b' = b + \varepsilon_y \cdot b = 50\,(1 - 0{,}00643) = 49{,}68\text{ mm}$$

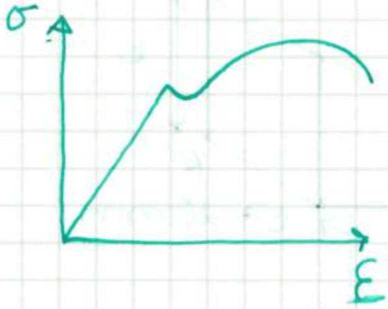
$$t' = 20(1+\varepsilon_z) = 20(1-0{,}00085)$$
$$= \underline{\underline{19{,}98\,mm}}$$

10,7 Theories of Faliure

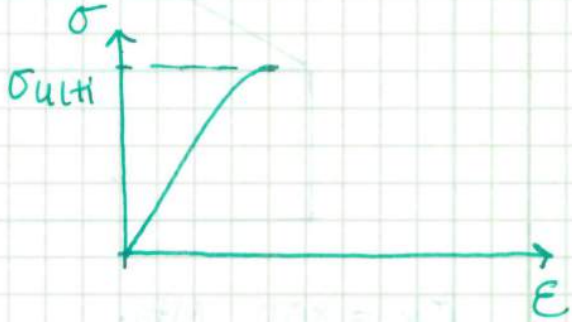
Objective to understand how faliure will occure

Revision: For uniaxial (1D)

For ductile material



For brittle material



Not to yield = $\sigma \leq \sigma_y$ due loading

not to fracture $\sigma \leq \sigma_{ulti}$

For Biaxial (2D)

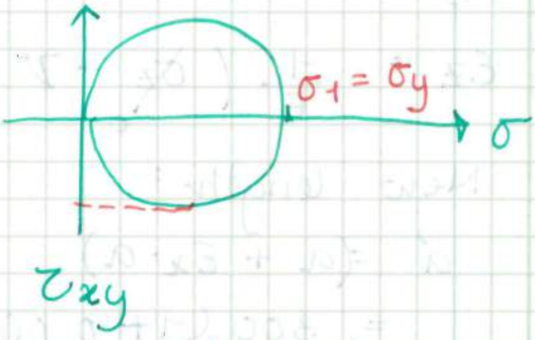
For ductile material

(I) Maximum shear stress theory (1868)

Tressa yield criterion) (von mises yield criterion)

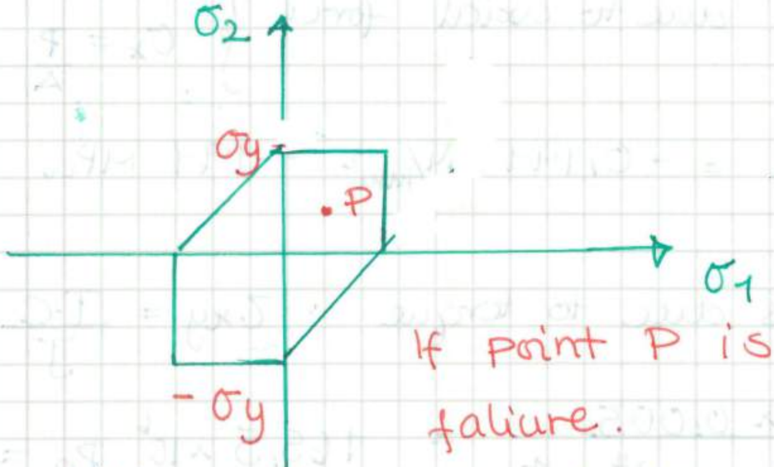
To avoid faliure

$$\tau_{absmax} \leq \frac{\sigma_y}{2}$$
$$\downarrow$$
$$\frac{\sigma_{max}-\sigma_{min}}{2} \leq \frac{\sigma_y}{2}$$



uniaxial tensile testing

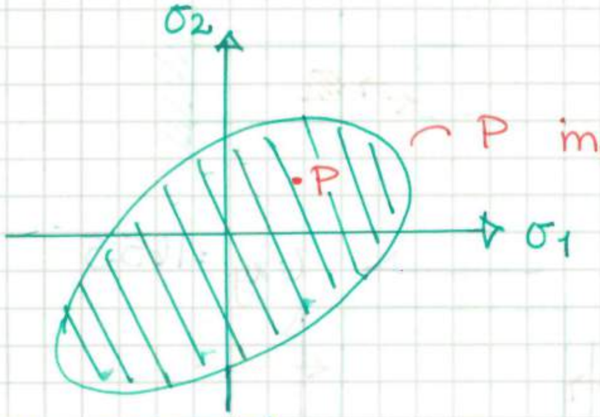
$$|\sigma_1| \leq \sigma_y$$
$$|\sigma_2| \leq \sigma_y \qquad |\sigma_1-\sigma_2| \leq \sigma_y$$



If point P is inside, no faliure.

max. distorsion energy ≤ distorsion energy at yielding

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_y^2$$



P material will fail

Brittle material

maximum normal stress theory

$|\sigma_1| \leqslant \sigma_{ult}$

$|\sigma_2| \leqslant \sigma_{ult}$

example (7)

$\sigma_{yield} = 360$ MPa

Find, wheather the shaft will fail due to this loading?

* Material → steel → ductile

* How will it fail (faliure criteria)

* In which point (σ_1, σ_2)

} $\sigma_1; \sigma_2$

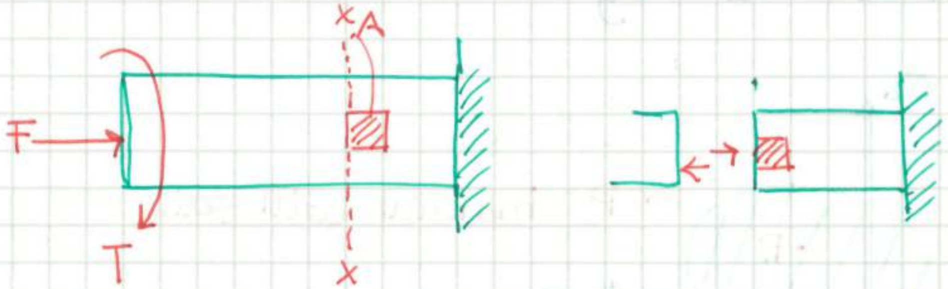
F = 15 KN Torque = 3,25 KN·cm

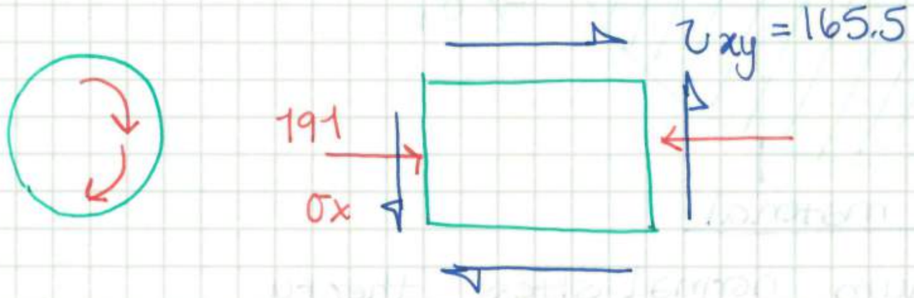
axial stress due to axial force $\Big\}$ $\sigma_x = \frac{P}{A}$

$$\sigma_x = \frac{15\ KN}{\pi(5)^2} = -0{,}191\ KN/mm^2 = -191\ MPa$$

shear stress due to torque : $\tau_{xy} = \frac{T \cdot C}{J}$

$$\tau_{xy} = \frac{3{,}25 \times 0{,}005}{\frac{\pi}{2}(0{,}005)^4\ m^4} = 165{,}5 \times 10^6\ Pa = 165{,}5\ MPa$$





$$\sigma_{1,2} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$\sigma_1 = -95 \pm 191{,}9$

$\underline{\sigma_1 = 95{,}6\ MPa}$

$\underline{\sigma_2 = -286{,}6\ MPa}$

(1) Maximum shear stress theory $|\sigma_1 - \sigma_2| \leq \sigma_y$

$$|\sigma_1 - \sigma_2| = |95{,}6 - (-286{,}6)| = 382.2\ MPa$$

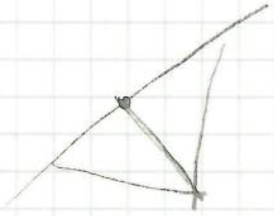
$\sigma_y = 360\ MPa$

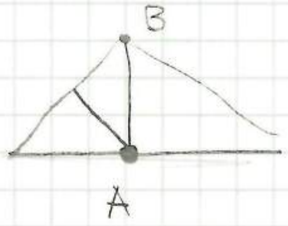
shear falicure of the material will occure.

Max distortion theory

$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 < \sigma_y^2$

$344.5 \leqslant 360$ (not fail)





$F_{AB} = 0$

No support
No external for