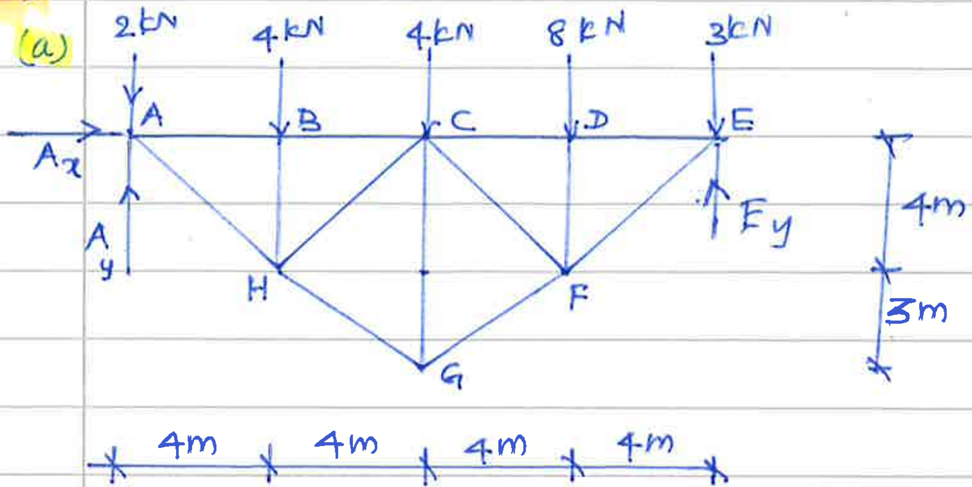
BYG 140: Konstruksjonsmekanikk 1

May 22, 2018

(P1)

(Q1)

(a)



considering equilibrium of the entire truss

$\rightarrow \Sigma F_x \qquad \underline{\underline{A_x = 0}}$

$\uparrow \Sigma F_y \qquad A_y + E_y - (2+4+4+8+3) = 0$

$$A_y + E_y = 21$$

$\circlearrowleft M_A = 16 E_y - (3 \times 16) - (8 \times 12) - (4 \times 8) - (4 \times 4) = 0$

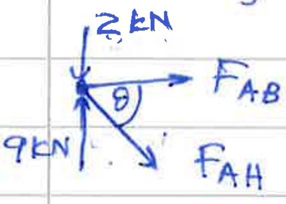
$$E_y = 192/16$$

$$\underline{\underline{E_y = 12 \text{ kN}}}$$

$$A_y = \underline{\underline{9 \text{ kN}}}$$

(b) Axial forces - by method of joints (F_{AB}, F_{AH}, F_{BC} & F_{BH})

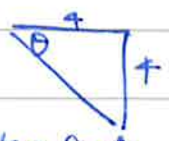
considering equilibrium of joint A



$\uparrow \Sigma F_y = 9 - 2 - F_{AH} \sin\theta = 0$

$$F_{AH} = 7/\sin\theta = \frac{7}{\sin 45} = 7\sqrt{2}$$

$$F_{AH} = \underline{\underline{9.9 \text{ kN (Tension)}}}$$



$\tan\theta = 1$

$\theta = 45^\circ$

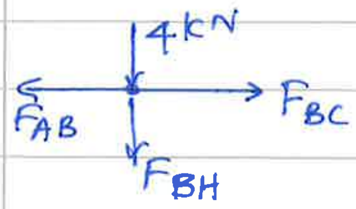
$\rightarrow \Sigma F_y = F_{AB} + F_{AH} \cos\theta =$

$$F_{AB} = -7\sqrt{2} \cdot \cos 45 = -7\sqrt{2} \frac{1}{\sqrt{2}}$$

$$F_{AB} = -7 \text{ kN}$$

$$F_{AB} = \underline{\underline{7 \text{ kN}}} \text{ (compression)}$$

considering joint B



$F_{AB} = F_{BC} = 7\ kN$ (compression)

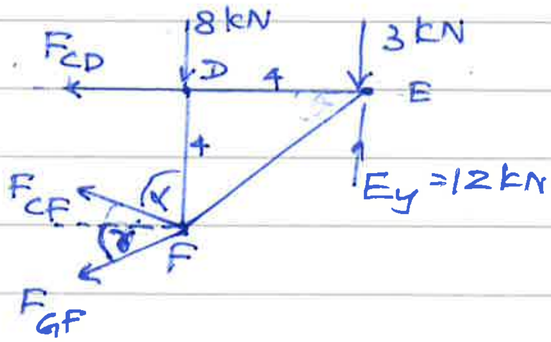
$F_{BH} + 4 = 0$

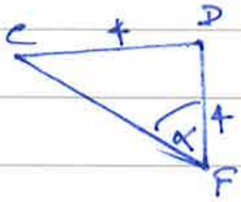
$F_{BH} = -4kN$

$F_{BH} = 4kN$ (compression)

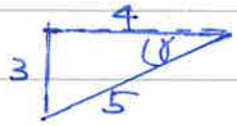
(c) No zero-force members.

(d) Axial forces - By method of sections. (F_{CD}, F_{CF}, F_{GF})





$\alpha = 45^\circ$



$\sin\gamma = 3/5$

$\cos\gamma = 4/5$

$\gamma = 36.87$

Considering equilibrium of the segment

$\curvearrowleft M_F \quad (F_{CD} \times 4) + (12 \times 4) - (3 \times 4) = 0$

$F_{CD} = -36/4 = -9\ kN$

$F_{CD} = 9\ kN$ (compression)

$\uparrow \Sigma F_y = F_{CF}\cos\alpha - 8 - 3 + 12 - F_{GF}\sin\gamma = 0$

$F_{CF} \cdot \frac{1}{\sqrt{2}} - F_{GF} \cdot \frac{3}{5} = -1$

$5F_{CF} - 3\sqrt{2}\,F_{GF} = -5\sqrt{2}$ —— ①

$\leftarrow \Sigma F_y = F_{CD} + F_{CF}\sin\alpha + F_{GF}\cos\gamma = 0$

$F_{CF}\ \frac{1}{\sqrt{2}} + F_{GF}\ 4/5 = 9$

$5F_{CF} + 4\sqrt{2}\ F_{GF} = 45\sqrt{2}$ —— ②

② - ① $\quad 7\sqrt{2}\,F_{GF} = 50\sqrt{2}$

$F_{GF} = 50/7 = 7.14\ kN$ (Tension)

① ⇒ $5F_{CF} = -5\sqrt{2} + 3\sqrt{2} \cdot \frac{50}{7}$

$= \sqrt{2}(115/7)$

$F_{CF} = 4.65\ kN$ (Tension)

(e) Required cross-sectional areas (for member AB & BH)

$F_{AB} = -7\,kN$

$F_{BH} = -4\,kN$

For member AB

$$\sigma_{all} \geq \frac{F_{AB}}{area} = \frac{7\times10^3}{area}$$

$$155\times10^6 \geq \frac{7\times10^3}{area}$$

$$area \geq \frac{7\times10^3\,N}{155\times10^6\,N/m^2} = 4.516\times10^{-5}\,m^2 = 45.16\,mm^2$$

area of the member AB $= 46\,mm^2 \sim 50\,mm^2$

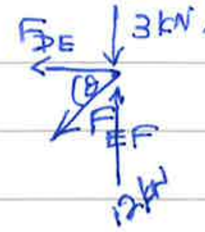
For membe BH

$$\sigma_{all} \geq \frac{F_{BH}}{area}$$

$$area \geq \frac{4\times10^3}{155\times10^6} = 2.58\times10^{-5}\,m^2 = 25.81\,mm^2$$

area of the member BH $= 26 \sim 30\,mm^2$

If area AB is used for DE



$12 - EF\sin\theta - 3 = 0$

$EF = 9/\sin\theta = 9\sqrt{2}$

$F_{DE} + F_{EF}\cos\theta = 0$

$F_{DE} = 9\,kN$

$$\frac{F_{DF}}{area} = \frac{9\times10^3}{46\times10^{-6}} = 195\,MPa > \sigma_{all}$$

member fails.

If area BH is used for DF

$$\frac{F_{DF}}{Area} = \frac{8\times10^3\,N}{26\times10^{-6}\,m^2} = 307.69\,MPa > \sigma_{all}$$

member fails.

(f) change in length in member BH (Δ)

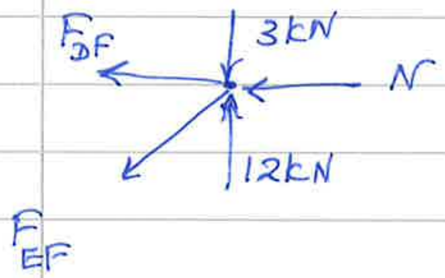
$$\frac{F}{A} = E\frac{\Delta}{L} \Rightarrow \Delta = \frac{FL}{EA}$$

$$\Delta = \frac{-4\times10^{3}N \times 4m}{26\times10^{-6}m^2 \times 200\times10^{9}N/m^2}$$

$= -3.057\ mm$ (member contracted).

No, the change of length is not equal to the vertical displacement of point H. as B is also displaced. Above calculated Δ value is relative displacement between B & H towards to the direction of BH.

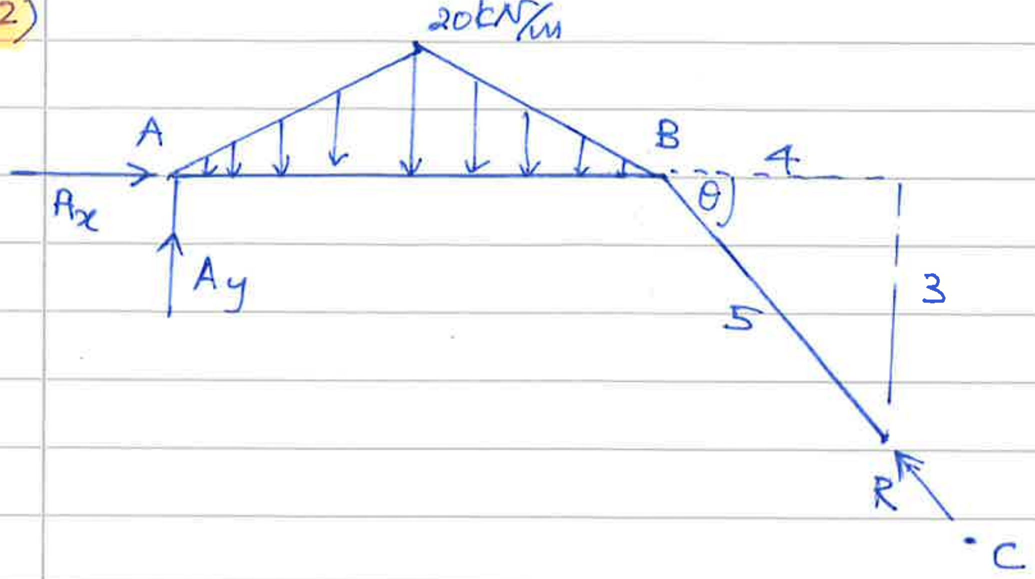
(g) Due to horizontal force at point E,



F_{EF} will not subjected to changes as there is no effect from new axial force N to the vertical upward equilibrium equation $(+\uparrow \Sigma F_y = 0)$

F_{DE} will increase (compression) as new axial force N affects to horizontal equilibrium equation $(\rightarrow \Sigma F_x = 0)$

(Q2)

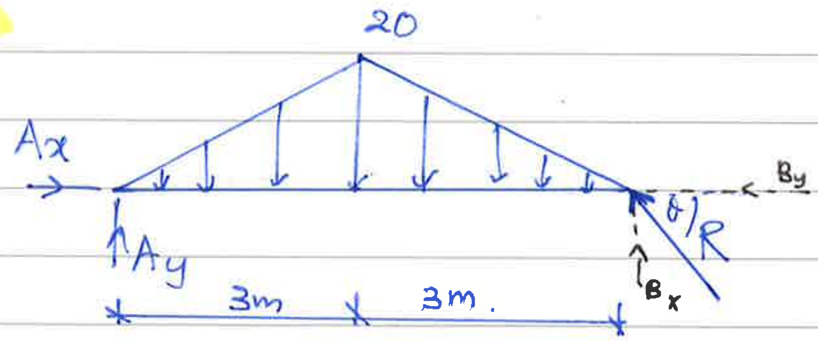


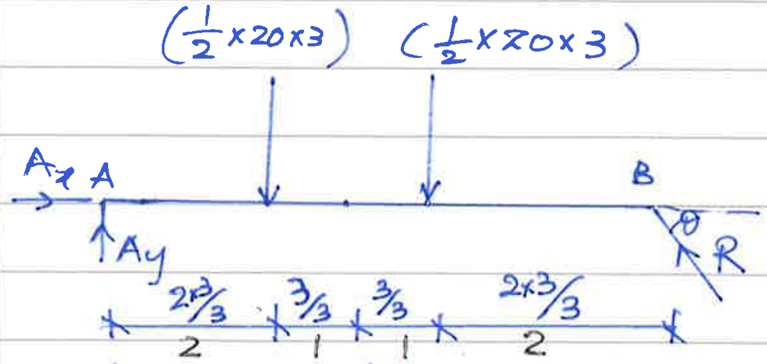
$\sin\theta = 3/5$

$\cos\theta = 4/5$

(a) BC is a two force member

(b)





considering equilibrium,

$\rightarrow \; A_x - R\cos\theta = 0$

$\uparrow \; A_y + R\sin\theta - 30 - 30 = 0.$

$\circlearrowright M_A \; (R\sin\theta \times 6) - (30\times4) - (30\times2) = 0$

$$R = \frac{(30\times6)}{6\times 3/5} = 50\,kN$$

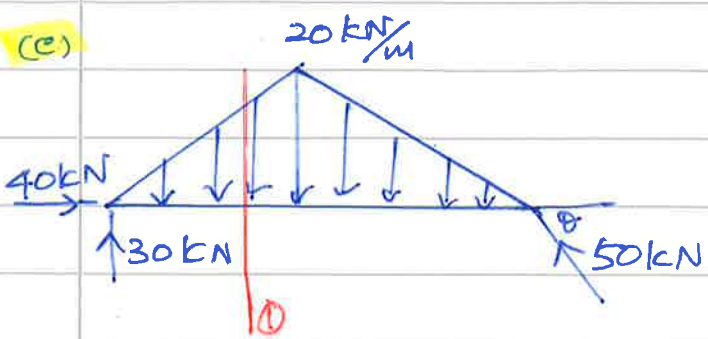
$R = 50kN$

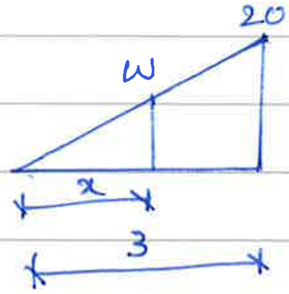
$B_x = 40kN$

$B_y = 30kN$

$A_x = 50\times\frac{4}{5} = 40kN$ $\quad A_y = 60 - (50\times 3/5) = 30kN$

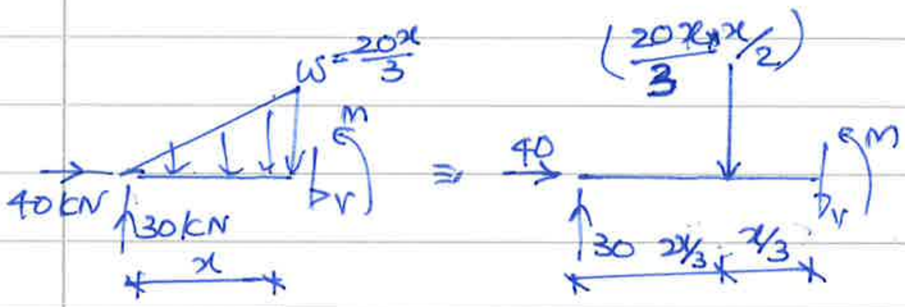
(c)





$$\frac{w}{x} = \frac{20}{3}$$

$$w = \frac{20x}{3}.$$



considering equilibrium.

$$\uparrow \quad 30 - \left(\frac{10x^2}{3}\right) - V = 0$$

$$V = -\frac{10x^2}{3} + 30$$

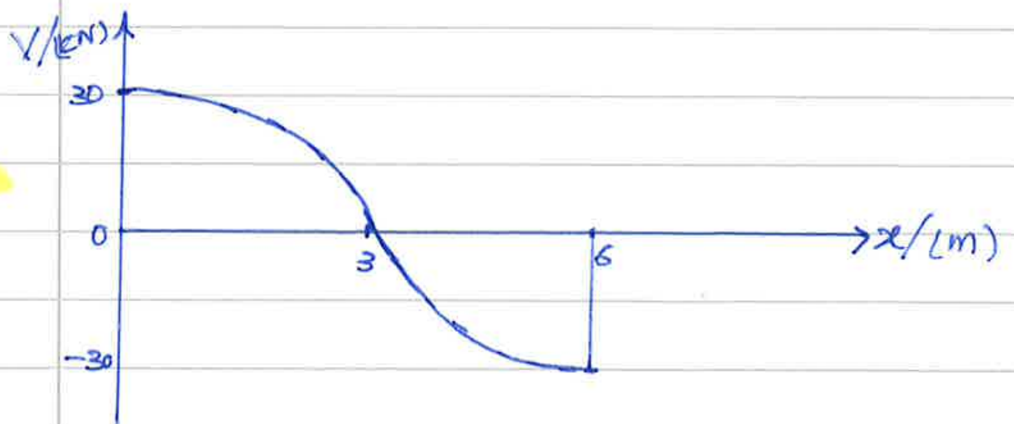
x	V
0	30
3	0

$$\circlearrowleft \quad M + \left(\frac{10x^2}{3} \cdot \frac{x}{3}\right) - 30x = 0$$

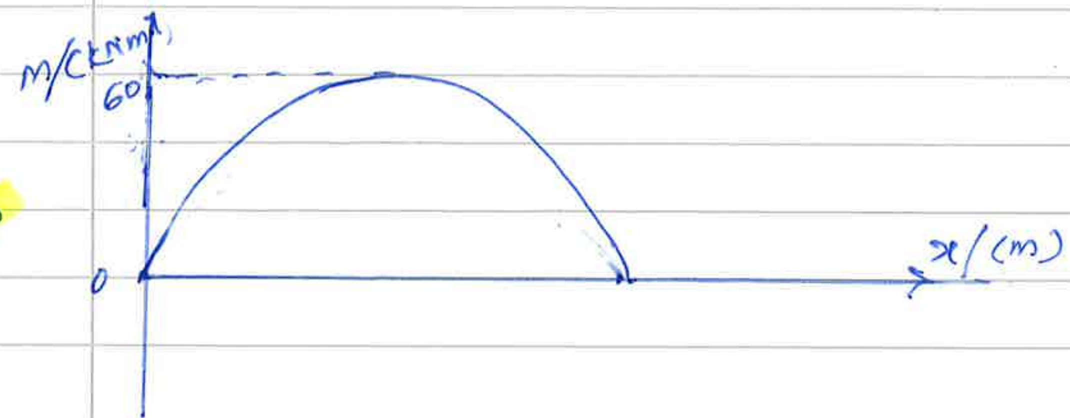
$$M = 30x - \frac{10x^3}{9}$$

x	m
0	0
3	30

SFD



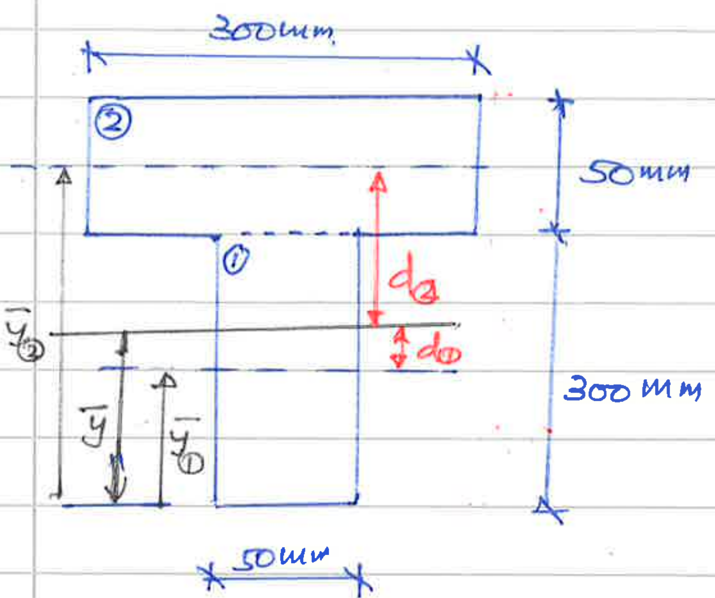
BMD



(d) cross sectional area.

$(300 \times 50) + (300 \times 50) = 30000\,mm^2$

moment of inertia (2^{nd} moment of area)



$$\bar{y} = \frac{A_1 \bar{y}_1 + A_2 \bar{y}_2}{A_1 + A_2}$$

$A_1 = 300 \times 50$

$A_2 = 300 \times 50$

$\bar{y}_1 = 300/2$

$\bar{y}_2 = (300 + 50/2)$

$$\bar{y} = \frac{(300 \times 50) \cdot \frac{300}{2} + (300 \times 50)(300 + \frac{50}{2})}{(300 \times 50) + (300 \times 50)} = \frac{150 + 325}{2} = 237.5\,m$$

$$I = \left[\left(\frac{1}{12} \cdot 50 \times 300^3\right) + (300 \times 50) \cdot (237.5 - 150)^2\right] + \left[\left(\frac{1}{12}\, 300 \times 50^3\right) + (300 \times 50)(325 - 237.5)^2\right]$$

$= 3.453 \times 10^8\,mm^4 \quad (= 3.453 \times 10^{-4}\,m^4)$

(e) maximum normal stress (σ)

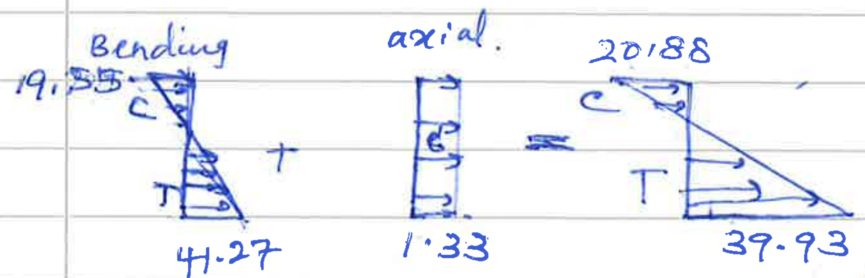
Normal stress due to axial force.

$$\sigma_N = \frac{P}{A} = \frac{-40\times10^{3}\,N}{(30000\times10^{-6}\,m^2)} = -1.33\ \text{Mpa.}$$

Stress due to bending (at $x = 3\,m$)

$$\sigma_m = \frac{-My_{max}}{I} = \frac{-60\times10^{3}\,Nm\,(-237.5\times10^{-3})\,m}{3.453\times10^{-4}\,m^4} = +41.27\ \text{Mpa.}$$

$$\sigma_m = \frac{-60\times10^{3}\,Nm\,(350-237.5)10^{-3}}{3.453\times10^{-4}} = -19.55\ \text{mpa.}$$



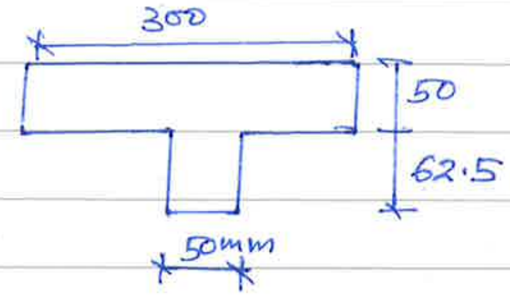
$$\sigma = (41.27 - 1.33) = \underline{39.93\ \text{mpa}}$$

maximum shear stress (τ_{max}) (at $x = 0$ or $x = 6m$)

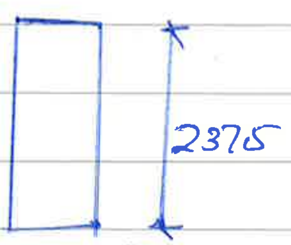
$$\tau_{max} = \frac{V_{max}Q_{max}}{It}$$

$I = 3.453\times10^{-4}\,m^4$

$t = 50mm\ (50\times10^{-3}\,m)$



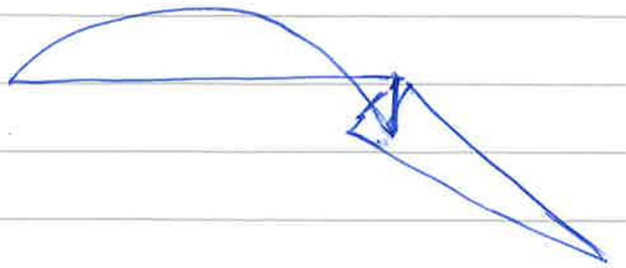
$$Q_{max} = (50\times62.5)\cdot\frac{62.5}{2} + (300\times50)\left(62.5+\frac{50}{2}\right)$$
$$= 1.410156\times10^{6}\,mm^3$$



$$Q_{max} = (50\times237.5)\cdot\frac{237.5}{2}$$
$$= 1.410156\times10^{6}\,mm^3.$$

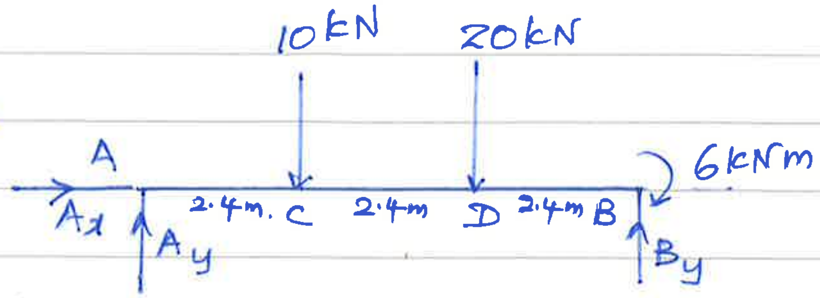
$$\tau_{max} = \frac{30\times10^{3}\,N \times 1.410156\times10^{6}\times10^{-9}}{3.453\times10^{-4}\times50\times10^{-3}} = \underline{2.4502\,\text{Mpa.}}$$

(f) moment diagram changes
Beading moment reduces
Normal stress may reduce



(Q3)

(a)



considering equilibrium

$\rightarrow \Sigma F_x = 0 \qquad \underline{\underline{A_x = 0}}$

$\uparrow \Sigma F_y = 0 \qquad A_y + B_y = 30$

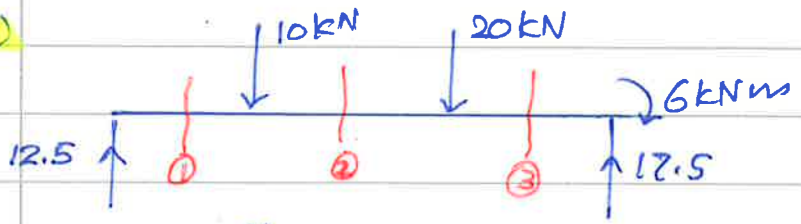
$\circlearrowleft M_A \quad (B_y \times 7.2) - (20 \times 4.8) - (10 \times 2.4) - 6 = 0$

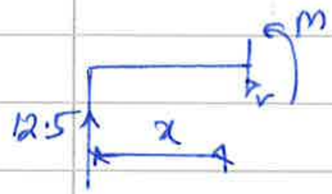
$B_y = (24 + 96 + 6)/7.2$

$B_y = \underline{\underline{17.5\ kN}}$

$A_y = \underline{\underline{12.5\ kN}}$

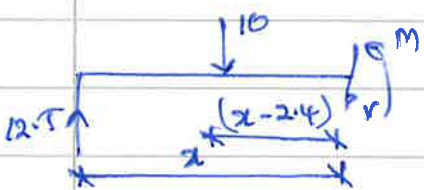
(b)





$\circlearrowleft m - 12.5x = 0 \qquad m = 12.5x$

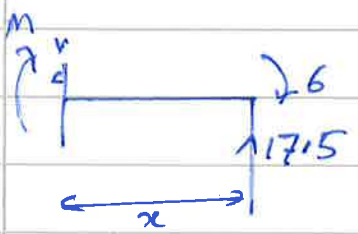
x	m
0	0
2.4	30



$\circlearrowleft m + 10(x - 2.4) - 12.5x = 0$

$m = 2.5x + 24$

x	m
2.4	30
4.8	36



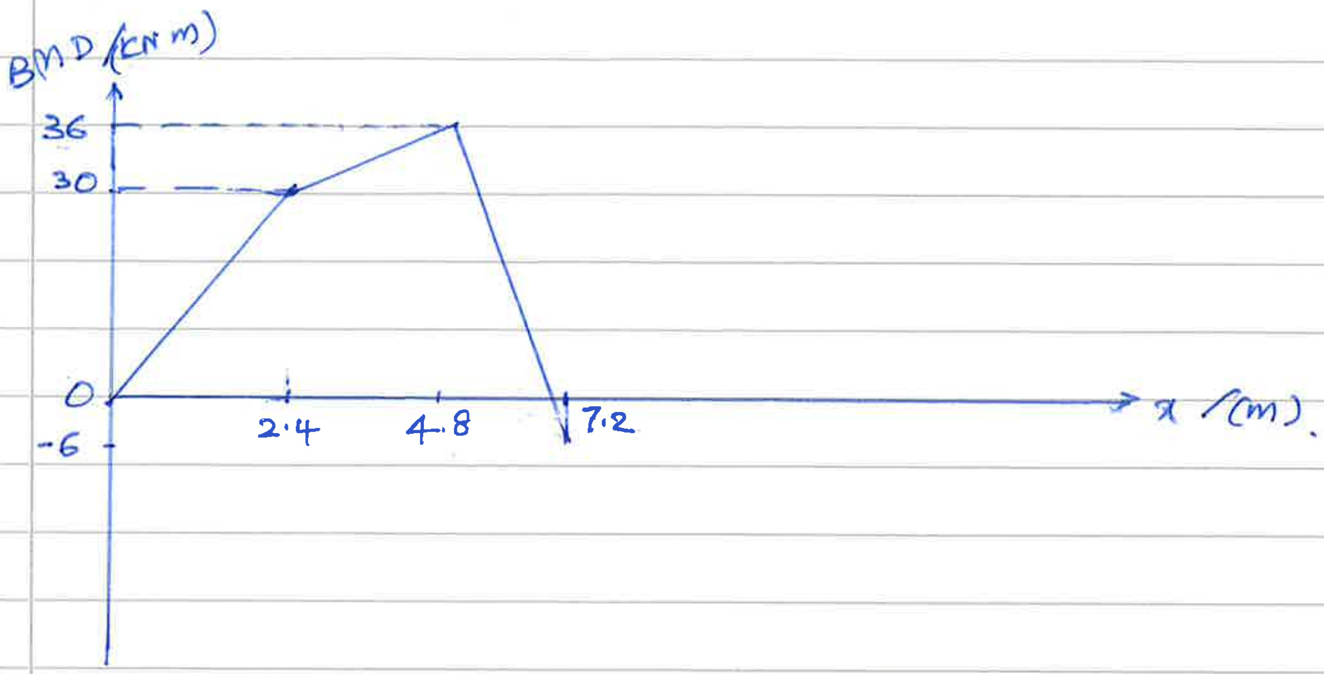
$M + 6 - (17.5x) = 0$

$M = 17.5x - 6$

x	M
0	-6
2.4	36.

when $M = 0 \qquad 17.5x - 6 = 0$

$x = 6/17.5 = 0.343m.$

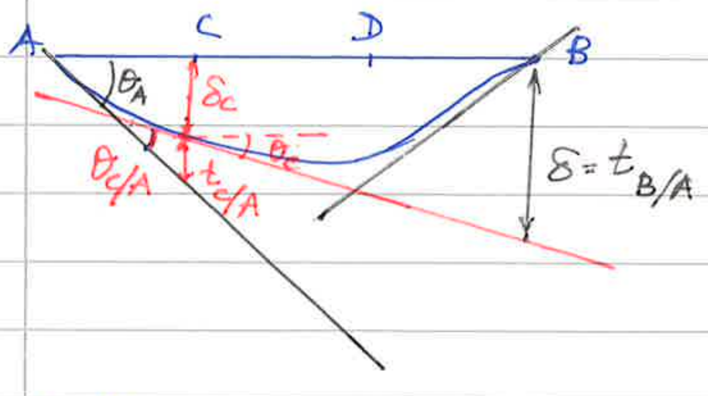


(c) moment of inertia

$$I_{xx} = \left(\frac{1}{12} \times 150 \times 300^3\right) - \left(2 \times \frac{1}{12} \frac{(150-20)}{2} (300-40)^3\right)$$

$$= 1.47 \times 10^8 \text{ mm}^4 \quad (\text{or } 1.47 \times 10^{-5} \text{ m}^4).$$

(d) Deflection and slope at c – By moment area method.



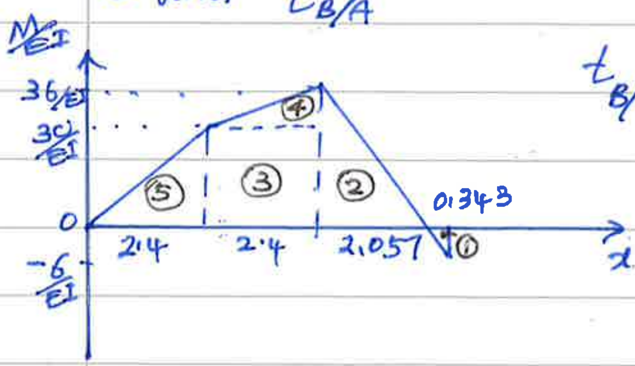
$$\theta_A = \theta_c + \theta_{c/A}$$

$$\theta_c = \theta_A - \theta_{c/A}$$

$$\theta_A = \frac{\delta}{L} = \frac{\delta}{7.2}$$

$$\delta = t_{B/A}$$

To find $t_{B/A}$



$$t_{B/A} = \left[\left(\frac{1}{2} \times \frac{-6}{EI} \times 0.3429\right)\left(\frac{0.3429}{3}\right)\right] +$$

$$\left[\left(\frac{1}{2} \cdot \frac{36}{EI} \times 2.057\right)\left(\frac{2.057}{3} \times 2 + 2.4\right)\right] +$$

$$\left[\left(\frac{30}{EI} \times 2.4\right)\left(2.4 + \frac{2.4}{2}\right)\right] +$$

$$\left[\left(\frac{1}{2} \cdot \frac{6}{EI} 2.4\right)\left(2.4 + \frac{2.4}{3}\right)\right] +$$

$$\left[\left(\frac{1}{2} \cdot \frac{30}{EI} \times 2.4\right)\left(2.4 + 2.4\right) + \frac{2.4}{3}\right)\right]$$

$$t_{B/A} = \frac{547.202}{EI} \ kNm^3$$

$$\theta_A = \frac{547.202}{EI \quad 7.2} = \frac{76.000}{EI}$$

$$\theta_C = \theta_A - \theta_{C/A}$$

$$= \frac{76.0}{EI} - \frac{36}{EI} = \frac{39.9995}{EI} \ kNm^2 = \frac{39.9995 \times 10^3 \ Nm^2}{200 \times 10^9 \times 1.47 \times 10^8 \times 10^{-12}}$$

$$\theta_C = 0.001361$$

$$\frac{t_{B/A}}{7.2} = \frac{\delta_C + t_{C/A}}{2.4}$$

$$t_{C/A} = \left(\frac{1}{2} \times 2.4 \times \frac{30}{EI}\right) \frac{2.4}{3} = \frac{28.8}{EI}$$

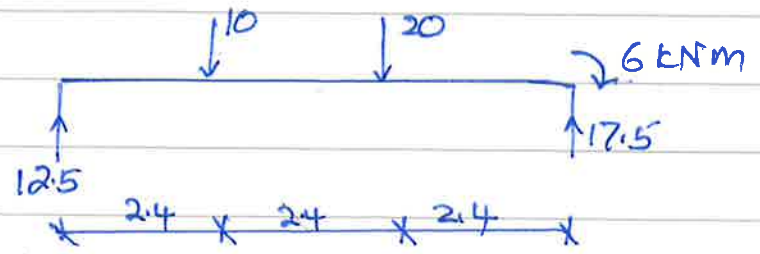
$$2.4 \times \frac{547.202}{EI \times 7.2} = \delta_C + t_{C/A}.$$

$$\delta_C = \frac{547.202}{3EI} - \frac{28.8}{EI} = \frac{153.6007}{EI} \ kNm^3$$

$$= \frac{153.6007 \times 10^3 \ Nm^3}{\underset{N/m^2}{200 \times 10^9} \times \underset{m^4}{1.47 \times 10^8 \times 10^{-12}}} = 0.005225 \ m$$

$$\delta_C = 5.225 \ mm.$$

deflection and slope by discontinuity fuction.



$$M = 12.5\langle x-0\rangle^1 - 10\langle x-2.4\rangle^1 - 20\langle x-4.8\rangle^1$$

as $x \geq 0$

$$M = 12.5x - 10\langle x-2.4\rangle^1 - 20\langle x-4.8\rangle^1$$

$$EI\frac{d^2v}{dx^2} = M = 12.5x - 10\langle x-2.4\rangle^1 - 20\langle x-4.8\rangle^1$$

By integrating

$$EI\frac{dv}{dx} = \frac{12.5x^2}{2} - \frac{10}{2}\langle x-2.4\rangle^2 - \frac{20}{2}\langle x-4.8\rangle^2 + C_1 \quad -(1)$$

$$EI\,v = \frac{12.5x^3}{2\times3} - \frac{10}{2\times3}\langle x-2.4\rangle^3 - \frac{20}{2\times3}\langle x-4.8\rangle^3 + C_1x + C_2 \quad (2)$$

using boudary conditions

when $x=0$, $v=0$; from equation (2) $\Rightarrow C_2 = 0$

when $x = 7.2$, $v=0$

$$0 = \left(\frac{12.5\times7.2^3}{6}\right) - \frac{10}{6}(7.2-2.4)^3 - \frac{20}{6}(7.2-4.8)^3 + C_1(7.2)$$

$$= 547.2 - 7.2C_1$$

$$C_1 = -76 \ m^2$$

Then $$v = \frac{1}{EI}\left(\frac{12.5}{6}x^3 - \frac{10}{6}\langle x-2.4\rangle^3 - \frac{20}{6}\langle x-4.8\rangle^3 - 76x\right)$$

when $x = 2.4$

deflection. $$v = \frac{1}{EI}\left[\left(\frac{12.5\times2.4^3}{6}\right) - \frac{10}{6}(0) - 76(2.4)\right]$$

$$= \frac{-153.6\ kNm^3}{EI} = \frac{-153.6\times10^3\ Nm^3}{200\times10^9\ N/m^2 \times 1.47\times10^8\times10^{-12}\ m^4}$$

$$\underline{\underline{v = 5.22\,mm}}$$

Slope

$$EI\frac{dv}{dx} = \frac{12.5x^2}{2} - \frac{10}{2}\langle x-2.4\rangle^2 - \frac{60}{6}\langle x-4.8\rangle^2 - 76$$

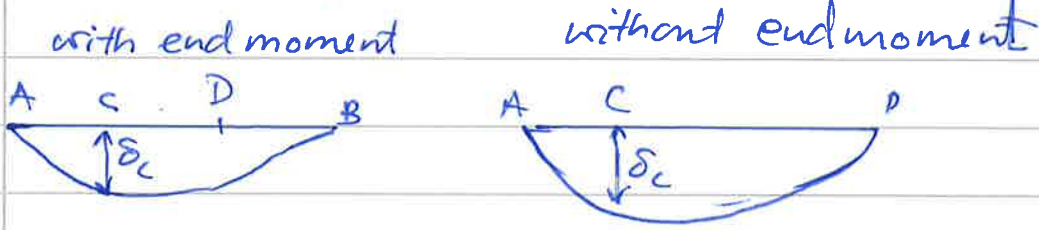
$$\frac{dv}{dx} = \frac{1}{EI}\left[\left(\frac{12.5\times 2.4^2}{2}\right)-76\right] = -\frac{40}{EI}$$

$$= \frac{-40\times 10^3\ \mathrm{Nm}^2}{\left(200\times 10^9\ \mathrm{N/m^2} \times 1.47\times 10^8 \times 10^{-12}\ \mathrm{m^4}\right)} = 0.00136$$

$$\frac{dv}{dx} = \underline{\underline{0.00136}}$$

(c) if the moment of 6 kNm at end B is removed. deflection at point C increases

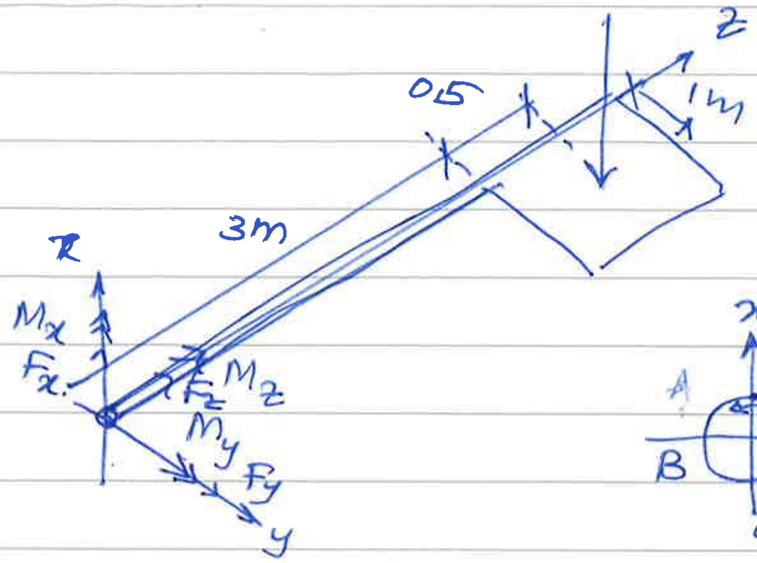
deflected shape

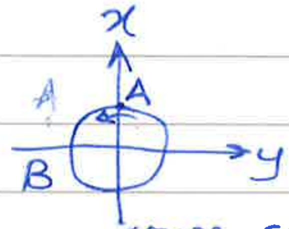


(Q4)

kPa
$(1.5 \times 2 \times 1) = 3\,kN.$

(a)





cross sectional area.

considering equilibrium.

$F_z = 0$

$M_z + (3 \times 1) = 0$

$M_z = -3\ kNm$ (torsion $T = 3\,kNm$)

$F_y = 0$

$M_y - (3 \times 3.5) = 0$

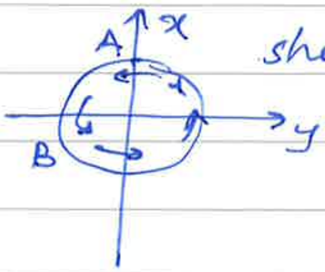
$M_y = 10.5\ kNm$

$F_x - 3 = 0$

$F_x = 3\ kN$

$M_x = 0.$

(b) At A



shear stress due to torsion

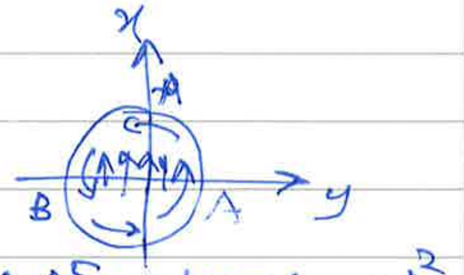
$$\tau_A = \frac{TC}{J} = \frac{(3 \times 10^3)(0.05)}{\pi/2\,(0.05)^4}$$

$$= 15.27 \times 10^{+6} = 15.3\ mpa$$

Normal stress

$$\sigma_A = \frac{MC}{I} = \frac{(10.5 \times 10^3) \times 0.05}{\pi/4\,(0.05)^4} = 107\ Mpa.$$

At B



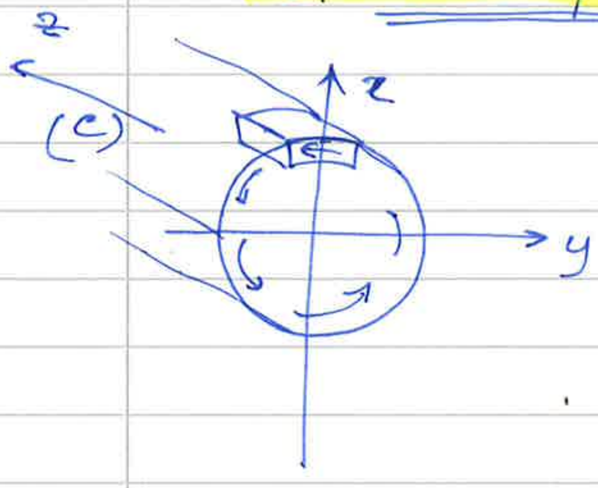
$$\sigma_B = 0$$

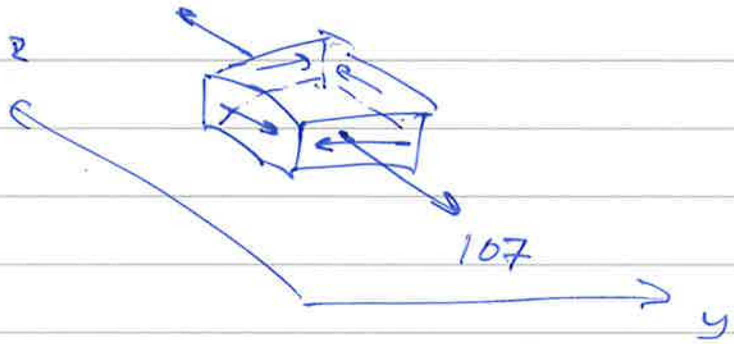
$$\tau_B = \frac{TC}{J} - \frac{VQ}{It}$$

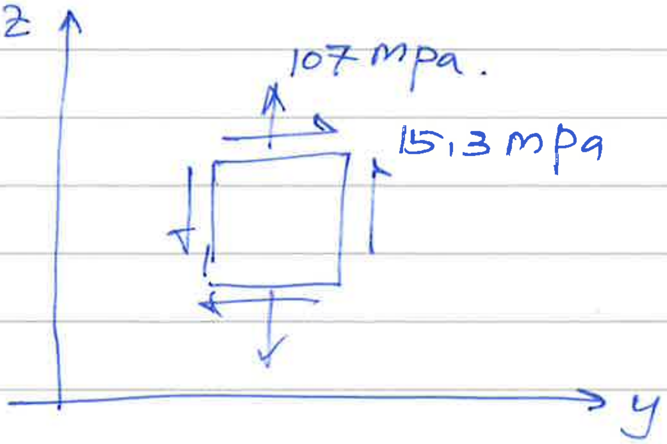
$$= \frac{(3\times10^3)\times0.105}{\frac{\pi}{2}\times(0.105)^4} - \frac{(3\times10^3)\left(\frac{4}{3}\times\frac{0.105}{\pi}\right)\frac{1}{2}\pi(0.105)^2}{\frac{\pi}{4}(0.105)^4\ (0.1)}$$

$$\underline{\underline{\tau_B = 14.8\ \text{Mpa}}}$$

(c)







(d) Principal stress and its orientation

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$

$\sigma_x = 0$

$\sigma_y = 107$ Mpa

$\tau_{xy} = 15.3$ Mpa

$$2\theta_p = \tan^{-1}\left[\underbrace{\frac{15.3}{(0-107)/2}}_{-0.28598}\right] = (-0.27855)$$

$\theta_p = -0.13927$ rad

$\theta_p = -7.9798^\circ$

$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \sigma_{xy}^2}$$

$$= \left(\frac{0+107}{2}\right) \pm \sqrt{\left(\frac{0-107}{2}\right)^2 + 15.3^2}$$

$$= 53.5 \pm \sqrt{3096.34}$$

$\sigma_1 = 109.145$ mpa

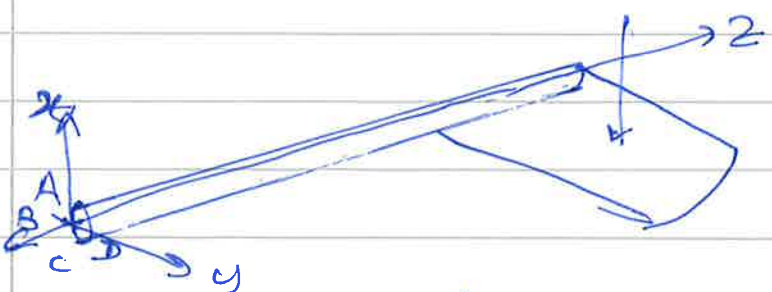
$\sigma_2 = -2.145$ mpa

(e) absolute maximum shear stress.

$$\tau_{\substack{abs\\max}} = \frac{\sigma_{max}-\sigma_{min}}{2} = \frac{109.145-(-2.145)}{2}$$

$= 55.645$ Mpa.

(f) when supporting post has considerable weight. The weight makes a compressive stress at the give cross section.
Then calculated normal stress due to bending will change.



Normal stress at A decrease (axial & bending)
That will affect the principle stresses.