Tentative solution

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## (1) Sampling and Aliasing:

a) In order to properly reconstruct an analog signal from its samples, it is necessary to have at least two samples per smallest period contained in the signal.

If $T_{min} = \frac{1}{f_{max}}$ is the smallest period: $\Delta t \leq \frac{1}{2} \cdot T_{min}$, or

frequency

$$\Delta t \leq \frac{1}{2 f_{max}} \cdot \quad (1)$$

Nyquist frequency

The Nyquist frequency is given by:

$$f_{Nyq} = \frac{1}{2\Delta t} \cdot \quad (2)$$

According to the Shannon/Nyquist sampling interval

$\Delta t$ must be chosen such that $f_{Nyq} \geq f_{max}$.

1b) Signal 1:

There is only one period in this signal. By inspection of the graph, the period of the signal is equal to $T = 0.005\,s$

By the sampling interval:

$$\Delta t \leq \frac{T_{min}}{2} \rightarrow \Delta t \leq 0.0025\,s$$

Signal 2:

Here we have an amplitude spectrum showing a broadband of frequencies. The maximum frequency is around 200 Hz. This corresponds to a minimum period of $T_{min} = \frac{1}{200} = 0.005\,s$

So, $\Delta t \leq \frac{T_{min}}{2} \rightarrow \Delta t \leq 0.0025\,s$

Signal 3:

Similar to signal 2, here we have an amplitude spectrum. This signal is in meters, so we have to change frequency and period for wavenumbers and wavelengths, respectively.

The minimum wavelength in this case is approximately $0.8\overline{33}$ m. So $\Delta x \leq \frac{0.8\overline{33}}{2} = 0.417\,m$

2. Elastic moduli and Reflection of Seismic waves:

a)

| Depth (km) | K (GPa) | $\mu$ (GPa) | $\nu$ |
|---|---|---|---|
| 100 | 129.8 | 66.9 | 0.2800 |
| 500 | 218.6 | 104.9 | 0.2932 |
| 1000 | 352.5 | 186.4 | 0.2754 |
| 2000 | 514.6 | 245.1 | 0.2944 |
| 2890 | 654.8 | 293.8 | 0.3048 |
| 2900 | 644.7 | 0 | 0.50 |
| 4000 | 1023.8 | 0 | 0.50 |
| 5000 | 1285.8 | 0 | 0.50 |
| 5500 | 1382.6 | 165.5 | 0.4424 |
| 6470 | 1424.6 | 176.3 | 0.4406 |

b) Important observations:

- There is a general increase in incompressibility and rigidity with depth until 2890 km. After this depth there is a sharp change in properties, The shear modulus becomes zero indicating a fluid (outer core), at the same time density increases dramatically indicating a likely change in material (constitutive elements).

Deeper than 5500 km the shear modulus becomes non-zero indicating that the inner core is solid.

2c. $R_0 = \dfrac{\rho_2 V_{p2} - \rho_1 V_{p1}}{\rho_2 V_{p2} + \rho_1 V_{p1}}$

$\rho_1 = 5560\ kg/m^3$

$V_{p1} = 13720\ m/s$

$\rho_2 = 9900\ kg/m^3$

$V_{p2} = 8070\ m/s$

$R_0 = 0.023$

2d. Snell's law:

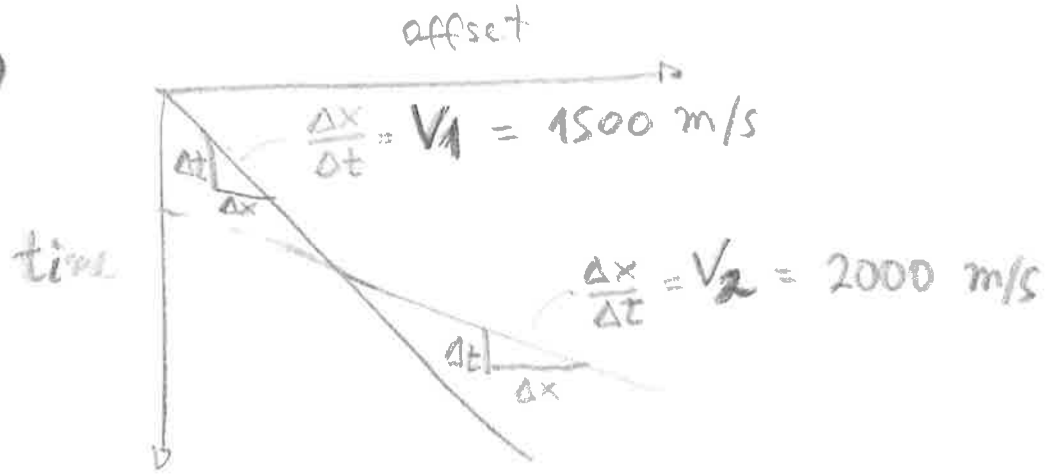
$\dfrac{\sin\theta_1}{V_{p1}} = \dfrac{\sin\theta_2}{V_{p2}}$  $\theta_1 = 30°$

$\theta_2 = \sin^{-1}\left[\sin\theta_1 \cdot \dfrac{V_{p2}}{V_{p1}}\right] = 17.1°$

3. Refraction seismic.

3a). Two slopes can be identified from the first arrivals. This means two layers can be detected.

3b)



3c) To determine the thickness of the topmost layer (5)
we can use the refraction traveltime formula, or
the zero-offset traveltime of the reflection:

Using the refraction traveltime equation:

$$t(x) = \frac{x}{V_2} + \frac{2z_1 \cos\theta_c}{V_1}$$

$$\cos\theta_c = \sqrt{1-\sin^2\theta_c} = \sqrt{1-\left(\frac{V_1}{V_2}\right)^2}$$

$$z_1 = \left(t(x) - \frac{x}{V_2}\right) \cdot \frac{V_1}{2 \cdot \cos\theta_c}$$

Using $x = 2470\,m$ and that $t(2470) = 1.5$

$$z_1 = \left(1.5 - \frac{2470}{2000}\right) \cdot \frac{1500}{2 \cdot \sqrt{1-\left(\frac{1500}{2000}\right)^2}} = \underline{\underline{300\,m}}.$$

4. Seismic velocities:

<u>Lower</u>. Due to gas: $V_p$ decreases

$V_s$ increases

5. Reflection seismic

5a) The RMS velocities and zero-offset traveltimes
can be directly picked in the semblance map.

$V_{RMS_1} = 1500\,m/s$ $\quad T_{0,1} = 0{,}4\,s$

$V_{RMS_2} = 1710\,m/s$ $\quad T_{0,2} = 0.766\,s$

5a cont.)

An alternative way is to pick the traveltimes in the CMP gather and use traveltime equations.

$$t_1^2(x) = T_{0,1}^2 + \frac{x^2}{V_{RMS_1}^2}, \text{ and}$$

$$t_2^2(x) = T_{0,2}^2 + \frac{x^2}{V_{RMS_2}^2}.$$

But this leads to more work.

5b.) Using the Dix formula:

$$V_2^2 = \frac{V_{RMS,2}^2 \cdot T_{0,2} - V_{RMS,1}^2 \cdot T_{0,1}}{T_{0,2} - T_{0,1}} = \frac{1710^2 \cdot 0.766 - 1500^2 \cdot 0.4}{0.766 - 0.4}$$

$\underline{\underline{V_2 = 1913 \text{ m/s}}}$

and,

$\underline{\underline{V_1 = V_{RMS,1} = 1500 \text{ m/s}}}$

$$Z_1 = \frac{V_1 \cdot T_{0,1}}{2} = \underline{\underline{300 \text{ m}}}$$

$$Z_2 = \frac{T_{0,2} - T_{0,1}}{2} \cdot V_2 = \underline{\underline{350 \text{ m}}}$$

$$T_{0,n} = \sum_{i=1}^{n} \frac{2 \cdot Z_i}{V_i}$$

6. Gravity 1

6a) 1. Effect of rotation: The centripetal acceleration consumes some of the gravity and causes a <u>reduction</u> of gravity around the Equator where the centripetal acceleration is highest.

6a cont.)

- Extra radius around the Equator causes a decrease in gravity due to the inverse relation of gravity and distance from the center of the Earth.

- Extra Mass around the Equator due to the extra radius. This effect causes an increase in gravity around the Equator.

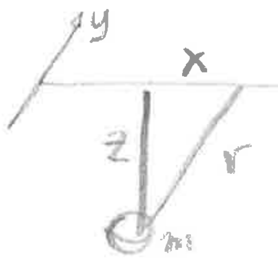
6b. Possible choices are:

- Free air correction (Elevation correction)
- Bougher correction (Extra mass due to topography correction)
- Terrain Correction (Detailed extra mass due to topography)
- Drift correction (correction for the effect of tides and instrument drift)
- Eötvös correction (correction due to moving platform)

7. To determine if the source is a sphere or cylinder we need to check which gravity anomaly fits the observations best.

The formula for a gravity anomaly due to a sphere is

$$\Delta g_z^{(1)}(r) = \frac{G\, m \cdot z}{|r|^3}$$




For a cylinder:

$$\Delta g_z^{(2)}(r) = \frac{2G\, m \cdot z}{|r|^2}$$


Testing if it is a sphere:

$$\Delta g_z^{(1)}\Big|_{x=0} = \frac{G \cdot m \cdot 500}{500^3} = 1 \Rightarrow Gm = 500^2$$

$$\Delta g_z^{(2)}\Big|_{x=1000} = \frac{Gm \cdot 500}{\sqrt{1000^2+500^2}^{\,3}} = \frac{500^3}{\sqrt{1000^2+500^2}^{\,3}} = \underline{0.0894}$$ (It does not fit)

Testing if it is a cylinder:

$$\Delta g_z^{(2)}\Big|_{x=0} = \frac{2Gm \cdot 500}{500^2} = 1 \Rightarrow 2Gm = 500$$

$$\Delta g_z^{(2)}\Big|_{x=1000} = \frac{2Gm \cdot 500}{\sqrt{1000^2+500^2}^{\,2}} = \frac{500^2}{\sqrt{1000^2+500^2}^{\,2}} = \underline{0.2}$$ (it fits the observations)

7. cont) We conclude it is a cylinder.

8. Archie's law and Resistivity logs:

Which factors determine electrical resistivity of porous formations?

Answer: all the points.

Which statements are correct:
Answer:
2. and 3.

Which statements are correct?

Answer:

All correct

9. Gamma ray log:

Answer: 1, 2, 4, 5

10. Log interpretation:

10a) There are signs of cavings and washouts on the top part of the log above 631m, and below 636m. Between 631m and 635m there are signs of mudcake formation.

10b) The key to identify permeable zones is to note that they are associated with mudcake formation.

The zone between 631m and 635m is associated with mudcake and possibly permeable.

10c) This interval is characterized by:

- Low Gamma ray
- Permeable (mudcake formation)
- High relative seismic velocities, but not very high.
- Slight crossover on the NPHI-Rhob log.

This characteristics are indicative of a sandstone. Another more unlikely possibility is a limestone, however the seismic velocities are on the low end and the NPHI-Rhob does not fit the limestone hypothesis. But if the rock is gas saturated, both lithologies are possible.

(11)

10d. Yes. There is a higher resistivity (~ 50 ohm m) interval that correlates with the permeable (Sandstone/limestone) interval.

10e.

632 m (Assuming a sandstone) HC saturated

Using the sonic:

$$\phi = \frac{\frac{1}{V_{ma}} - \frac{1}{V}}{\frac{1}{V_{ma}} - \frac{1}{V_{fl}}} = \frac{\frac{1}{6050} - \frac{1}{3825}}{\frac{1}{6050} - \frac{1}{1055}} = 0.12 = 12\%$$

Using the density:

$$\phi = \frac{\rho_{ma} - \rho}{\rho_{ma} - \rho_{fl}} = \frac{2.65 - 2.3}{2.65 - 0.6} = 0.17 = 17\%$$

638 m (Assuming a shale). water saturated

Using the sonic:

$$\phi = \frac{\frac{1}{5810} - \frac{1}{3300}}{\frac{1}{5810} - \frac{1}{1585}} = 28\%$$

Using the density:

$$\phi = \frac{2.82 - 2.5}{2.82 - 1.146} = 19\%$$

10f. $S_w = \sqrt{\frac{F \cdot R_w}{R_t}}$

$F = \frac{a}{\phi^m} = \frac{0.81}{0.18^{2.8}} =$

$R_w = 0.08\ \Omega m$

$R_t \approx 50\ \Omega m$

$S_w = 0.397 \approx \underline{\underline{40\%}}$

10g. $V_{sh} = \frac{GR - GR_{sand}}{GR_{shale} - GR_{sand}}$

Sand line $\approx$ 37
Shale line $\approx$ 105

at 630 m:

$GR \sim 85$

$V_{sh} = 0.7 = 70\%$

at 632 m
At the sand line,
$V_{sh} \sim \underline{\underline{0\%}}$

[signature]
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