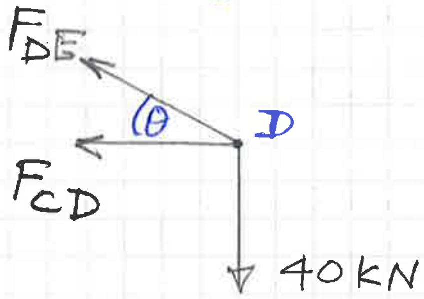
BYG 140: Konstruksjonmekanikk 1 ①
May 18, 2016.

(Q1) (i)



$\theta = 45^\circ$

using method of joints, considering equilibrium of joint D,

$$\uparrow \Sigma F_y = F_{DE} \sin 45^\circ - 40 = 0$$

$$F_{DE} = \frac{40}{\sin 45^\circ}$$

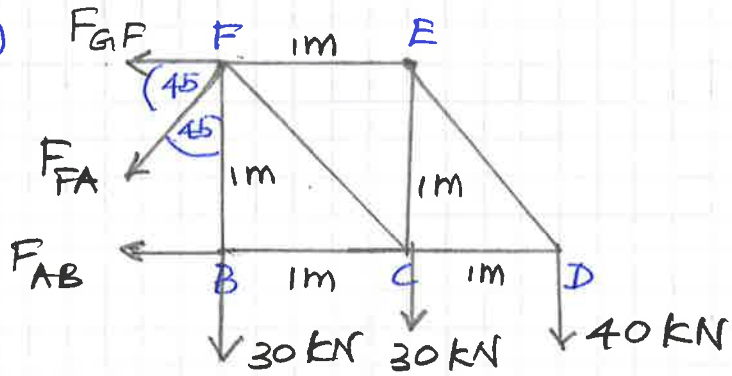
$$\underline{F_{DE} = 56.568 \text{ kN}} \text{ (Tension)}$$

$$\rightarrow \Sigma F_x = -F_{CD} - F_{DE} \cos 45^\circ = 0$$

$$F_{CD} = -F_{DE} \cos 45^\circ$$

$$\underline{F_{CD} = 40 \text{ kN}} \text{ (compression)}$$

(ii)



axial forces of members using method of section.
Considering equilibrium of the segment.

$$\downarrow \Sigma F_y \quad 30 + 30 + 40 + F_{FA} \cos 45^\circ = 0$$

$$F_{FA} = \frac{-100}{\cos 45^\circ}$$

$$\underline{F_{FA} = 141.42 \text{ kN}}$$

(compression)

taking moment about F

$$\circlearrowleft\ (F_{AB}\times 1) + (30\times 1) + (40\times 2) = 0$$

$$F_{AB} = -110$$

$$F_{AB} = \underline{\underline{110\ kN}}\ \text{(compression)}$$

$$\leftarrow \Sigma F_x = F_{GF} + F_{FA}\sin 45^\circ + F_{AB} = 0$$

$$F_{GF} = -F_{AB} - F_{FA}\sin 45^\circ$$

$$F_{GF} = \underline{\underline{210\ kN}}\ \text{(Tension)}$$

(iii) Required cross sectional area.

$F_{CD} = 40\,kN$ $\quad F_{DE} = 56.568\,kN$ (calculated (i))

For member CD

$$\sigma_{all} \geq \frac{F_{CD}}{area} = \frac{40\times 10^3}{area}.$$

$$area \geq \frac{40\times 10^3\ N}{200\ N/mm^2} = 200\,mm^2.$$

$$\text{area of } CD = \underline{\underline{200\ mm^2}}$$

For member DE

$$area \geq \frac{56.568\times 10^3\,N}{200\ N/mm^2} = 282.84\,mm^2.$$

$$\text{area of } DE = \underline{\underline{283\ mm^2}}.$$

(iv) Change in length CD. (Δ)

$$\frac{F}{A} = E\,\frac{\Delta}{L}$$

$$\frac{40\times 10^3\,N}{200\,mm^2} = 200\times 10^3\,N/mm^2 \times \frac{\Delta}{1\times 10^3\,mm}$$

$$\Delta = \underline{\underline{1\,mm}}\ \text{(contracted)}$$

(v) $\frac{F}{A} = E\frac{\Delta}{L}$

- geometry → L → same.
- loading & supports → same.

$\Delta = \frac{LF}{AE}$

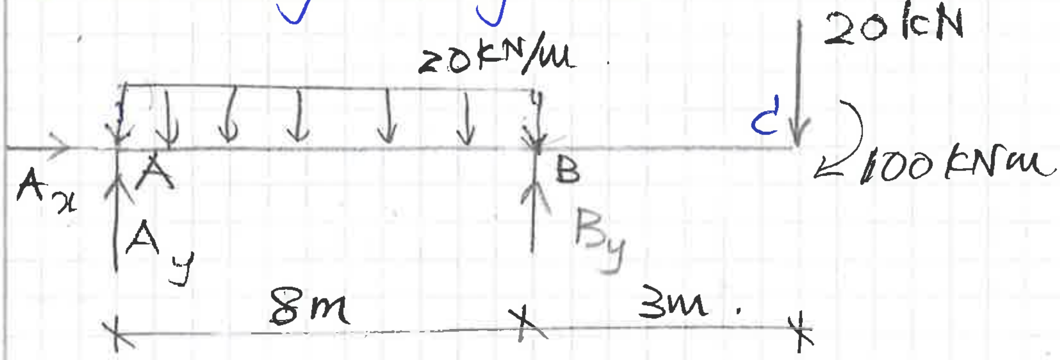
F remains same
(as statically determinat structure)

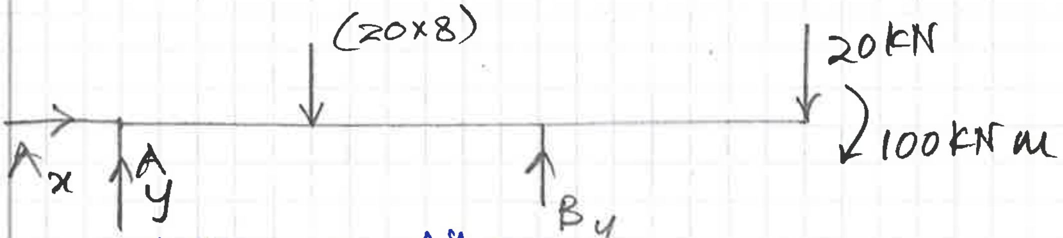
when L, & F remain same. and
E is constant,
when A is increased, Δ decreases,
when Δ decreases, downward displacement
decreases.

(vi) $F_{CD} = F_{DE} = F_{EF} = 0$
all become zero force members. when
applied load at D is zero. (ie: CD, DE, CE, & EF
become zero force members)

Q2)

(i) Free body diagram.





considering equilibrium;

taking moment about A,

$$8B_y - (20\times8\times4) - (20\times11) - 100 = 0$$

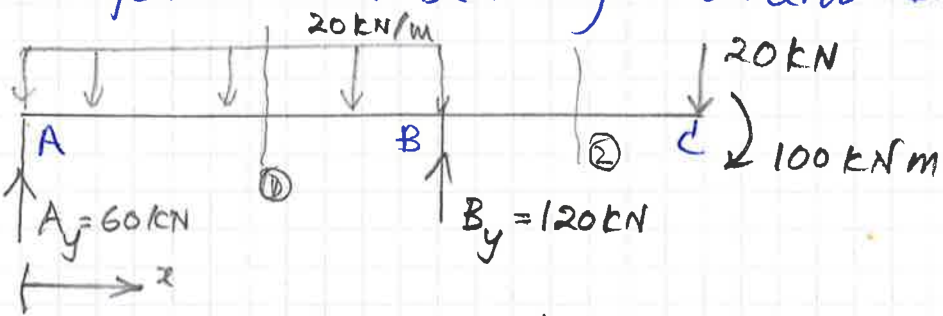
$$B_y = \frac{960}{8} = \underline{120\,kN}$$

$$\uparrow \sum F_y = A_y + B_y - 20 - 160 = 0$$

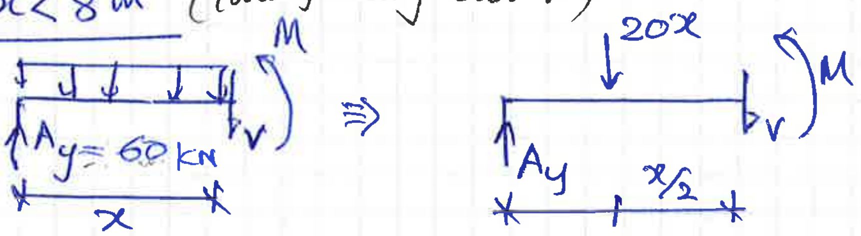
$$A_y = \underline{60\,kN}$$

$$\rightarrow \sum F_x = \qquad A_x = \underline{0}$$

(ii) Shear force and bending moment diagram.



$0 < x < 8\,m$ (imaginary cut ①)



considering equilibrium of the segment.

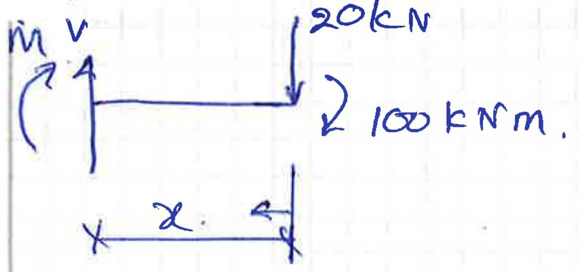
$\downarrow \quad V + 20x - 60 = 0$

$V = 60 - 20x.$

$\curvearrowleft \quad M + \left(20x \cdot \frac{x}{2}\right) - 60x = 0$

$M = 60x - 10x^2$

imaginary cut 2



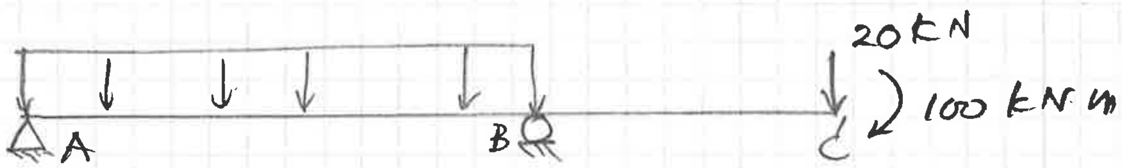
considering equilibrium of the segment.

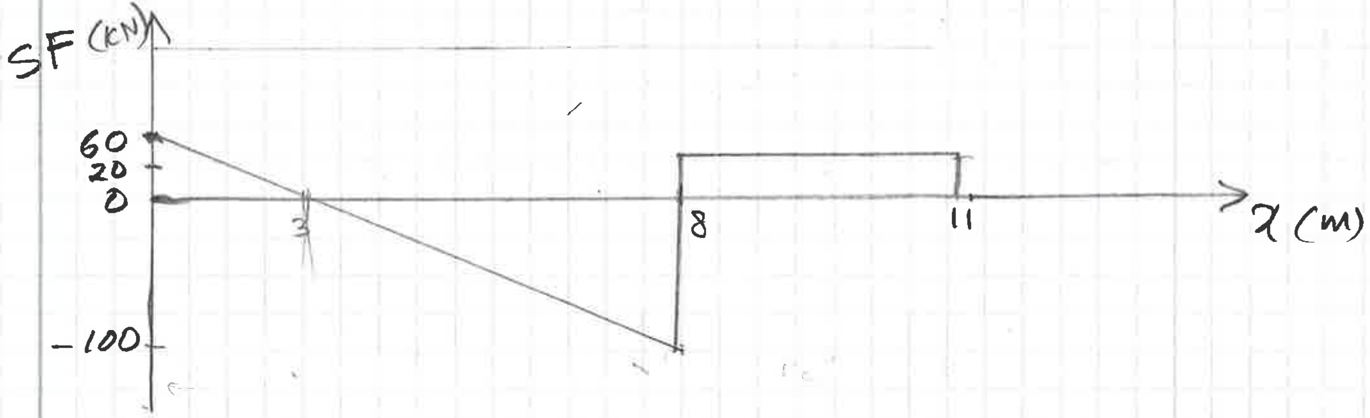
$\uparrow \quad V - 20 = 0$

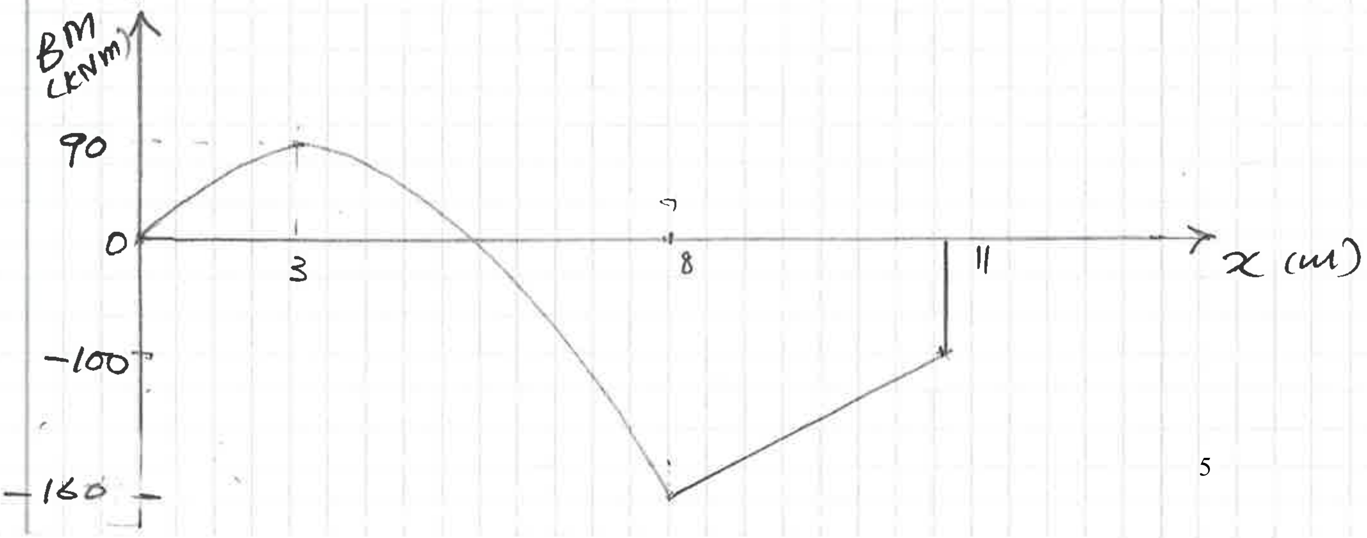
$V = 20$

$\curvearrowright \quad M + 100 + 20x = 0$

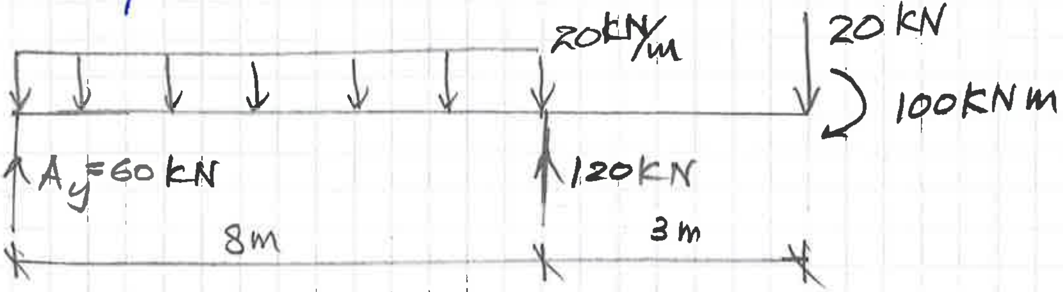
$M = -20x - 100$



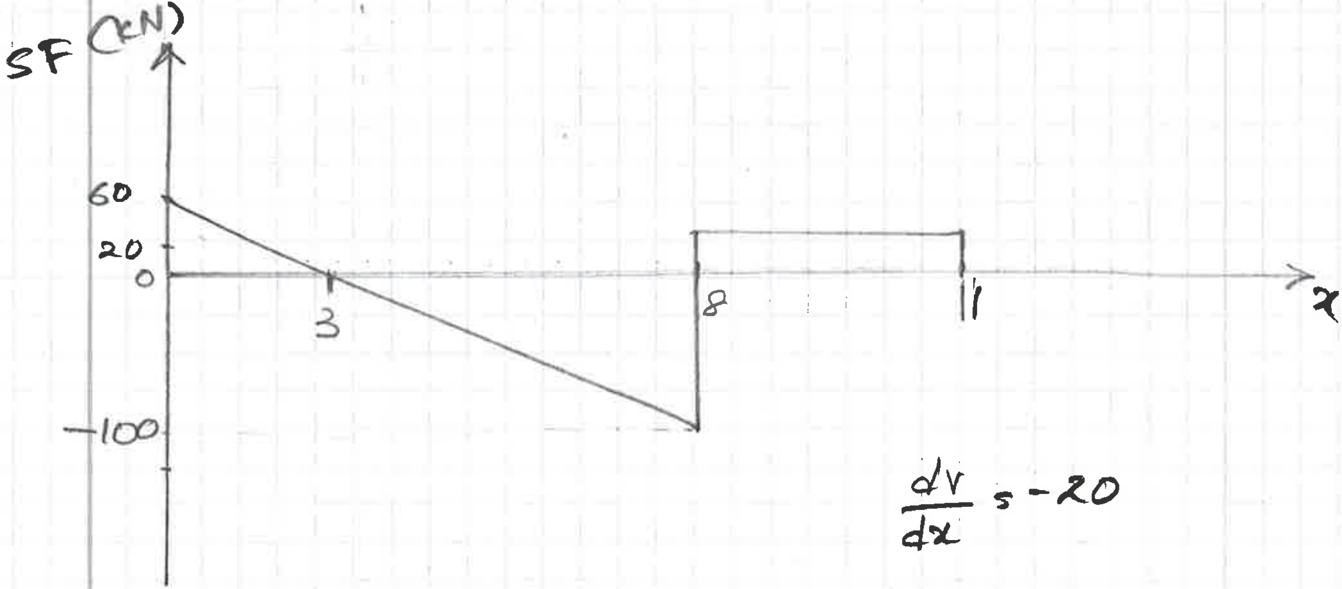




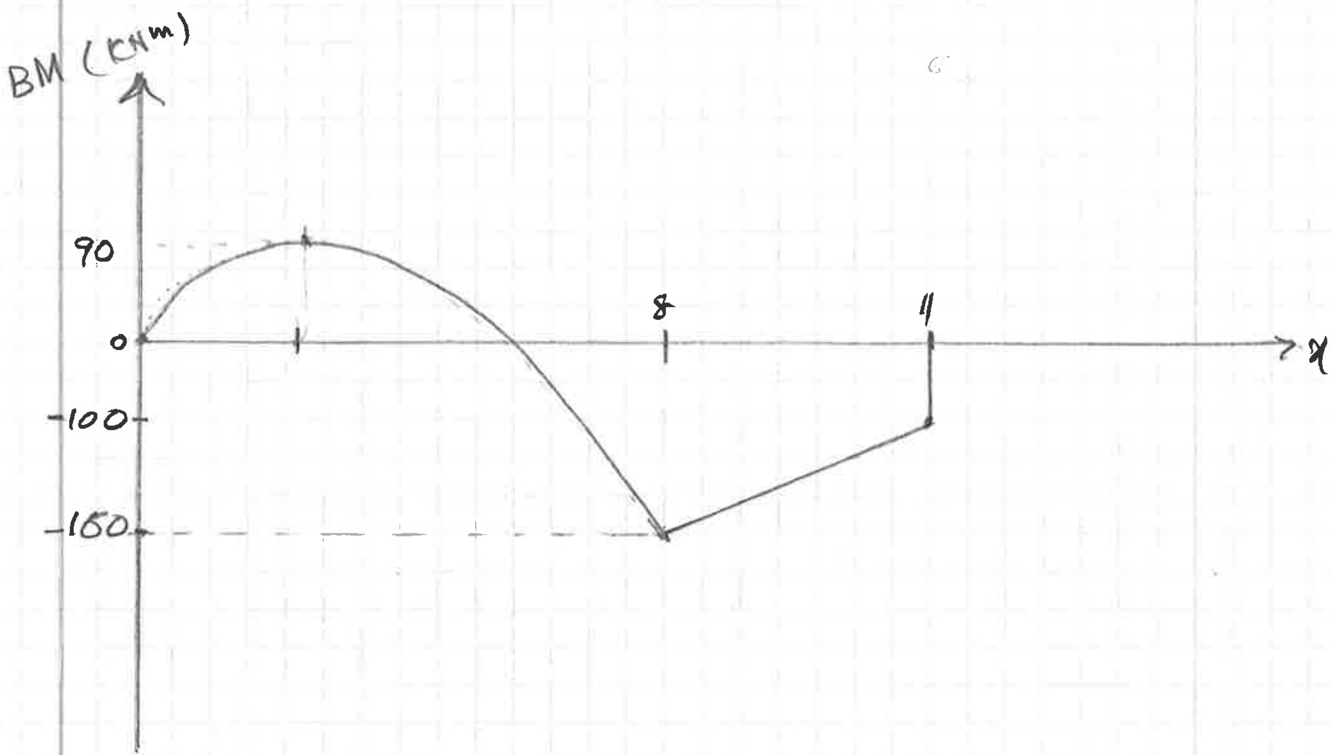
Shear force and bending moment diagram by graphical method.



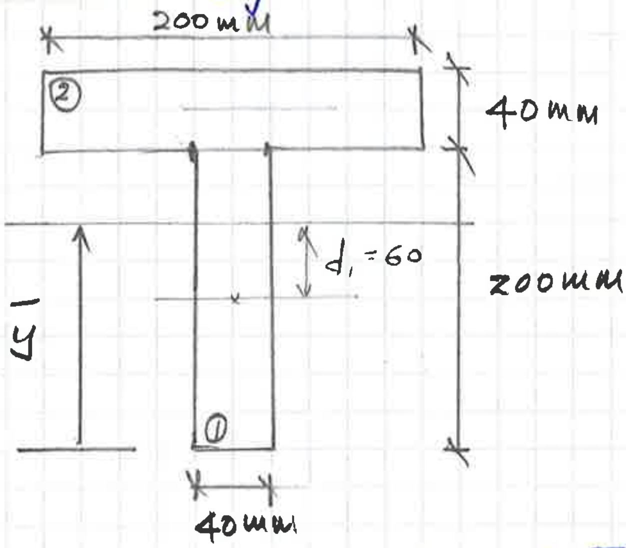
slope $\dfrac{dV}{dx} = -w(x)$ $\dfrac{dM}{dx} = V(x)$



$\dfrac{dV}{dx} = -20$



(iii) moment of inertia.



$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{A_{①}\bar{y}_{①} + A_{②}\bar{y}_{②}}{A_{①} + A_{②}}$$

$A_{①} = 200 \times 40 \ mm^2$

$\bar{y}_{①} = 100 \ mm$

$A_{②} = 200 \times 40 \ mm^2$

$\bar{y}_{②} = 220 \ mm.$

$$\bar{y} = \frac{(200\times40)100 + (200\times40)220}{2\times(200\times40)}$$

$\bar{y} = 160 \ mm.$

$$I = \Sigma(I_i + A_i d_i^2)$$

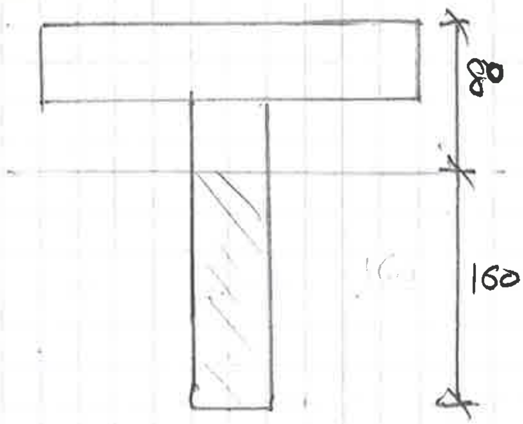
$$= \left[\left(\frac{1}{12}\times 40\times200^3\right) + (200\times40)60^2\right]$$

$$+ \left[\left(\frac{1}{12}\times200\times40^3\right) + (200\times40)60^2\right]$$

$$I = \underline{\underline{8.533\times10^7 \ mm^4}}$$

(iv) maximum shear stress (τ_{max})

$$\tau_{max} = \frac{V_{max}Q_{max}}{It}$$



$V_{max} = 100$ kN (from SFD)

$Q_{max} = (160\times40)\times\frac{160}{2} = 512000 \ mm^3$

$$\tau_{max} = \frac{(100\times10^3)\times(512000)}{8.533\times10^7\times40}$$

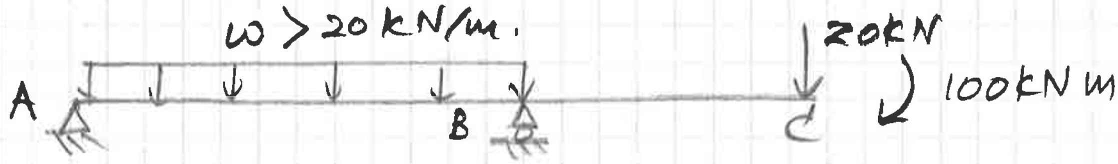
$$= \underline{\underline{15.00 \ Mpa.}}$$

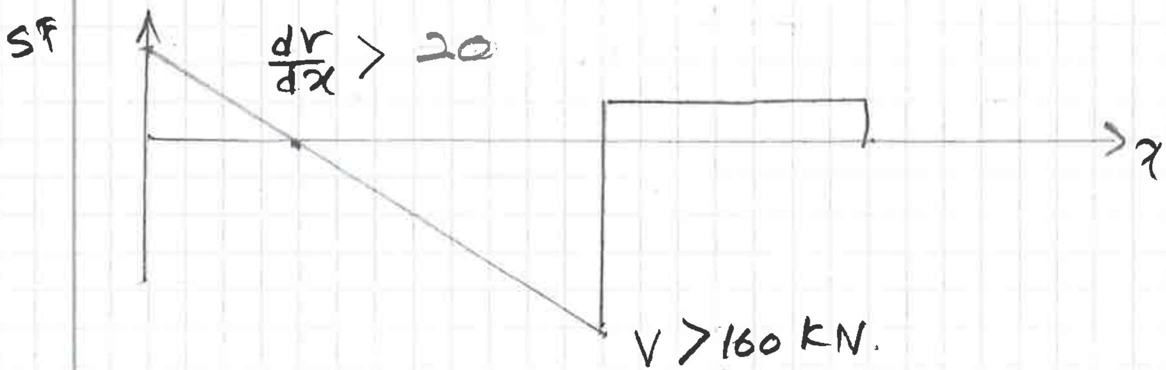
Maximum normal stress (σ)

$$\sigma_{max} = \frac{M_{max} y_{max}}{I} = \frac{160 \times 10^6 \times 160}{8.533 \times 10^7} = \underline{\underline{300.\ MPa}}$$

(v) when magnitude of uniform distributed load is increased, slope of the shear force diagram is increased between A & B. (ie $\frac{dV}{dx} = -w(x)$)
There is no any change in shear force between B & C as there is no any change of the loading. ($\therefore \frac{dv}{dx} = 0$, V is same as before.)

Shear force at B is increased. therefore shear stress at B is increased.



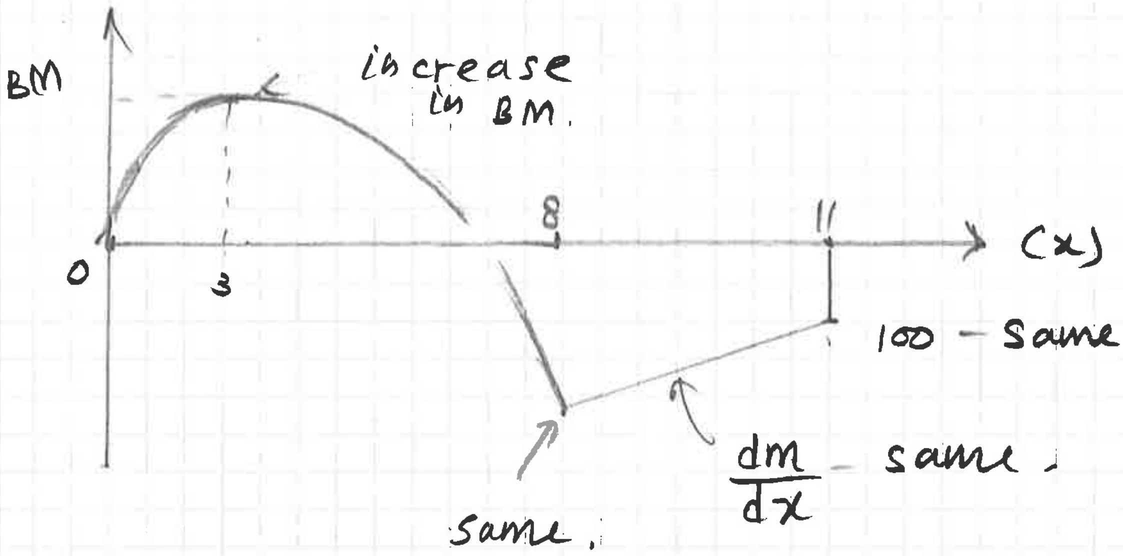


$$\tau_{max} = \frac{V_{max} Q_{max}}{It}$$

Q_{max} – same
I – same
t – same

$V_{max} \uparrow \rightarrow \tau_{max} \uparrow$ – shear stress at B increases.

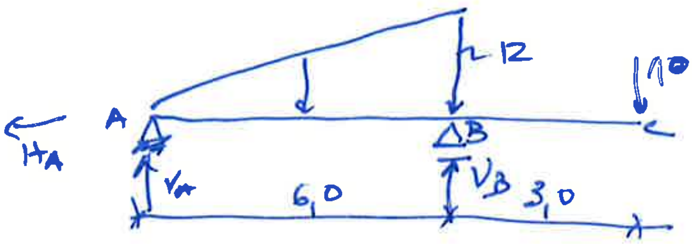
Similarly; $\frac{dM}{dx} = V(x)$ increases between A and B. Magnitude of bending moment increases between A & B. There is no any changes of BM between B & C. BM at C is 100 kNm.



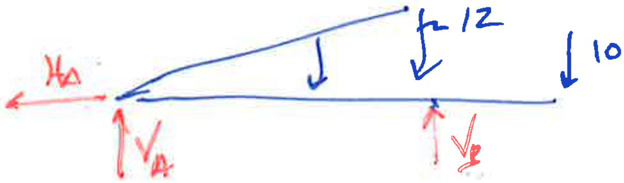
Therefore

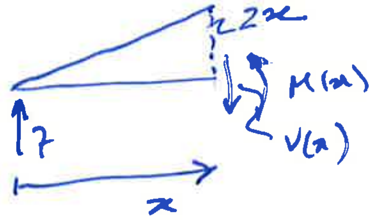
$$\sigma_{max} = \frac{MY}{I} \quad \text{– remain same.}$$

Problem 3 :



I Support reactions





$\Sigma F_H = 0 \Rightarrow H_A = 0$ ✓

$\Sigma F_V = 0 \quad V_A + V_B - 10 - \frac{12 \times 6}{2} = 0$ ✓

$\Sigma M_A = 0 \quad 6 V_B - 10 \times 9 - \frac{12 \times 6}{2} \times \frac{2}{3} \times 6 = 0$

$V_B = \frac{144 + 90}{6} = 39 \text{ kN}$

Hence $V_A = 46 - 39 = 7 \text{ kN}$

[From A to B]

$V(x) = 7 - 2\frac{x^2}{2} = 7 - x^2$ ✓

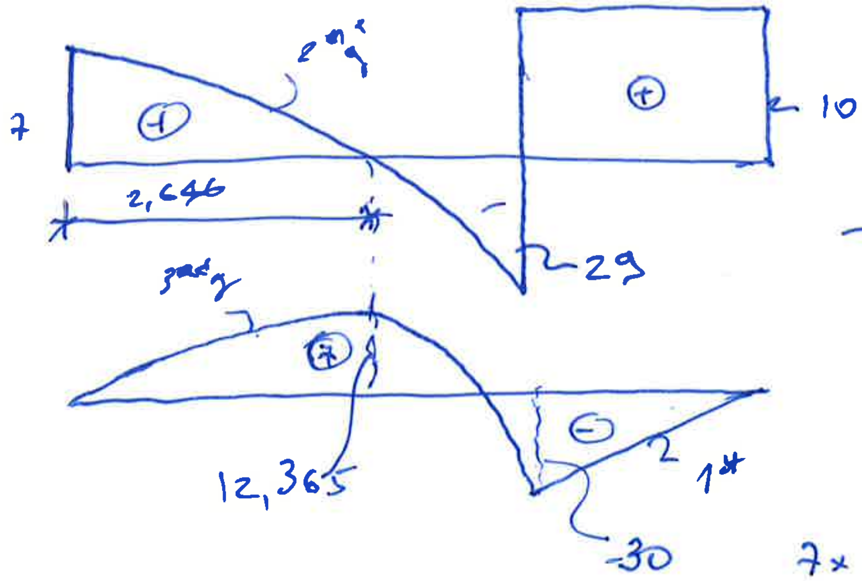
$V = 0 \Rightarrow x = 2{,}646 \text{ m}$

$M(x) = 7x - 2\frac{x^3}{6}$

$M(x = 2{,}646) = 7 \times 2{,}646 - 2 \times \frac{2{,}646^3}{6} = 12{,}365 \text{ kNm}$ ✓

II

V



III

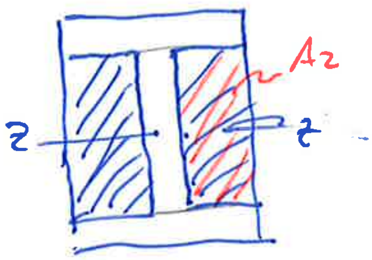
Inertia

Because the Section is symetrical in both axis, the center of gravity is located in the center point of the cross section.

$$I_z = \frac{200^3 \times 100}{12} - 2 \times \underbrace{\frac{160^3 \times 42}{12}}_{A_2} =$$

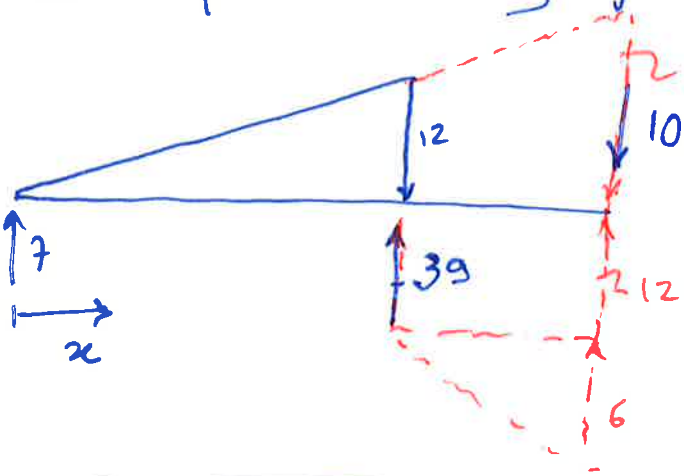
$$= 37994667 \ mm^4 =$$

$$= 3799 \ cm^4$$



IV The rotation at point A is equal to the slope of the deformed shape at that point.

The best approach to this problem comes through the use of discontinuity functions.



because loads in discontinuity functions have to go until the end, we need to compensate the extra loading

this equation is valid for the entire beam

$$\boxed{M(x) = 7x - \frac{12}{6}\frac{\langle x-0\rangle^3}{6} + 39\langle x-6\rangle + 12\frac{\langle x-6\rangle^2}{2} + \frac{6}{3}\frac{\langle x-6\rangle^3}{6}}$$

$$EI \frac{d^2 z}{dx^2} = M(x)$$

$$EI.\frac{dz}{dx} = \int M(x)\,dx = \frac{7x^2}{2} - 2\frac{x^4}{24} + 39\frac{\langle x-6\rangle^2}{2} + 12\frac{\langle x-6\rangle^3}{6} + 2\frac{\langle x-6\rangle^4}{24} + C_1$$

$$EI.z(x) = \frac{7x^3}{6} - 2\frac{x^5}{120} + 39\frac{\langle x-6\rangle^3}{6} + 12\frac{\langle x-6\rangle^4}{24} + 2\frac{\langle x-6\rangle^5}{120} + C_1 x + C_2$$

compatibility equations:

$$z(x=0) = 0 \Rightarrow C_2 = 0$$

$$z(x=6) = 0 \rightarrow$$

$$7\times\frac{6^3}{6} - \frac{6^5}{60} + 6C_1 = 0 \Rightarrow C_1 = -20{,}4 \checkmark$$

therefore, the rotation at A is:

$$EI.\frac{dz}{dx}(x=0) = C_1 \Rightarrow \theta_A = \frac{C_1}{EI} = \frac{-20{,}4}{200\times10^6\times3799\times10^{-8}} =$$

$$= -2{,}685\times10^{-3}\ \text{rad}$$

the vertical displacement at C can be determined using $EI.z(x=9)$. Therefore

$$EI.z(x=9) = \frac{7\times9^3}{6} - \frac{9^5}{60} + 39\times\frac{3^3}{6} + 12\times\frac{3^4}{24} + \frac{3^5}{60} - 20{,}4\times9 =$$

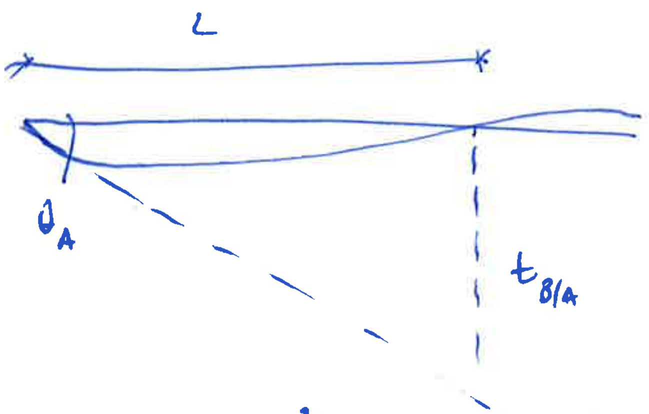
$$EI \cdot z(x=9) = 850.5 - 984{,}15 + 175{,}5 + 40{,}5 + 4{,}05 - 183{,}6$$

$$= -97{,}2$$

So, the vertical displacement is

$$z = \frac{-97{,}2}{200 \times 10^6 \times 3799 \times 10^{-8}} = -0{,}0128 \text{ m} \downarrow$$

Alternative method: using Area-moment



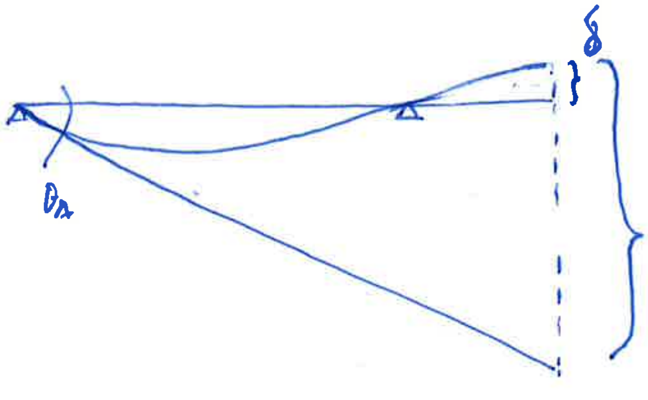
$$\theta_A \times L = t_{B/A} = \frac{1}{EI}\int_A^B M(x)\, x\, dx$$

$$M(x) = 7x - \frac{2x^3}{6}$$ Between A and B

$$-\frac{1}{EI}\int_0^6 \left(7x - \frac{x^3}{3}\right)(6-x)\, dx = -\frac{1}{EI}\left[42x - 7x^2 - 2x^3 + \frac{x^4}{3}\right]_0^6 = -\frac{122{,}4}{EI} = t_{B/A}$$

$$\theta_A = \frac{t_{B/A}}{6} = \frac{-122{,}4}{6\,EI} = \frac{-20{,}4}{EI} = \underline{-2{,}685 \times 10^{-2} \text{ rad}}$$

vi)



$$9\theta_A - t_{c/A} = \delta$$

③

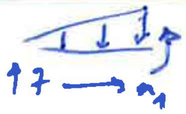
$$t_{CA} = -\frac{1}{EI}\left[\int_0^6 \left(7x - \frac{x^3}{3}\right)(9-x)\,dx + \int_6^9 10\,(x-9)(9-x)\,dx\right] =$$

$$= \frac{-1}{EI} \times \left[176{,}4 - 90\right] = -\frac{86{,}4}{EI}$$

$$\delta = -\frac{9 \times 20{,}4 - (-86{,}4)}{EI} = -\frac{97{,}2}{EI} = -0{,}0128 \text{ m} \downarrow$$

Alternative solution, without using singularity functions

$M(x)$ A-B : $7x_1 - \dfrac{x_1^3}{3}$



$M(x)$ B-C : $-10\,x_2$

$$\frac{d^2z}{dx^2} = \frac{M}{EI} \quad \xrightarrow{A-B} \quad EI.\frac{dz}{dx} = \frac{7x_1^2}{2} - \frac{x_1^4}{12} + C_1$$

$$EI.z(x) = \frac{7x_1^3}{6} - \frac{x_1^5}{60} + C_1 x_1 + C_2$$

B-C

$$EI.\frac{dz}{dx} = -\frac{10\,x_2^2}{2} + C_3$$

$$EI.\,z(x) = -10\,\frac{x^3}{6} + C_3 x + C_4$$

Compatibility equations:

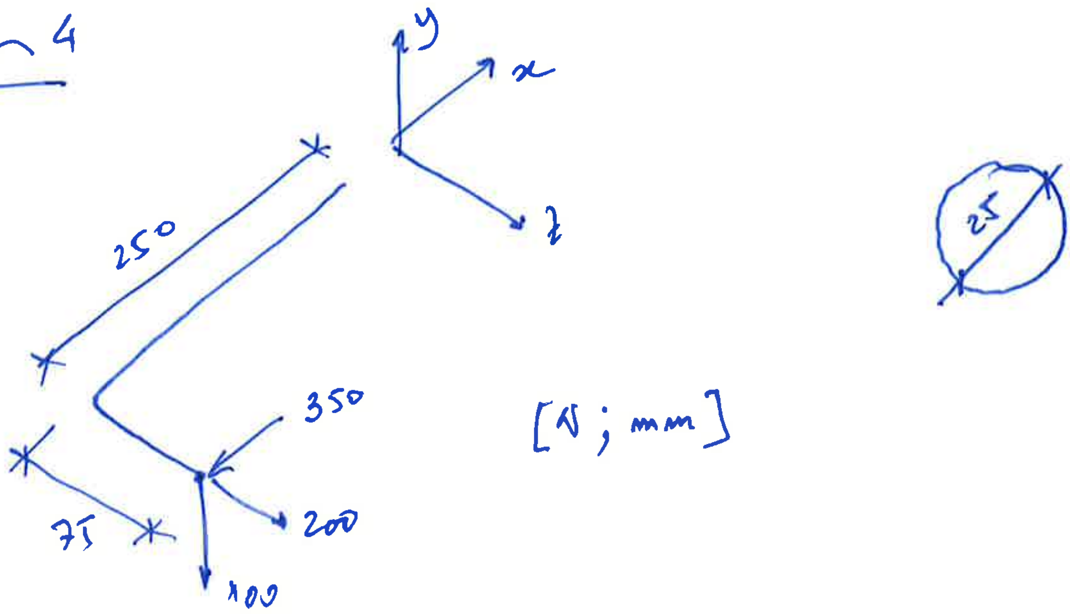
A-B $\quad z(A) = 0 \quad ; \quad z(B) = 0$

$\hookrightarrow C_2 = 0$

$\hookrightarrow \dfrac{7 \times 6^3}{6} - \dfrac{6^5}{60} + 6\,C_1 = 0 \quad \rightarrow \quad C_1 = -20{,}4$

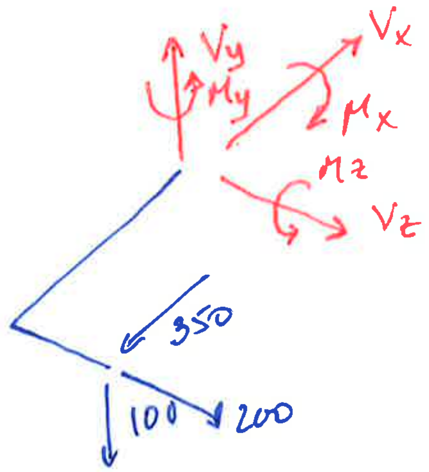
$$\theta_A = \frac{C_1}{EI} = -\frac{20{,}4}{EI} = -2{,}685 \times 10^{-2} \text{ rad}$$

Problem 4



[N ; mm]

Ⓘ Determine the internal forces at section A-B Using Free body diagram and the 6 equations of equilibrium.



$\Sigma F_x = 0 \Leftrightarrow$

$V_x - 350 = 0 \Rightarrow V_x = 350\ N$

$\Sigma F_y = 0 \Leftrightarrow$

$V_y - 100 = 0 \Rightarrow V_y = 100\ N$

$\Sigma F_z = 0 \Leftrightarrow 200 + V_z = 0 \Leftrightarrow$

$V_z = -200\ N$

Equilibrium also include moments at A-B

$\Sigma M_x = 0 \qquad 100 \times 75 + M_x = 0 \Leftrightarrow M_x = -7500\ N.mm$

$\Sigma M_y = 0 \qquad M_y + 200 \times 250 - 350 \times 75 = 0 \Rightarrow M_y = -23750\ N.mm$

$\Sigma M_z = 0 \qquad M_z + 100 \times 250 = 0 \Leftrightarrow M_z = -25000\ N.mm$

Ⓘⓘ At section A-B there are 6 internal forces

V_x = Axial Force $\quad V_z$ = Horizontal shear

V_y = Vertical Shear $\quad M_x$ = Torsion

$M_y ; M_z \rightarrow$ Bending moment along y and z

At point A

Normal stresses come from Axial Force and bending along $\underline{z}$

$$\sigma = \frac{N}{A} + \frac{M_z}{I} \times y =$$

$$= \frac{350}{490{,}87} + \frac{25000}{19174{,}76} \times 12{,}5 =$$

$$= 17{,}01 \ N/mm^2 = 17{,}01 \ MPa$$

Section properties

$$A = \pi r^2 =$$
$$= \pi \times 12{,}5^2 =$$
$$= 490{,}87 \ mm^2$$

$$I = \frac{\pi R^4}{4} = \frac{\pi \times 12{,}5^4}{4} =$$
$$= 19174{,}76 \ mm^4$$

For shear stresses: At point A only torsion and Horizontal shear produce stress

$$\tau = \frac{T \times c}{J} + \frac{V_z Q}{I b} =$$

$$= \frac{7500 \times 12{,}5}{38349{,}52} + \frac{200 \times 1302{,}08}{19174{,}76 \times 25} =$$

$$= 2{,}99 \ N/mm^2 = 2{,}99 \ MPa$$

Section properties

$$(Q_y)_A = 0$$

$$(Q_z)_A = \frac{\cancel{\pi} R^2}{2} \times \frac{4}{3} \frac{R}{\cancel{\pi}}$$

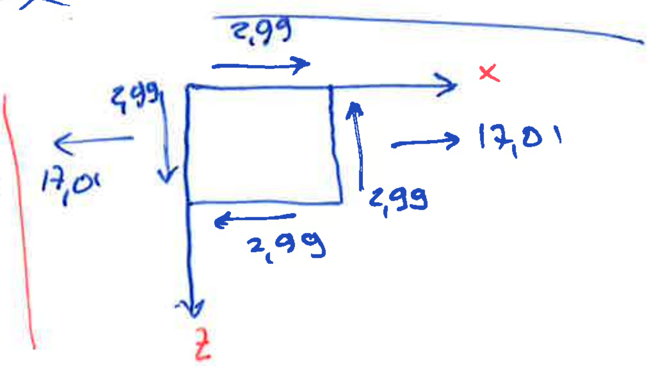
$$= \frac{2}{3} R^3 = 1302{,}08 \ mm^3$$

$$J = \frac{\pi R^4}{2} = 38349{,}52 \ mm^4$$

III Principal Stresses

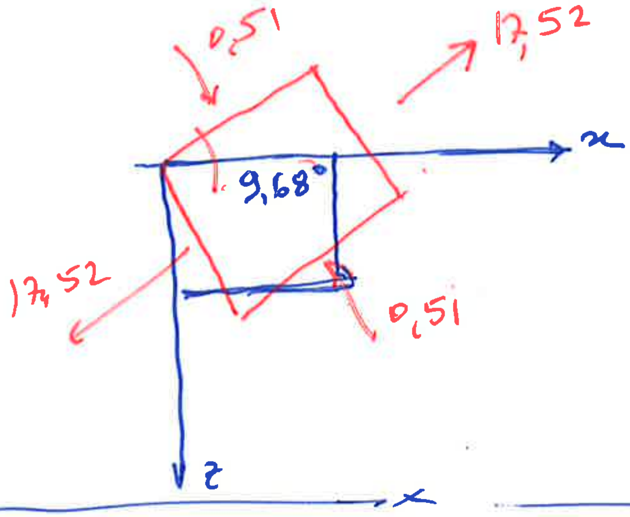
$$tg \ 2\theta_p = \frac{\tau_{xz}}{(\sigma_x - \sigma_z)/2} = \frac{-2{,}99}{\cancel{17{,}01}\ 8{,}055} =$$

$$\Leftrightarrow \theta_p = \cancel{-4{,}98^\circ} \ 9{,}68^\circ$$



$$\sigma_{1,2} = \frac{\sigma_x + \sigma_z}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2} =$$

$$= 8{,}505 \pm 9{,}01 = \begin{cases} +17{,}52 \text{ MPa} \\ -0{,}51 \text{ MPa} \end{cases}$$



IV

Maximum Shear stress

$$\tau_{\substack{abs\\max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{17{,}52 + 0{,}51}{2} = 9{,}015 \text{ MPa}$$

V

$$\varepsilon_x = \frac{17{,}01 \text{ MPa}}{200 \times 10^3 \text{ MPa}} = -0{,}08505 \times 10^{-3}$$

$$\varepsilon_y = \varepsilon_z = -\frac{0{,}3 \times 17{,}01}{200 \times 10^3} = -0{,}0255 \times 10^{-3}$$

$$\gamma_{xz} = \frac{-2{,}99}{76{,}92 \times 10^3} = -0{,}03887 \times 10^{-3}$$

$$\begin{cases} \sigma_x = 17{,}01 \text{ MPa} \\ \sigma_y = 0 \\ \sigma_z = 0 \\ \tau_{xz} = 2{,}99 \text{ Pa} \end{cases}$$

$$G = \frac{E}{2(1+\nu)} =$$

$$= 76{,}92 \text{ GPa}$$

VI

For statically independent structures, the values of the internal forces at any given point are independent of the flexibility of the Structure. therefore, so are the Stresses. Hence, the correct answer is "there is no change"