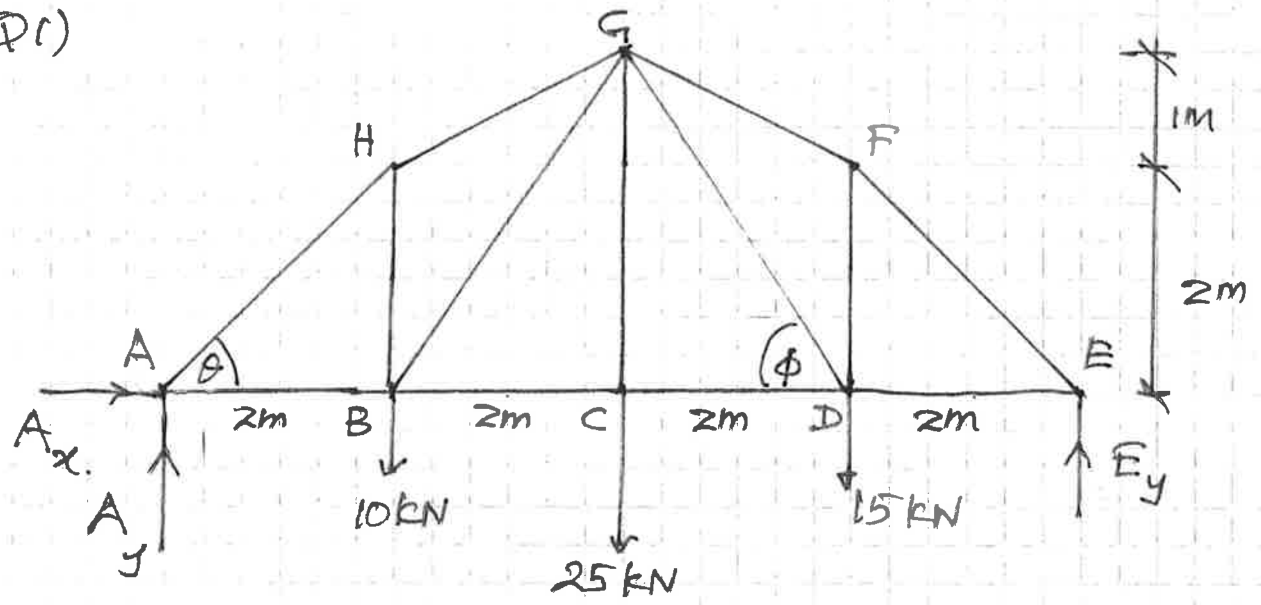
BYG 140 - Konstruksjonmekanikk 1

september 13, 2016.

(Q1)



(I) considering equilibrium of entire truss taking moment about A,

$$\circlearrowleft \; E_y 8 - (15 \times 6) - (25 \times 4) - (10 \times 2) = 0$$

$$E_y = \frac{90+100+20}{8}$$

$$E_y = \underline{\underline{26.25\ kN}}$$

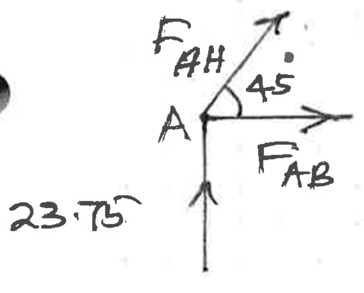
$\rightarrow \sum F_x \qquad A_x = \underline{\underline{0}}$

$\uparrow \sum F_y \qquad A_y + E_y - (10+25+15) = 0$

$$A_y = 50 - 26.25$$

$$A_y = \underline{\underline{23.75\ kN}}$$

(II) Forces F_{AB} and F_{AH} by method of joints.
considering equilibrium of joint A



$\uparrow \sum F_y \qquad F_{AH} \sin 45 + 23.75 = 0$

$$F_{AH} = \frac{-23.75}{\sin 45}$$

$$F_{AH} = \underline{\underline{33.58\ kN}}$$

(compression.)

$\theta = \tan^{-1}\left(\frac{2}{2}\right)$

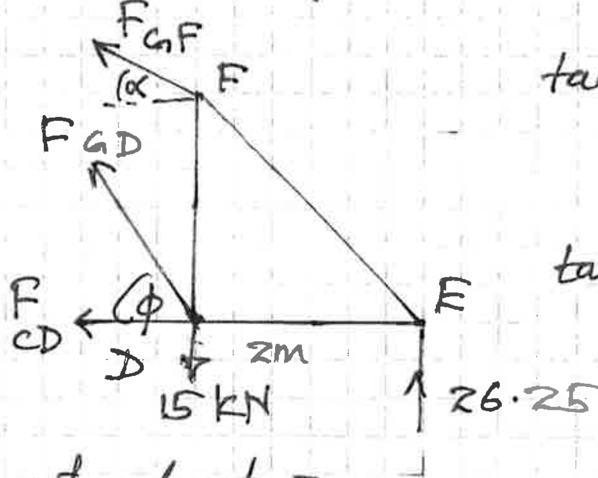
$\theta = 45°$

$\xrightarrow{} \Sigma F_x \quad F_{AB} + F_{AH} \cos 45 = 0$

$F_{AB} = -(-33.58 \cos 45)$

$F_{AB} = \underline{\underline{23.75\ kN}}$ (Tension)

(III) By using method of section



$\tan \alpha = (1/2)$

$\alpha = \tan^{-1}(1/2)$

$= 26.56°$

$\tan \phi = (3/2)$

$\phi = \tan^{-1}(3/2)$

$= 56.31°$

taking moment about D,

$\circlearrowleft \ F_{GF} \cos \alpha \cdot 2 + (26.25 \times 2) = 0$

$$F_{GF} = \frac{-26.25 \times 2}{2 \times \cos \alpha}$$

$F_{GF} = -29.35\ kN$

$F_{GF} = \underline{\underline{29.35\ kN}}$ (Compressio

$\uparrow F_{GF} \sin \alpha + F_{GD} \sin \phi + 26.25 - 15 = 0$

$$F_{GD} = \frac{-26.25 - (-29.35 \sin \alpha) + 15}{\sin \phi}$$

$F_{GD} = \underline{\underline{2.25\ kN}}$ (tension)

$\leftarrow F_{CD} + F_{GD} \cos \phi + F_{GF} \cos \alpha = 0$

$F_{CD} = -(2.25 \cos \phi - 29.35 \cos \alpha)$

$F_{CD} = \underline{\underline{25.0\ kN}}$ (tension)

(IV) No zero force members.

(V) Cross sectional area of GC

$$\sigma_{all} \geq \frac{F_{GC}}{Area}$$

$F_{GC} = 25\,kN$ at joint C (considering equilibrium of joint C)

$$250\ N/mm^2 \geq \frac{25 \times 10^3\ N}{area}$$

$$area \geq \frac{25 \times 10^3}{250} = \underline{\underline{100\ mm^2}}$$

(VI) change in length of GC member

$$\frac{F_{GC}}{A} = E\,\frac{\delta}{L}$$

$$\frac{25 \times 10^3\ N}{100\ mm^2} = 200 \times 10^3\ N/mm^2 \left(\frac{\delta}{3000\ mm}\right)$$

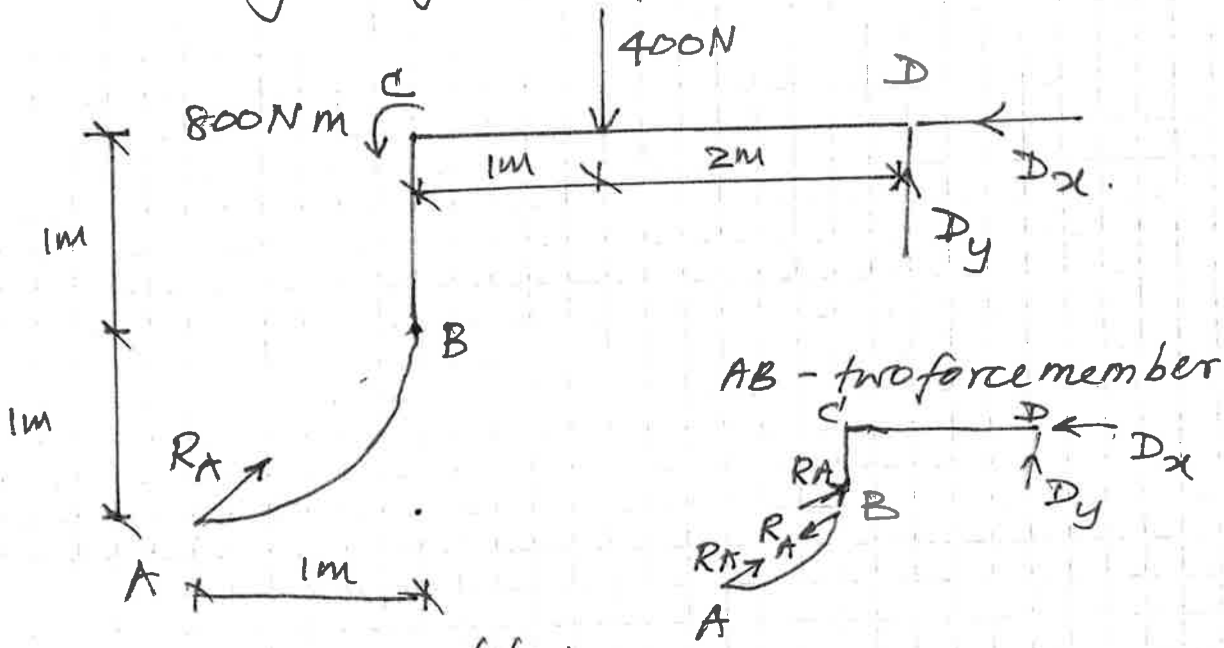
$$\delta = \underline{\underline{3.75\ mm}}\ \text{(elongation)}$$

(VII) If the FD is removed, joint F is not in equilibrium (ie: F joint starts to move until it is stable.)

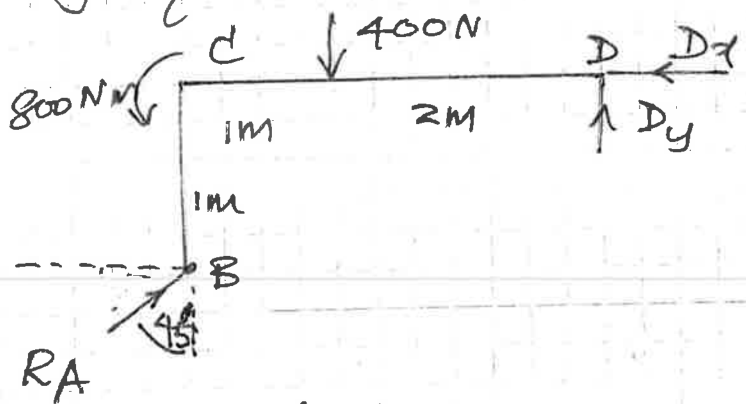
- unstable.

(Q2)

(i) Free body diagram of the structure.



considering equilibrium



taking moment about D

$$\circlearrowleft \; 800 + (400 \times 2) + (R_A \sin 45 \times 1) - (R_A \cos 45 \times 3) = 0$$

$$\therefore R_A = \frac{800}{\cos 45}$$

$$R_A = \underline{1131.37\,N}$$

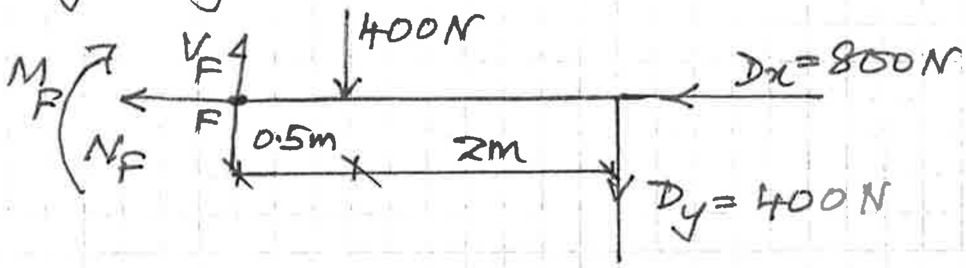
$$\uparrow \; D_y + R_A \cos 45 - 400 = 0$$

$$D_y = \underline{-400N}$$

$$\rightarrow \; R_A \sin 45 - D_x = 0$$

$$D_x = \underline{800\,N}$$

(ii) Imaginary cut at F



considering equilibrium

$\curvearrowleft$ $M_F + (400 \times 0.5) + (400 \times 2.5) = 0$

$M_F = -1200\,Nm$

$\leftarrow$ $N_F + 800 = 0$

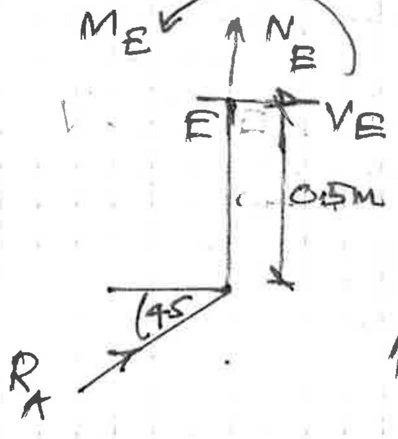
$N_F = -800\,N$

$\uparrow$ $V_F - 400 - 400 = 0$

$V_F = 800\,N$

Imaginary cut at E,

considering equilibrium,



$M_E + (R_A \cos 45 \times 0.5) = 0$

$M_E = -1131.37 \sin 45 \times 0.5$

$M_E = -400\,Nm$

$\uparrow$ $N_E + R_A \sin 45 = 0$

$N_E = -1131.37 \sin 45$

$N_E = -800\,N$

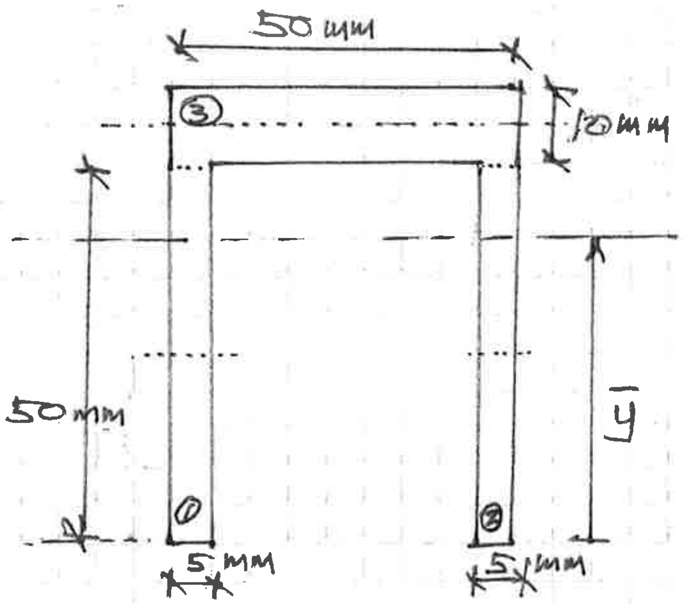
$\rightarrow$ $-V_E + R_A \cos 45 = 0$

$V_E = -1131.37 \cos 45$

$V_E = -800\,N$

(III) cross sectional area

$$A = (50\times60) - (40\times50) = \underline{1000\,mm^2}$$



$$\bar{y} = \frac{\sum A_i \bar{y}_i}{\sum A_i}$$

$$= \frac{[(50\times5)\times25\times2] + [(50\times10)55]}{(2\times50\times5) + (50\times10)}$$

$$\bar{y} = 40\,mm$$

$$I = (I_1 + A_1 d_1^2) + (I_2 + A_2 d_2^2) + (I_3 + A_3 d_3^2)$$

$$= 2\left[\left(\tfrac{1}{12}\times5\times50^3\right) + (50\times5)\times15^2\right] + \left[\left(\tfrac{1}{12}\times50\times10^3\right) + (50\times10)\times15^2\right]$$

$$= \underline{3.33\times10^5\,mm^4}$$

(IV) Maximum shear stress at point F

Shear stress $\tau_F = \dfrac{VQ}{It}$

$$(\tau_{max})_F = \frac{VQ_{max}}{It}$$

$V_F = 800N$

$I = 3.33\times10^3\,mm^4$

$t = 10\,mm$

$Q_{max} = \bar{y}A' = \dfrac{40}{2}(40\times5)\times2 = 8000\,mm^3$

$$(\tau_{max})_F = \frac{800\times8000}{3.33\times10^5\times10} = \underline{1.92\,Mpa}$$

Maximum normal stress at F.

$$(\sigma_{max})_F = \frac{-M_F y_{max}}{I} + \frac{N_F}{A}$$

$$= \frac{-(-1200\times10^3)(-40)}{3.33\times10^5} + \left(\frac{-800}{1000}\right)$$

$$(\sigma_{max})_F = \underline{\underline{-144.8 \text{ Mpa}}}$$ (compressive maximum normal stress = 144.8 Mpa)

Maximum shear stress at E

$$(\tau_{max})_E = \frac{V_E Q_{max}}{It}$$

$V_E = 800$ N $\qquad I = 3.33\times10^5$ mm^4

$Q_{max} = 8000$ mm^3 $\qquad t = 10$ mm

$$(\tau_{max})_E = \frac{-800\times8000}{3.33\times10^5\times10} = \underline{\underline{-1.92 \text{ Mpa}}}$$

Maximum normal stress at E

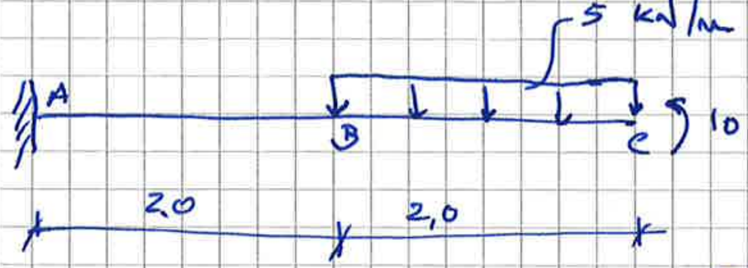
$$(\sigma_{max})_E = \frac{-M_E y_{max}}{I} + \frac{N_E}{A}$$

$$= \frac{-(-400\times10^3)(-40)}{3.33\times10^5} + \left(\frac{-800}{1000}\right)$$

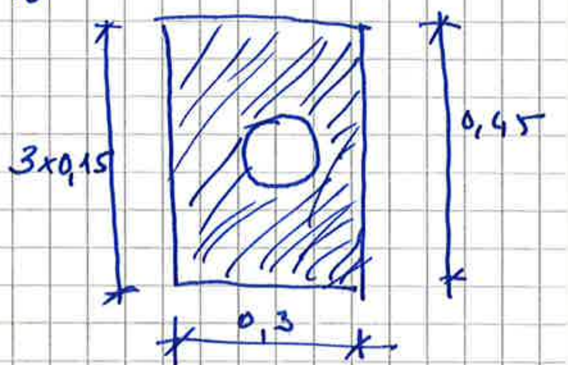
$$(\sigma_{max})_E = \underline{\underline{-48.8 \text{ Mpa}}}$$ (compressive maximum normal stress = 48.8 Mpa)

(V) If the concentrated load (400N) is removed.
magnitude of support reactions reduce. ($R_A \rightarrow$ half, $D_x \rightarrow$ half, $D_y \rightarrow$ same)
therefore all the internal forces reduce. Then stresses reduce.

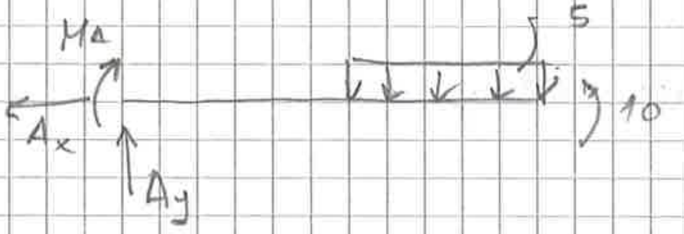
QUESTION 3

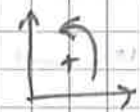


$E = 25\ GPa$



I) Support Reactions





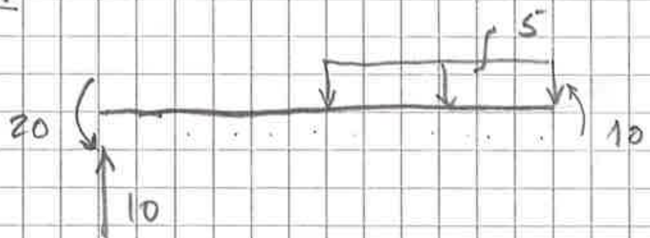
$\Sigma F_V = 0 \rightarrow \quad A_y - 5 \times 2 = 0 \quad \rightarrow A_y = 10\ kN$

$\Sigma F_H = 0 \rightarrow \quad -A_x = 0 \quad \rightarrow A_x = 0$

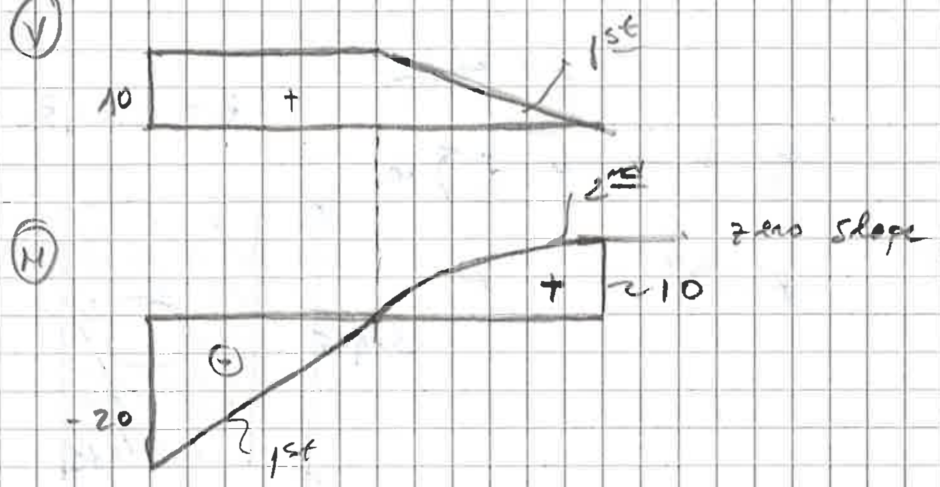
$\Sigma M_A = 0 \rightarrow \quad -M_A - 5 \times 2 \times (2+1) + 10 = 0$

$M_A = -20\ kN.m$

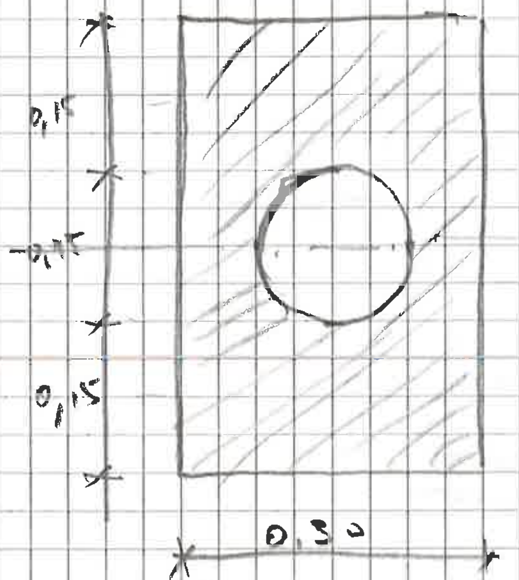
FBD



II) Shear force and bending moment Diagrams



III) Moment of Inertia along z-z



$$I = I_{\square} - I_{\bigcirc} = \frac{b\,h^3}{12} - \frac{\pi\, r^4}{4}$$

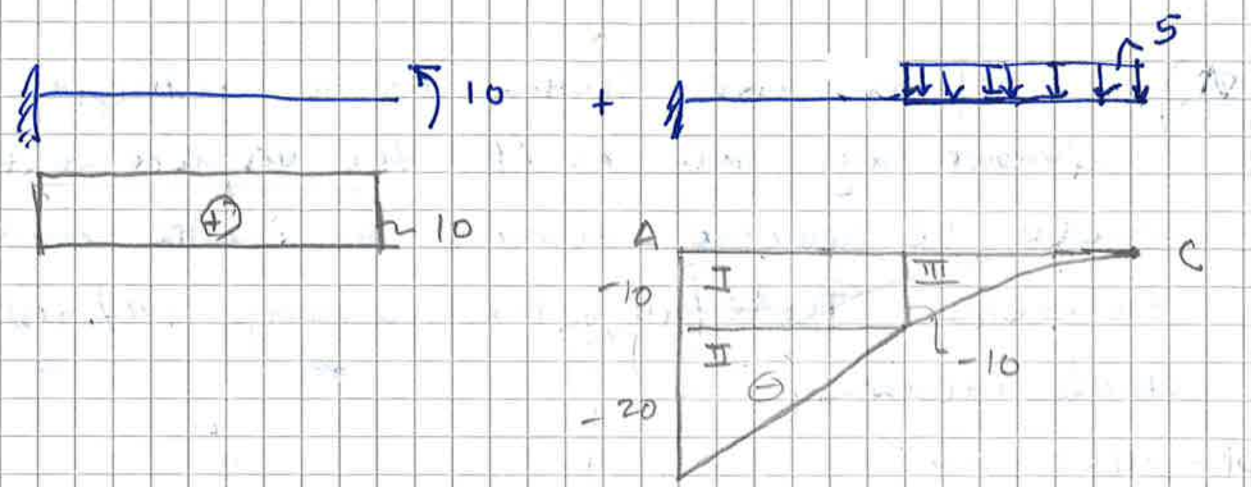
$$I_{\square} = \frac{bh^3}{12} = \frac{0{,}3 \times 0{,}45^3}{12} = 2{,}278 \times 10^{-3}\ m^4$$

$$I_{\bigcirc} = \frac{\pi\, r^4}{4} = \frac{\pi \times 0{,}075^4}{4} = 24{,}85 \times 10^{-6}\ m^4$$

$$\boxed{I = 2{,}253 \times 10^{-3}\ m^4}$$

IV) Vertical Displacement at C

The best option is to use Mohr's theorems, calculating the effects of each load separately.



Using Mohrs second theorem:

$$t_{C/A} = \delta_C = \frac{1}{EI}\int_A^C M(x)\, x\, dx =$$

$$t_{B/A} = \frac{1}{EI} \times \Big[+10\times4\times2 + \Big(\underbrace{-10\times2\times(1+2)}_{\mathrm{I}} \underbrace{- \frac{20\times2}{2}\times\left(\frac{2}{3}\times2+2\right)}_{\mathrm{II}} -$$

$$\underbrace{- \frac{10\times2}{3}\times\frac{3}{4}\times2}_{\mathrm{III}} \Big)\Big] = \frac{80-60-200/3-10}{EI} =$$

$$= \frac{-190\times10^3}{3\times(25\times10^9\times2{,}253\times10^{-5})} =$$

$$= 1{,}124\times10^{-3}\ \mathrm{m} \downarrow$$

(V) Rotation of point C

$$\theta_C - \overset{=0}{\theta_A} = \frac{1}{EI}\int_A^C M(x)\, dx =$$

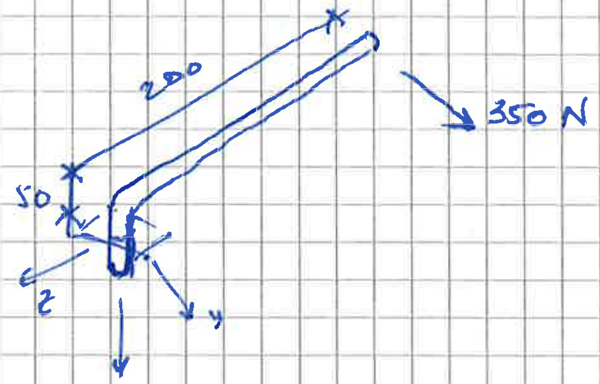
$$= \frac{1}{EI}\times\left[+10\times4 - 10\times2 - \frac{20\times2}{2} - \frac{10\times2}{3}\right] =$$

$$= \frac{40-20-20-20/3}{EI} = \frac{-20}{3EI} =$$

$$= -1{,}184\times10^{-4}\ \mathrm{rad}$$

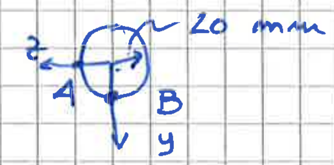
VI) If the cross section was a rectangle without any hole on it, the displacements would be reduced, because de inertia would increase. Therefore, the bending stiffness would increase.

Question 4

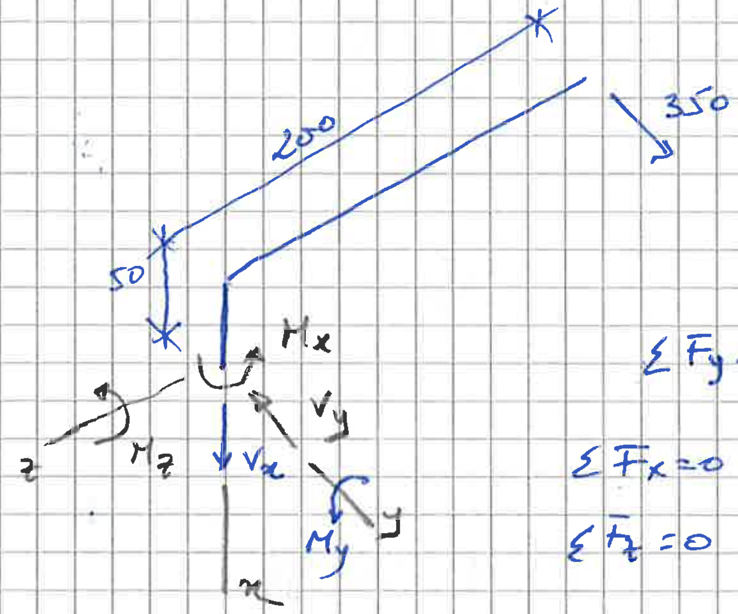


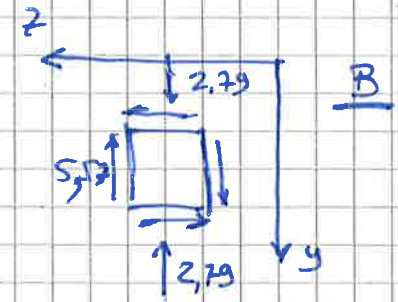
$E = 200$ GPa

$\nu = 0{,}30$



I) Internal Forces





$\Sigma F_y = 0 \quad 350 - V_y = 0 \rightarrow V_y = 350 \text{ N}$

$\Sigma F_x = 0 \quad \rightarrow V_x = 0$

$\Sigma F_z = 0 \quad \rightarrow V_z = 0$

$\Sigma M_x = 0 \quad \rightarrow 350 \times 200 - M_x = 0$

$M_x = 70\,000$ N.mm

$\Sigma M_y = 0 \quad \longrightarrow \quad M_y = 0$

$\Sigma M_z = 0 \quad \rightarrow \quad M_z - 350 \times 50 = 0 \rightarrow M_z = 17500$ N.mm

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II) At point B.

$$\sigma = \underbrace{\frac{N}{A}}_{=0} + \frac{M}{I} \times y$$

$$\tau = \underbrace{\frac{V_x Q}{I t}}_{=0} + \frac{T \times r}{J}$$

$$tg\ 2\theta_p = \frac{\tau_{zy}}{(\sigma_z - \sigma_y)/2} =$$

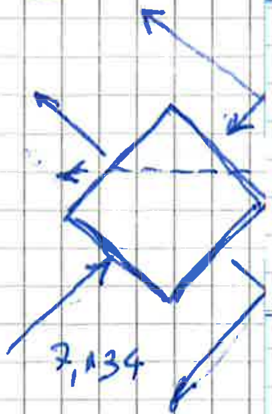
mm^4

$$2\theta_p = -75{,}94^\circ \rightarrow$$

mm^4

$$\sigma_{1,2} = \frac{+0 - 2{,}785}{2} \pm \sqrt{\ }$$

$$= -1{,}393 \pm 5{,}741 \quad Pa$$



$= 5{,}57$ MPa

$\sigma_z = 0$

Calculate I and J

$$\frac{-5{,}57}{[0-(-2{,}785)]/2} = -4$$

$$I = \frac{\pi R^4}{4} = 125\,663{,}7$$

$$\theta_p = -37{,}98^\circ$$

$$J = \frac{\pi R^4}{2} = 251\,327{,}4 \quad m$$

$$\left(\frac{0+2{,}785}{2}\right)^2 + (-5{,}57)^2$$

$$\sigma = \frac{17500 \times 20}{125663{,}7} = 2{,}79 \quad M$$

$=$

$\sigma_1 = 4{,}348$ MPa

$\sigma_2 = -7{,}134$ MPa

$$\tau = \frac{T \times R}{J} = \frac{70000 \times 20}{251327{,}4}$$

III) $\sigma_y = -2{,}785$ MPa

$\tau_{zy} = -5{,}570$ MPa

IV.)

$$\tau_{max} = \sqrt{\left(\frac{\sigma_z - \sigma_y}{2}\right)^2 + \tau_{zy}^2} =$$

$$= \sqrt{\left(\frac{0 + 2{,}785}{2}\right)^2 + (5{,}57)^2} = \underline{\underline{5{,}741 \text{ MPa}}}$$

V) strain at B

$$\varepsilon_x = \frac{1}{E}\left[\cancel{\sigma_x}^{=0} - \nu\left(\sigma_y + \cancel{\sigma_z}^{=0}\right)\right] = \frac{1}{200 \times 10^3} \times \left[0 - 0{,}3 \times (-2{,}785)\right]$$

$$= \underline{\underline{4{,}178 \times 10^{-6}}}$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left[\cancel{\sigma_x}^{=0} + \cancel{\sigma_z}^{=0}\right)\right] = \frac{-2{,}785}{200 \times 10^3} = \underline{\underline{-13{,}925 \times 10^{-6}}}$$

$$\varepsilon_z = \frac{1}{E}\left[\cancel{\sigma_z}^{=0} - \nu\left(\cancel{\sigma_x}^{=0} + \sigma_y\right)\right] = \underline{\underline{4{,}178 \times 10^{6}}}$$

$$\gamma_{zy} = \frac{\tau_{zy}}{G} = \frac{-5{,}57}{77 \times 10^3} = \underline{\underline{-72{,}34 \times 10^{-6}}}$$

$$\boxed{G = \frac{E}{2(1+\nu)} = \frac{200}{2(1+0{,}3)} = \underline{77 \text{ GPa}}}$$

VI) If the section was smaller, then the stresses would $\boxed{\underline{\underline{\text{increase}}}}$, because both the moment of Inertia (I) and the polar moment of inertia (J) would <u>decrease</u>

$$\tau = \frac{T \times R}{\frac{\pi R^4}{2}} = \frac{2T}{\pi R^3} \qquad \sigma = \frac{M \times R/2}{\frac{\pi R^4}{4}} = \frac{2M}{\pi R^3}$$