Construction mechanics I

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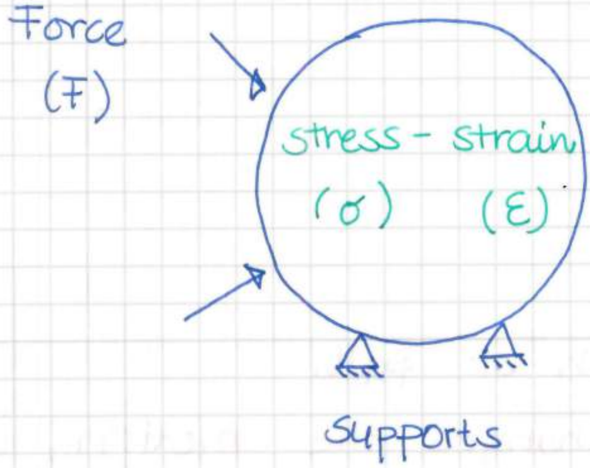
06.01.14



- deformation (d)
- velocity (v)
- acceleration (a)

why → F, σ, ε, d, v, a

Engineer → design (material / size / shape)
→ manufacture / produce / construction

To ensure - comfort, durability, safety

This course has been divided in two main parts

\# 1. Static (6 weeks)

\# 2. mechanic of material (MOM) (7 weeks)

Chapter 1 : General principles

physical science in the state of rest or motion, that are subjected to forces.

Mechanics → Rigid bodies, deformable bodies, fluid

fluid → compressive, incompressive

- Rigid bodies → statics
  → dynamics
- deformable bodie's - that change shapes.

Basic quantities

→ Length
→ time
→ Mass
→ Force

commonly, a push or pull.
Quantity that changes the motion, size or shape of a body.

Idealizations :

→ Partide (comparrativ) size can be neglected.
→ Rigid body (non detormable)
→ Concentrated forces act at a point on a body.

Newton's three laws :

First law : A particle originally at rest, or moving in a straight line with constant velocity tends to remain in this state. (no unbalaced force).

Second law : A particle of mass m, acted upon by an unbalanced force F experiences acceleration a that has same direction as the unbalanced force and a magnitude is directly porportional to the force.

$$F = ma$$

<u>Third law</u> - The mutual forces of action and reaction between two particles are equal, opposite & collinear.

For every action there is equal and opposite reaction.

Newton's law of gravitation attraction:

$$F = G \frac{m_1 m_2}{r^2}$$

G: universal constant of gravity
$66,73 \cdot 10^{-12}\ m^3/kg \cdot s^2$

Any two particles or bodies have a mutual attractive (gravitational) force acting between them.

$$W = G \frac{m M_e}{r^2} = mg$$

<u>Example</u>

$$g = G \frac{M_e}{r^2}$$

$$= \frac{66,73 \times 10^{-12}\ m^3/kgs^2 \cdot 5,97219 \times 10^{24}\ kg}{(6371.0 \times 10^3)^2\ m^2}$$

$$\underline{\underline{g_e = 9,81\ m/s^2}}$$

$$\frac{g_e}{g_{moon}} = 6$$

SI units :

Length (m)
Mass (kg)
Time (s)

$\left.\begin{matrix} \\ \\ \\ \end{matrix}\right\}$ base units

giga	10^9	G
mega	10^6	M
kilo	10^3	k
milli	10^{-3}	m
micro	10^{-6}	μ
nano	10^{-9}	n

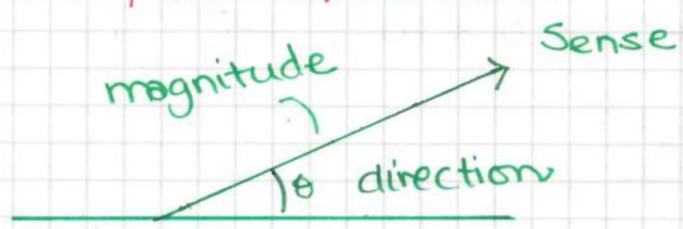
07.01 Chapter 2 - Force vectors

- Scalars

Any positive or negative physical quantity that can be specified by magnitude.
length, mass, time

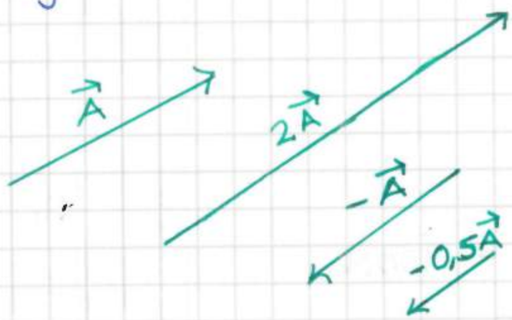
- Vector

A physical quantity that requires both magnitude and direction. ($\vec{V}$)
force, position, moment.

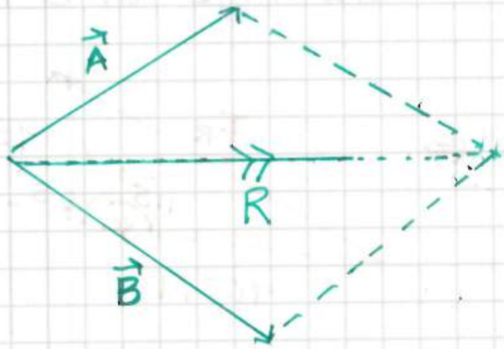


Actim force/ point - where the force is exerted.

Multiplication and division of a vector by a scalar :

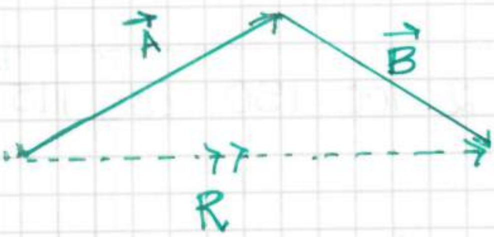


vector addtion :



R : Resultant force

← parallelogram

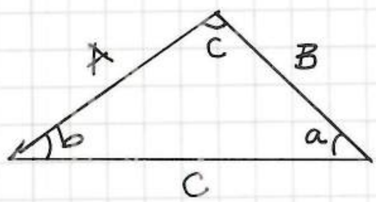


triangle law

Addition of collinear vectors :

$R = \vec{A} + \vec{B}$

$F_R = F_1 + F_2$



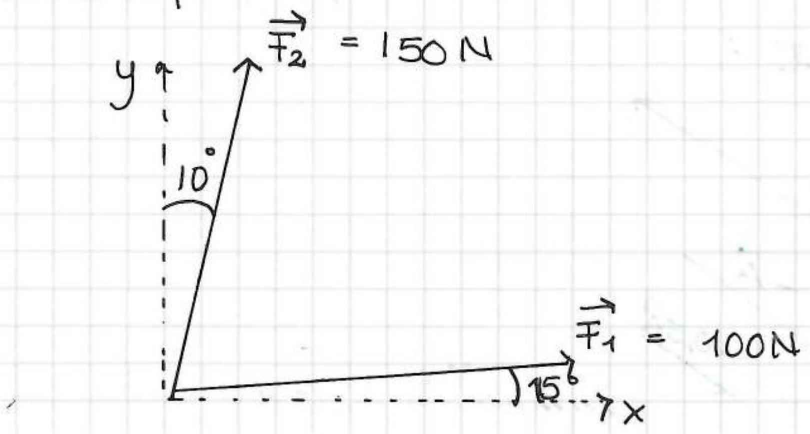
cosine law :

$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

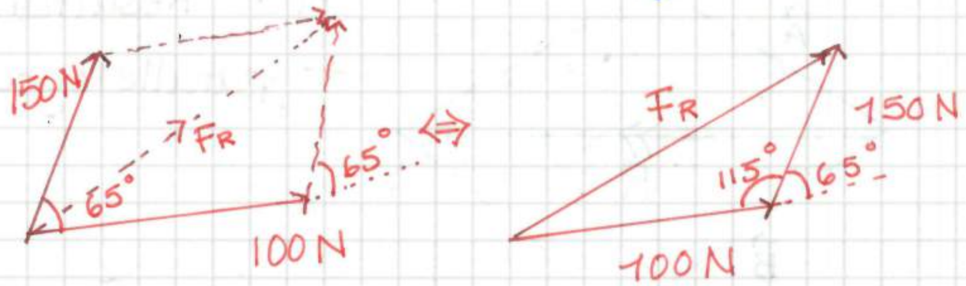
Sine law :

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

Example 1.



→ $90^\circ - 25^\circ = 65^\circ$ the angle in between



Using cosine Rule :

$$F_R = \sqrt{100^2 + 150^2 - 2 \cdot 100 \cdot 150 \cdot \cos 115^\circ}$$

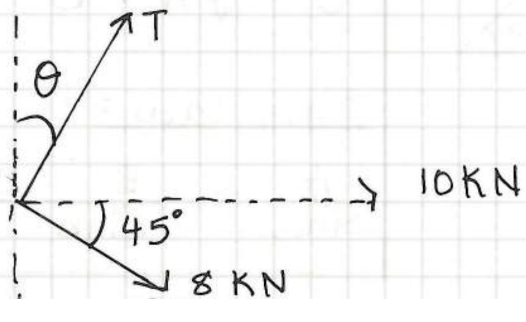
$$\underline{\underline{F_R = 213\ N}}$$

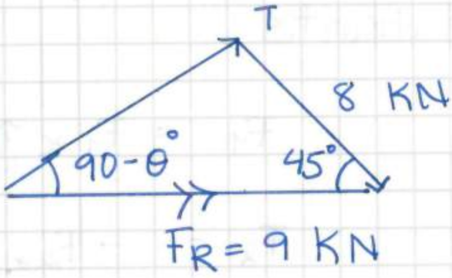
Using sine Rule :

$$\frac{150}{\sin\theta} = \frac{213}{\sin 115^\circ} \Rightarrow \sin\theta = \frac{150 \times \sin 115}{213}$$

$$\underline{\underline{\theta = 39{,}8^\circ}}$$

Example 2





By using the sin Rule:

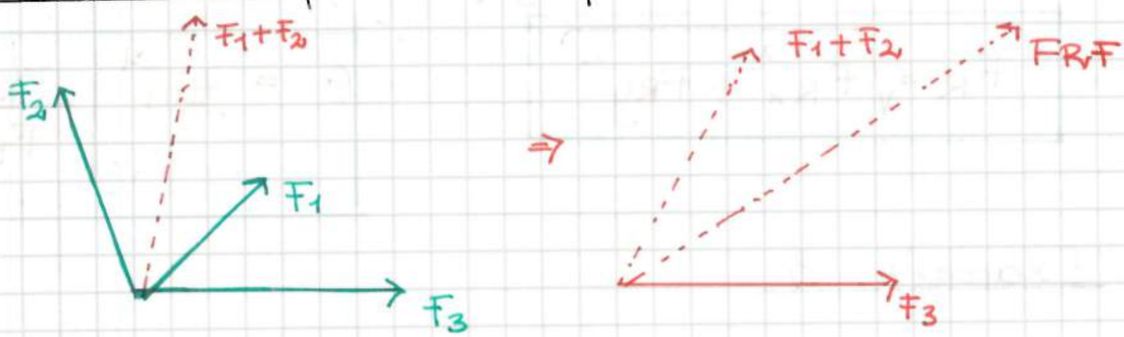
$$\frac{T}{\sin 45} = \frac{8}{\sin(90-\theta)} = \frac{9}{\sin(180-90-45+\theta)}$$

By using the cosine Rule:

$$T = \sqrt{8^2 + 9^2 - 2 \times 9 \times 8 \times \cos 45^\circ} \qquad T = \underline{\underline{6,57\ KN}}$$

$$\frac{T}{\sin 45} = \frac{8}{\sin(90-\theta)} \Rightarrow \underline{\underline{\theta = 30,6^\circ}}$$

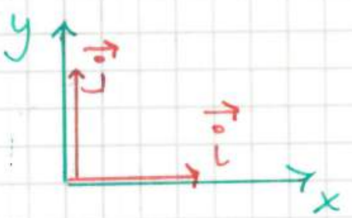
Addition of several forces:



Scalar Notation

$$F_x = F \cdot \cos\theta$$
$$F_y = F \cdot \sin\theta$$

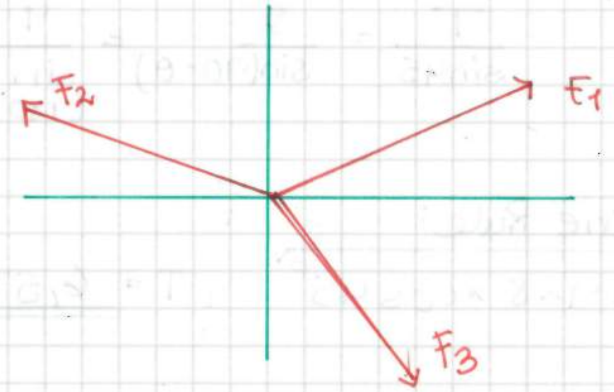
Cartesian Vector Notation



$$\vec{F} = F_x\,\hat{i} + F_y\,\hat{j}$$
$$\vec{F} = F\cos\,\hat{i} + F\sin\theta\cdot\hat{j}$$

unit vector: - magnetude 1
- direction of i is + x axis
- direction of j is + y-axis.

Coplaner Force Resultants



$F_1 = F_{1x}\hat{\imath} + F_{1y}\hat{\jmath}$

$F_2 = -F_{2x}\hat{\imath} + F_{2y}\hat{\jmath}$

$F_3 = F_{3x}\hat{\imath} - F_{3y}\hat{\jmath}$

$F = F_1 + F_2 + F_3$

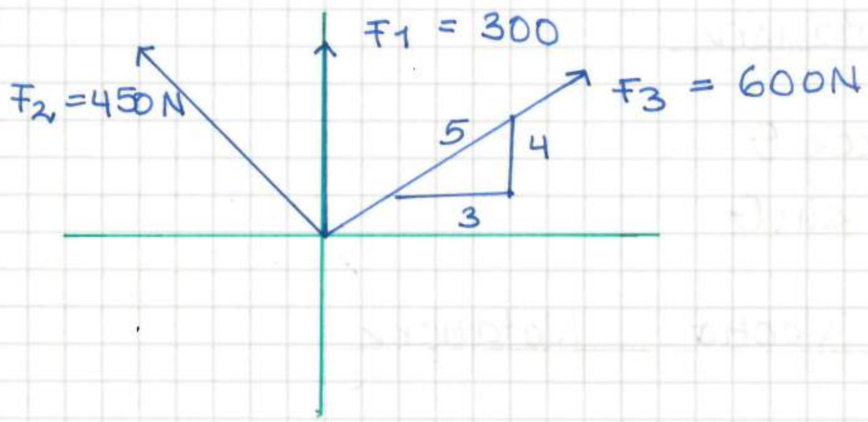
$$F_R = (F_{1x} - F_{2x} + F_{3x})\hat{\imath} + (F_{1y} + F_{2y} - F_{3y})\hat{\jmath}$$

$$F_R = F_{Rx}\hat{\imath} + F_{Ry}\hat{\jmath}$$

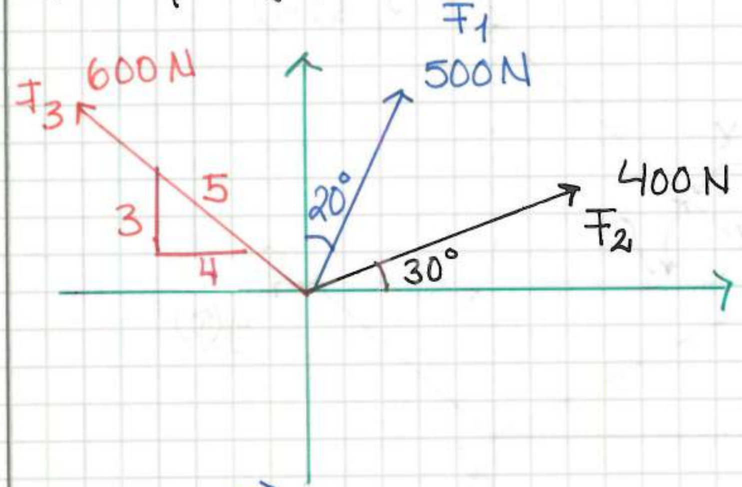
$$\boxed{F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}}$$

$$\boxed{\theta = \tan^{-1}\left|\frac{F_{Ry}}{F_{Rx}}\right|}$$

Example 3



10/1/14 Example (4)



Resolving forces in x and y directions.

$$\vec{F_1} = 500 \sin 20^\circ \hat{i} + 500 \cos 20^\circ \hat{j}$$

$$\vec{F_2} = 400 \cos 30^\circ \hat{i} + 400 \sin 30 \hat{j}$$

$$\vec{F_3} = -600 \cos \phi \hat{i} + 600 \sin \phi \hat{j}$$

$$F_R = \hat{F}_1 + \hat{F}_2 + \hat{F}_3$$

$$= (500 \sin 20 + 400 \cos 30 - 600 \times 4/5)\hat{i}$$

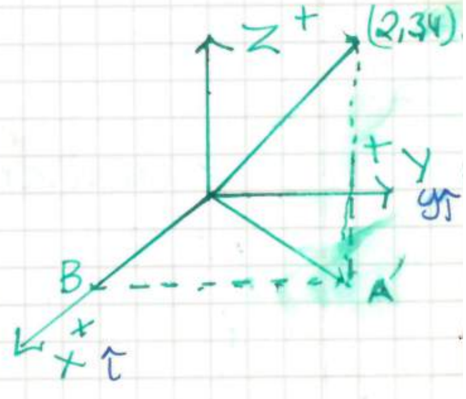
$$+ (500 \cos 20 + 400 \sin 30 + 600 \times 3/5)\hat{j}$$

$$F_R = 37.42 \hat{i} + 1029.8 \hat{j}$$

$$F_R = \sqrt{37{,}42^2 + 1029.8^2} = 1030{,}5 \text{ N} = \underline{\underline{1.03 \text{ kN}}}$$

$$\theta = \tan^{-1} \left| \frac{1029.8}{37.42} \right| = \underline{\underline{87.9^\circ}}$$

2,5 Cartesian vectors

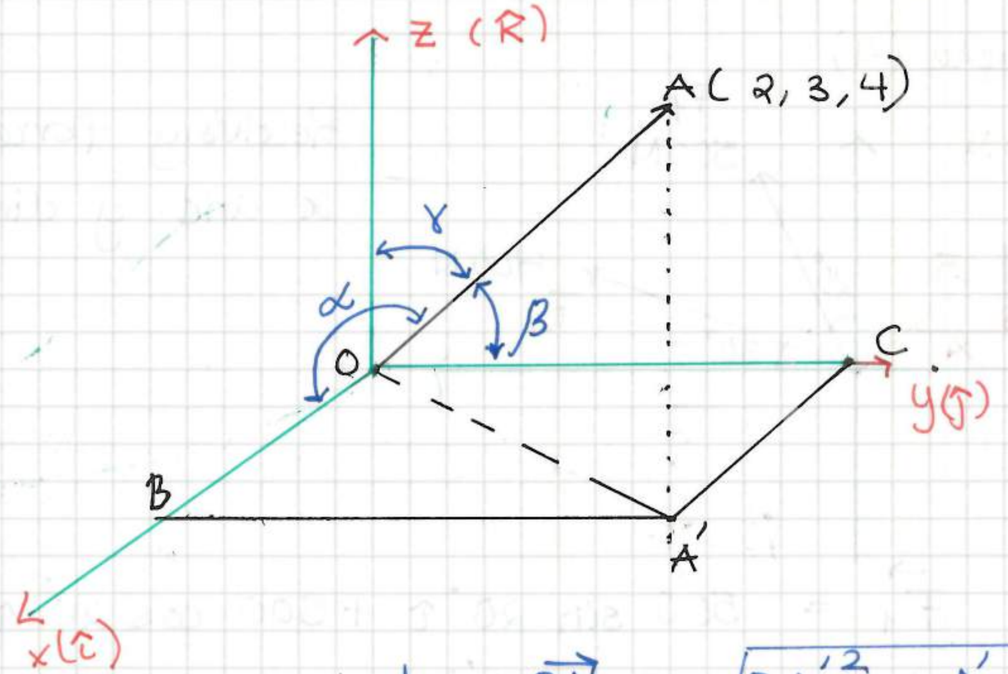


$$BA' = OC = 3$$

$$A'A = 4$$

$$\vec{OA} = \vec{OB} + \vec{BA'} + \vec{A'A}$$

$$\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$



magnetude : $\vec{OA} = \sqrt{OA'^2 + A'A^2}$

$= \sqrt{OB^2 + BA'^2 + A'A^2} = 5.38$

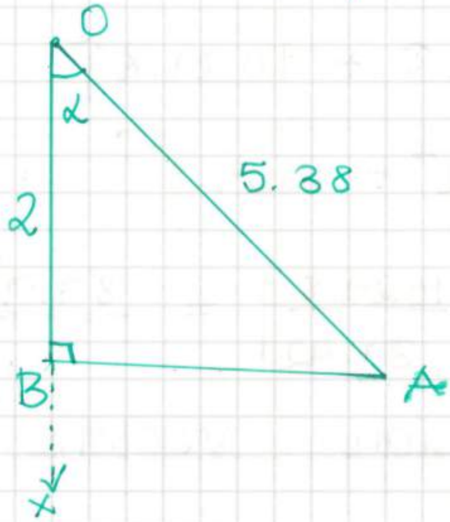
Define α β γ

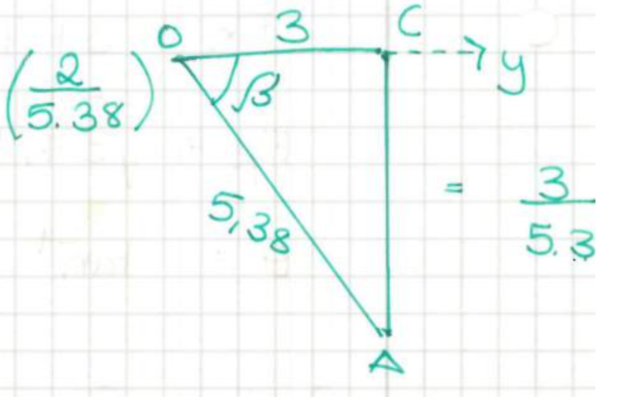
α → angle between x axis & OA

β → angle y axis & OA

γ → angle z axis & OA

directions cos α cos β cos γ

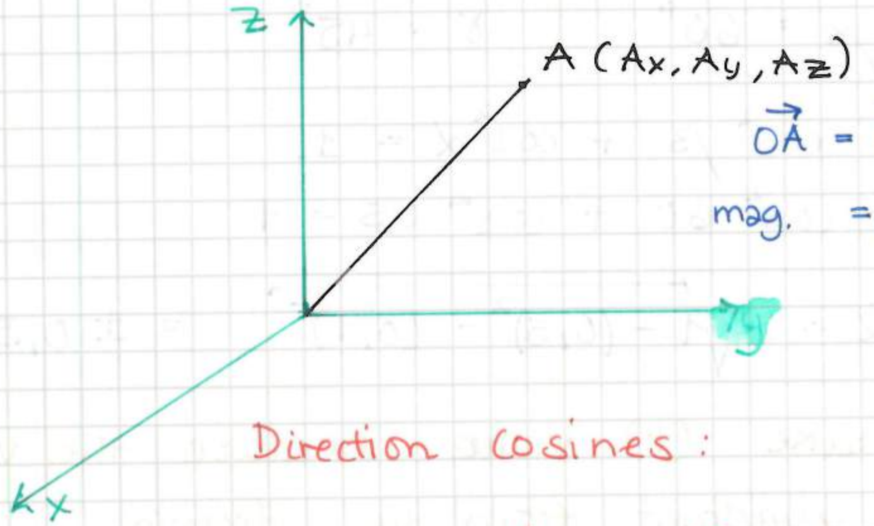




similar way : $\cos\alpha = 2/5.38$

$\cos\beta = 3/5.38$

$\cos\gamma = 4/5.38$



$$\vec{OA} = \vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\text{mag.} = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Direction cosines:

$$\left.\begin{aligned} \cos\alpha &= A_x / A \\ \cos\beta &= A_y / A \\ \cos\gamma &= A_z / A \end{aligned}\right\} \quad \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1. \; *$$

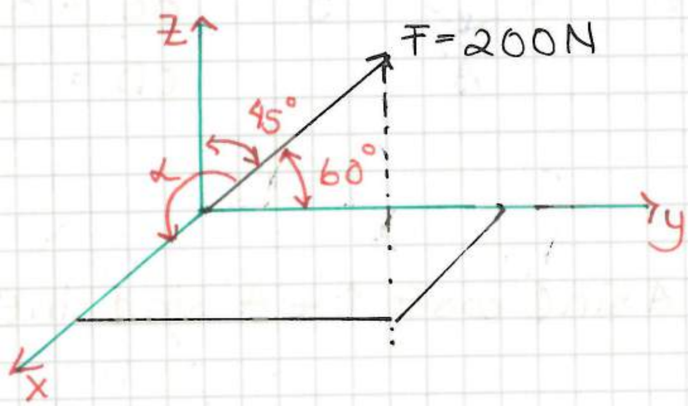
$$* = \frac{A_x^2}{A^2} + \frac{A_y^2}{A^2} + \frac{A_z^2}{A^2} = 1$$

unit vector $\vec{u_A} = \dfrac{\vec{OA}}{|OA|}$

$$\boxed{\vec{u_A} = \frac{A_x \hat{i}}{A} + \frac{A_y}{A}\hat{j} + \frac{A_z}{A}\hat{k}}$$

example 2.8 (5)

Express the force F in the figure as a Cartesian vector.



$\alpha = ?$ $\beta = 60^\circ$ $\gamma = 45^\circ$

$$\cos^2\alpha + \cos^2\beta + \cos^2\alpha = 1$$

$$\cos^2\alpha + \cos^2 60 + \cos^2 45 = 1$$

$$\cos\alpha = \sqrt{1-(0{,}5)^2-(0{,}1)^2} = \pm 0{,}5 = 60^\circ$$

we will take (+) value because the vector is in posetiv quadrant, from the drawing.

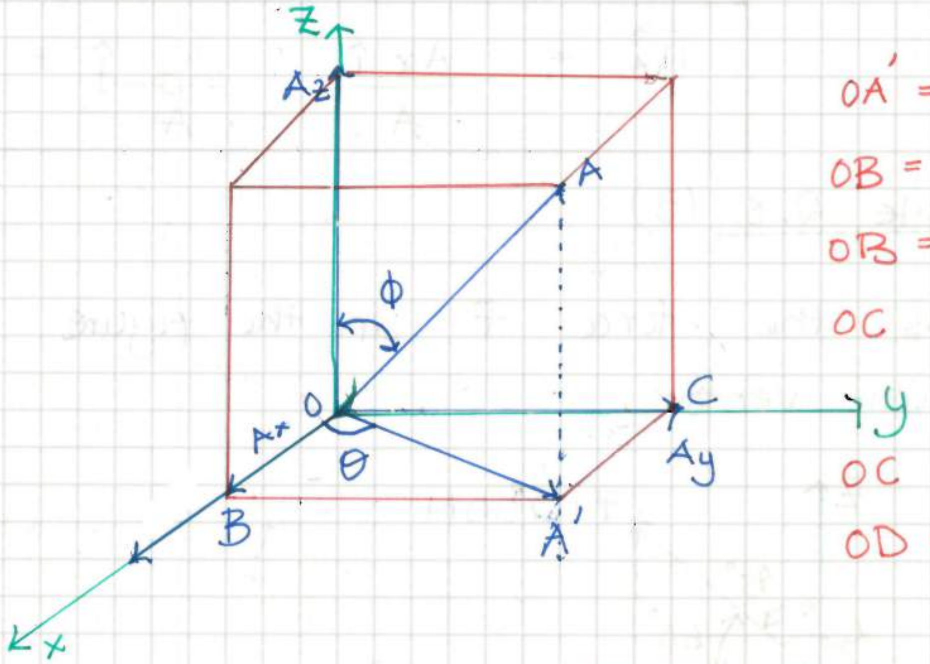
$$\vec{F} = F\cos\alpha\,\hat{\imath} + F\cos\beta\,\hat{\jmath} + F\cos\gamma\,\hat{k}$$

$$\vec{F} = F(\cos\alpha\,\hat{\imath} + \cos\beta\,\hat{\jmath} + \cos\gamma\,\hat{k})$$

$$= 200\cdot(\cos 60^\circ\,\hat{\imath} + \cos 60^\circ\,\hat{\jmath} + \cos 45\,\hat{k})$$

$$\underline{\underline{\vec{F} = (100\,\hat{\imath} + 100\,\hat{\jmath} + 141.4\,\hat{k})\,N}}$$

<u>Cartesian vector representation:</u>



$OA' = OA\sin\phi$

$OB = OA'\cos\theta$

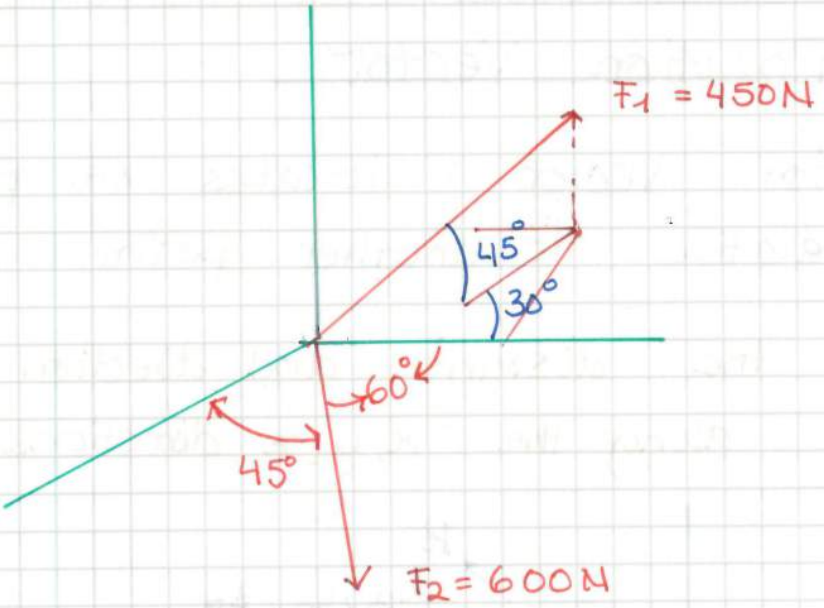
$OB = OA\sin\phi\cos\theta$

$OC = OA'\sin\theta$

$OC = OA\sin\phi\sin\theta$

$OD = OA\cos\theta$

$$\vec{OA} = \hat{A} = A\sin\phi\cos\theta\,\hat{\imath} + A\sin\phi\sin\theta\,\hat{\jmath} + A\cos\phi\,\hat{k}$$



$$F_1 = -F_1 \cos 45° \sin 30° \hat{i} + F_1 \cos 45° \cos 30° \hat{j} + F_1 \sin 45° \hat{k}$$

$$F_2 = 600 \cos 45° \hat{i} + 600 \cos 60 \hat{j} + \underbrace{600 \cos \gamma \hat{k}}_{?}$$

$$\Rightarrow \cos^2 45° + \cos^2 60 + \cos^2 \gamma = 1$$

$$\cos \gamma = \pm 0,5 \qquad \underline{\underline{\gamma = 120°}}$$

$$\vec{F_R} = \vec{F_1} + \vec{F_2}$$

$$\underline{\underline{\vec{F_R} = (265.16 \hat{i} + 575.57 \hat{j} + 18,20 \hat{k}) \ N}}$$

$$F_R = \sqrt{265.16^2 + 575.57^2 + 18,20^2} = 634 N$$

$$\alpha = \cos^{-1}\left(\frac{F_{Rx}}{F_R}\right) = \left(\frac{265.16}{634}\right) . \cos^{-1} = 65,3°$$

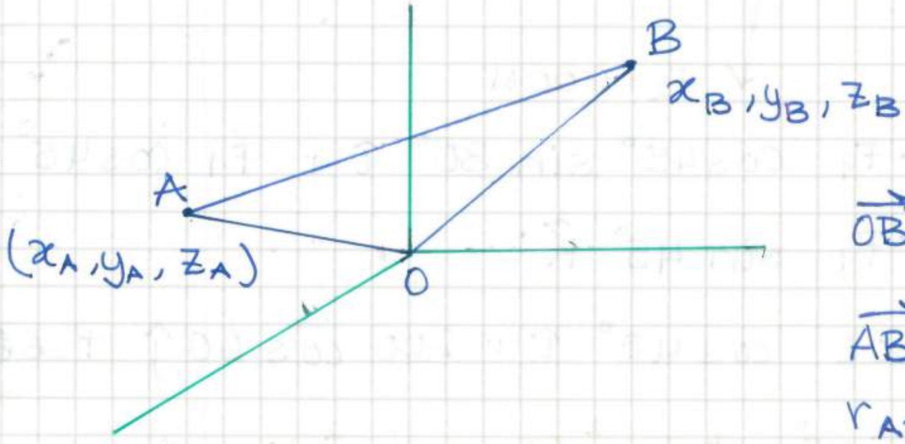
$$\beta = \cos^{-1}\left(\frac{F_{Ry}}{F_R}\right) = \underline{\underline{24,8°}}$$

$$\gamma = \cos^{-1}\left(\frac{F_{Rz}}{F_R}\right) = \underline{\underline{88,4°}}$$

2.7 Position Vector

A position vector r locates one point in space relative to another point.

Determine the distance and direction that must be traveld along the x, y, z directions.



$$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$$

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

$$r_{AB} = r_B - r_A$$

$$\overrightarrow{AB} = \begin{pmatrix} x_B \\ y_B \\ z_B \end{pmatrix} - \begin{pmatrix} x_A \\ y_A \\ z_A \end{pmatrix} = \begin{pmatrix} x_B - x_A \\ y_B - y_A \\ z_B - z_A \end{pmatrix}$$

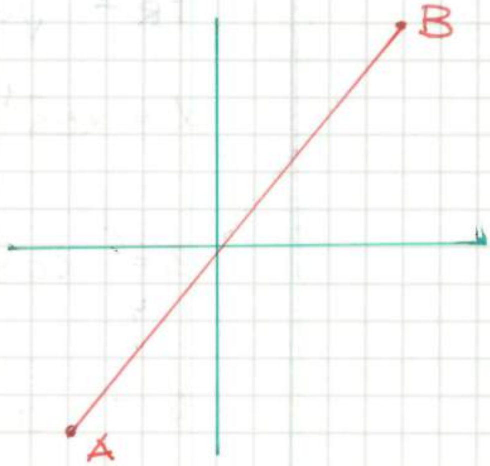
<u>example 7</u>

$$\vec{r}_A = 1\hat{\imath} - 3\hat{k}$$

$$\vec{r}_B = -2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$$

$$\vec{r}_{AB} = (-2-1)\hat{\imath} + 2\hat{\jmath} + 6\hat{k}$$

$$\underline{\underline{= -3\hat{\imath} + 2\hat{\jmath} + 6\hat{k}}}$$



2,8 Force vector directed on a line.

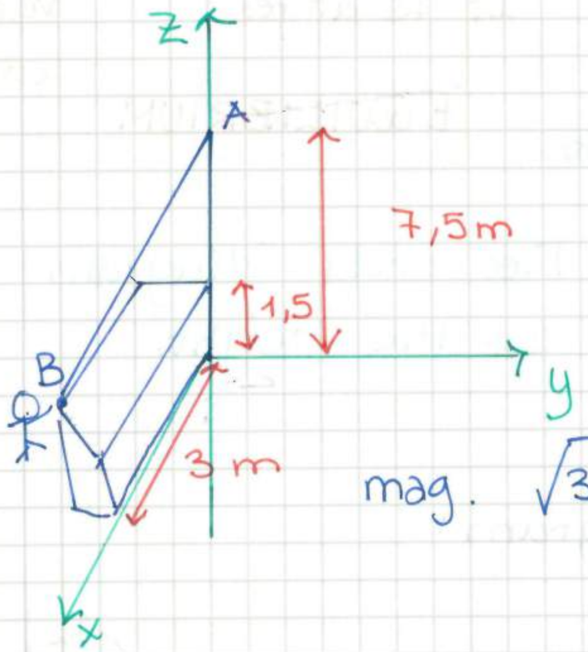
$\mathbf{F} = F \cdot u_{AB}$

u_{AB} : unit vector along a line

$\mathbf{F}$: force vector

F : magnitude of the force.

example (8) :



$r_{AB} = r_B - r_A$

$= (3\hat{i} - 2\hat{j} + 1{,}5\hat{k})$

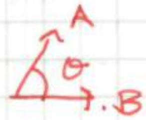
$- (0\hat{i} + 0\hat{j} + 7{,}5\hat{k})$

$r_{AB} = (3\hat{i} - 2\hat{j} - 6\hat{k})$

mag. $\sqrt{3^2 + 2^2 + 6^2} =$

2,9 Dot product

$A \cdot B = AB\cos\theta$

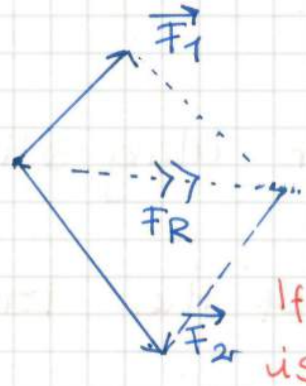


$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$= (A_x \cdot B_x)\hat{i} + (A_y \cdot B_y)\hat{j} + (A_z \cdot B_z)\hat{k}$

13/01 Chapter 3 - Equilibrium of a particle



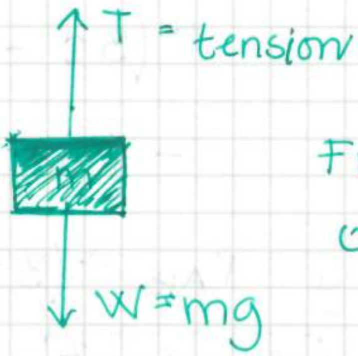
if resultant force $\vec{F_R} = 0$

$|\vec{F_R}| = 0$

If the particle is orginally at rest It is at rest

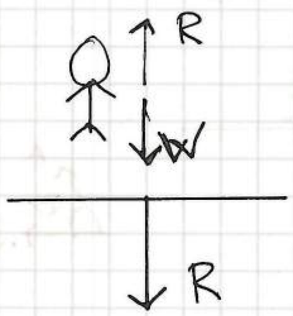
If the particle orginally has a const velocity, it has a const. velocitu

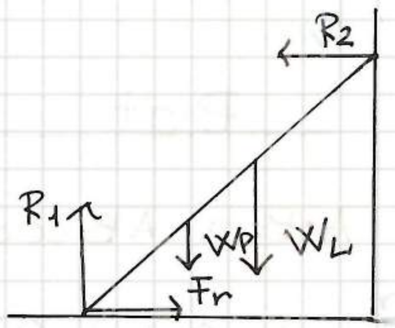
EQUILIBRIUM



Free body diagram (FBD) of the weight.

Free body diagram:





smooth roller

String or cable:

A mechanical device that can transmit tensile force along itself.

Linear spring

$F = K \cdot s$

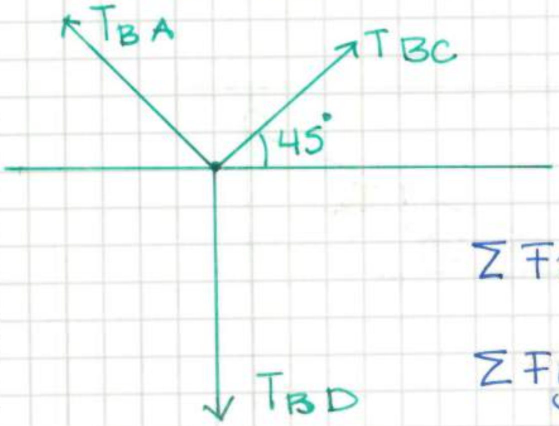
Frictionless pulleys

$T_1 = T_2$

3,3 Coplanar force systems

Example:

Determine T_{BA} and T_{BC}. 60 kg sylinder.

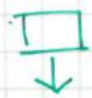


Considering equilibrium of particle B.

$$\Sigma F_x = T_{BC} \cos 45^\circ - T_{BA} \cos\theta = 0$$

$$\Sigma F_y = T_{BC} \sin 45^\circ + T_{BA} \cdot \sin\theta - T_{BD} = 0$$

considering mass D



$T_{BD} = (60 \times 9{,}81)\,N$ $\quad W = -(60 \times 9{,}81)\,N$

$$\Sigma F_y = T_{BC} \cdot 1/\sqrt{2} + T_{BA} \sin\theta - (60 \times 9{,}81)$$

$$\Rightarrow \Sigma F_x = T_{BC}\ 1/\sqrt{2} - T_{BA} \cdot \cos\theta = 0$$

$\hookrightarrow$ (1) $T_{BC} \times 1/\sqrt{2} - T_{BA}\, 4/5 = 0$

$$T_{BC} = \frac{4\sqrt{2}}{5} T_{BA}$$

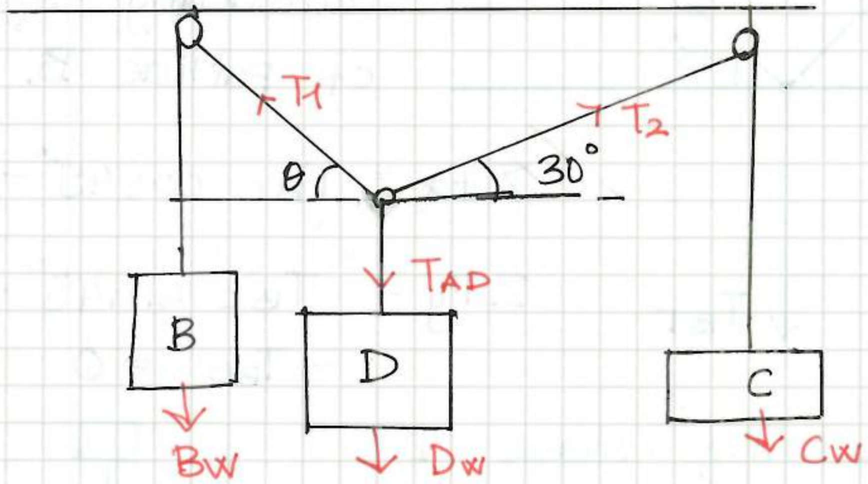
(2) $\frac{4\cancel{\sqrt{2}}}{5} T_{BA} \cdot \frac{1}{\cancel{\sqrt{2}}} - T_{BA} \cdot 3/5 = 0$

$$\underline{\underline{T_{BA} = 420\,N}}$$

$$T_{BC} = \frac{4\sqrt{2}}{5} \cdot 420 = \underline{476\,N}$$

Example :

If $D_w = 1{,}5$ KN $B_w = 1{,}375$ KN
determine weight of block C and angle θ.



considering A in equibrium :

$\Sigma F_x = T_2 \cos 30 - T_1 \cos\theta = 0$
$\Sigma F_y = T_2 \sin 30 + T_1 \sin\theta - T_{AD} = 0$

mass D $\uparrow T_{AD}$ $\downarrow 1500$ N $\Sigma F_y = T_{AD} = 1500$ N

mass B $\uparrow T_1$ $\downarrow 1375$ $\Sigma F_y \Rightarrow T_1 = 1375$ N

mass C $\uparrow T_2$ $\downarrow W_c$ $\uparrow F_y$ $T_2 = W_c$

① $(W_c \times \sqrt{3}/2) - (1375 \times \cos\theta) = 0$

② $(W_c \times 1/2) + (1375 \times \sin\theta) - 1500 = 0$

$W_c \times \sqrt{3}/2 = 1375 \cos\theta$ (4)

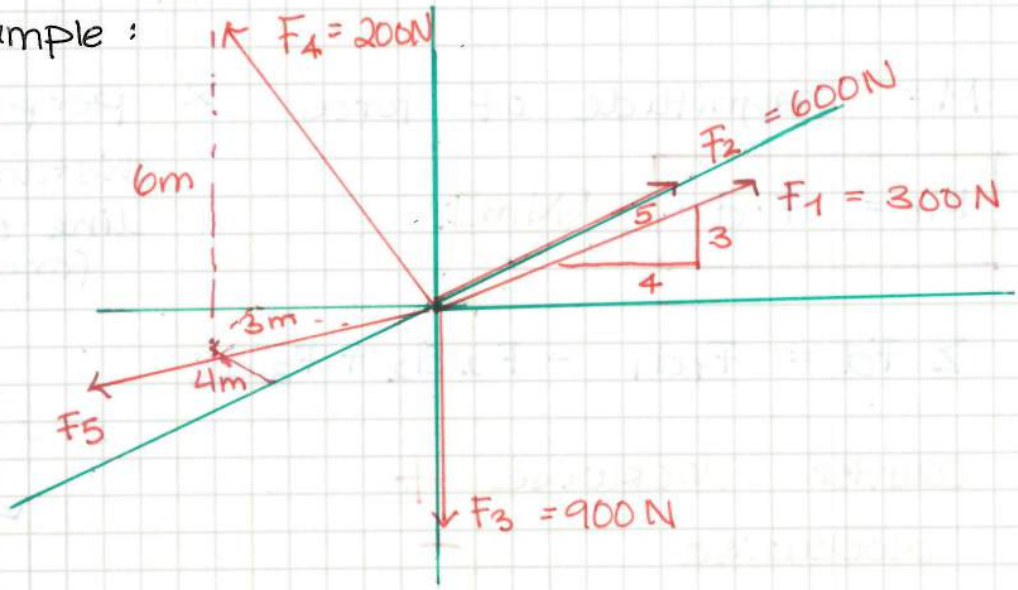
$1500 - W_c/2 = 1375 \cdot \sin\theta$ (5)

square both sides add

$\underline{\underline{W_c = 1200\,N}}$ $\theta = 40{,}9^\circ$

3,4 Three - Dimensional Force system

Example :



Find the force F_5 requried for equilibrium.

$\vec{F_1} = (300 \times 4/5)\,\hat{j} + (300 \times 3/5)\,\hat{k}$

$\vec{F_2} = -600\,\hat{i}$

$\vec{F_3} = -900\,\hat{k}$

$\vec{F_4} = 200 \times \dfrac{3\hat{i} - 4\hat{j} + 6\hat{k}}{\sqrt{3^2+4^2+6^2}}$

$\vec{F_5} = F_x\,\hat{i} + F_y\,\hat{j} + F_z\,\hat{k}$

Particle in equilibrium :

$\Sigma F_x = 0 = -600 + 76.8 + F_x = 0$

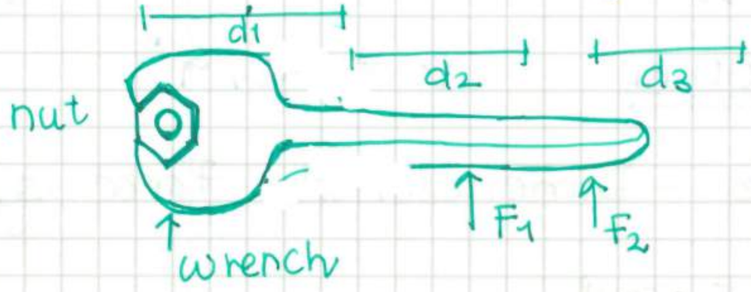
$\underline{F_x = 523.2\,N}$

$\Sigma F_y = 240 - 102{,}4 + F_y = 0 \Rightarrow F_y = -137.6\,N$

$\Sigma F_z = 566,4 \text{ N}$

$\vec{F_5} = 523\hat{i} - 138\hat{j} + 566,4\hat{k}$

17/01

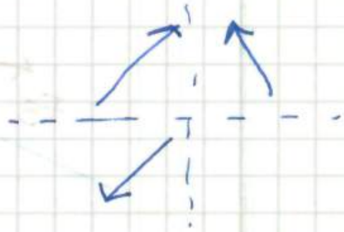


tendency of rotation
⇒ moment
$M = f(F, d)$

M = magnitude of force × perpendicular distance to the line of action of force.

$\boxed{M_O = F \cdot d}$ [Nm]

$\Sigma Fd = F_1 d_1 - F_2 d_2 + F_3 d_3$



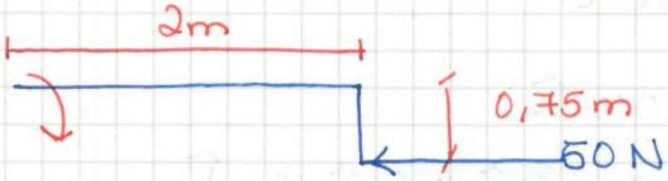
counter clockwise +
clockwise −

example

(a) $\overset{\curvearrowright}{M_O} = F \cdot d$

$= 100 \cdot N \cdot 2m = 200 \text{ Nm}$

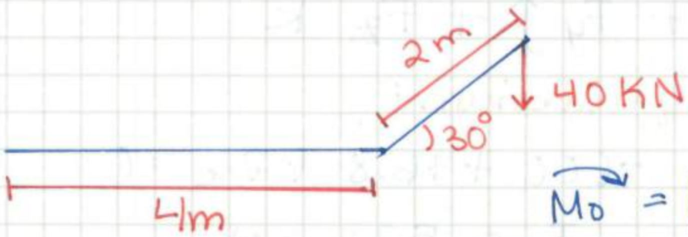
(b)



$\overset{\curvearrowright}{M_O} = F \cdot d$

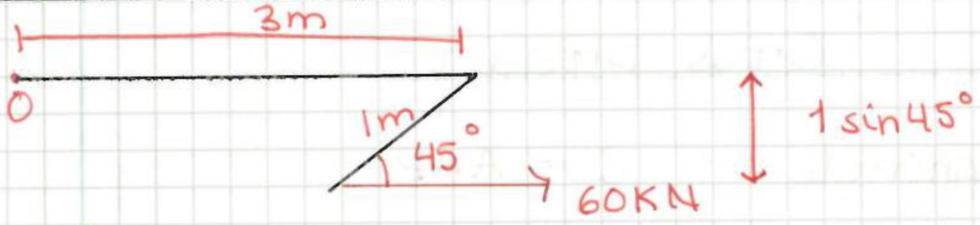
$= 50 \cdot 0,75 = \underline{37,5 \text{ Nm}}$

(c)



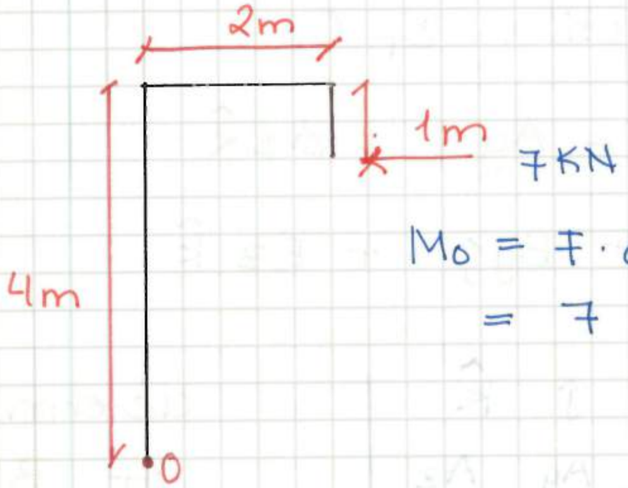
$\overset{\curvearrowright}{M_O} = (4 + 2 \cdot \cos 30^\circ) \cdot 40 \text{ KNm}$

d)



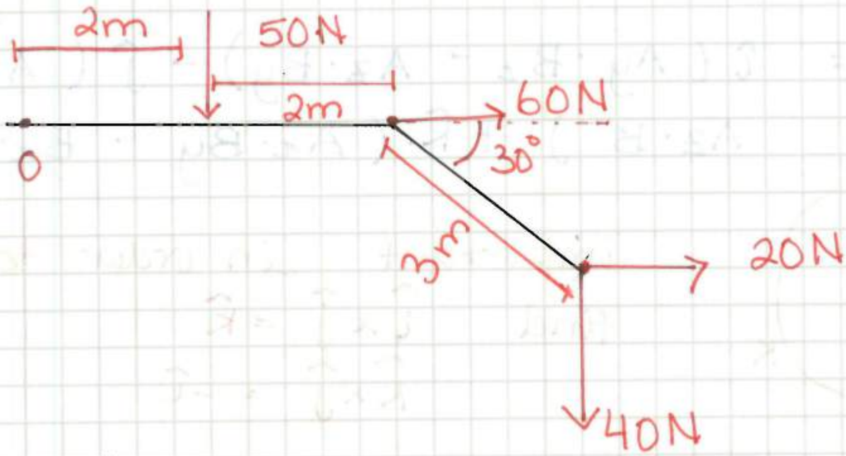
$$M_O = F \cdot d$$
$$= 60 \times 1 \sin 45^\circ = \underline{\underline{42{,}4 \text{ kNm}}}$$

(e)



$$M_O = F \cdot d$$
$$= 7 \cdot (4-1)\,m = \underline{\underline{21 \text{ Nm}}}$$

example:



$$M_{RO} = -(50N \cdot 2m) + (60 \cdot 0) - (20 \times 3 \sin 30)$$
$$-(40(4 + 3\cos 30^\circ))$$

$$= -324 \text{ Nm} = M_{RO} = 324 \text{ Nm}$$

4.2 Cross product

Magnitude = $C = A \times B$

Direction $C = A \times B = (AB\sin\theta)\, u_c$

u_c is the unit vector perpendicular to A & B vector and direction of C.

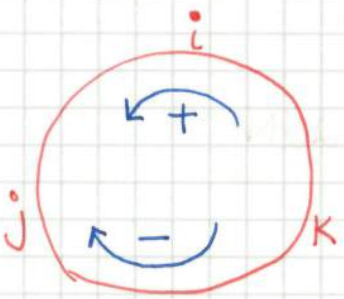
$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$A \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

determinant of 3x3

$$= \hat{i}\,(A_y \cdot B_z - A_z \cdot B_y) - \hat{j}\,(A_x \cdot B_z - A_z \cdot B\,) + \hat{k}\,(A_x \cdot B_y - B_x \cdot A_y)$$

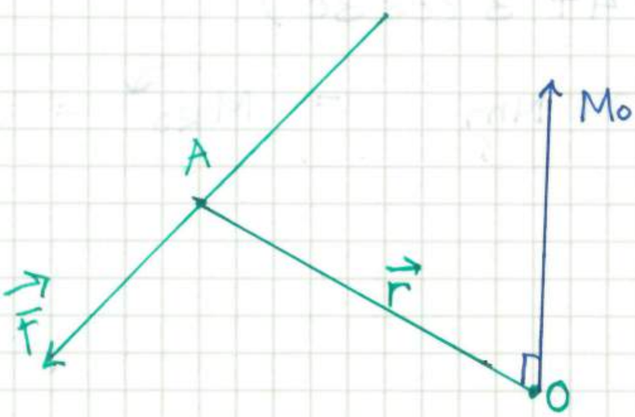


important in order to find $\hat{i} \times \hat{j} = \hat{k}$

$\hat{k} \times \hat{j} = -\hat{i}$

$M_O = rF\sin\theta$

$M_O = \vec{r} \times \vec{F}$



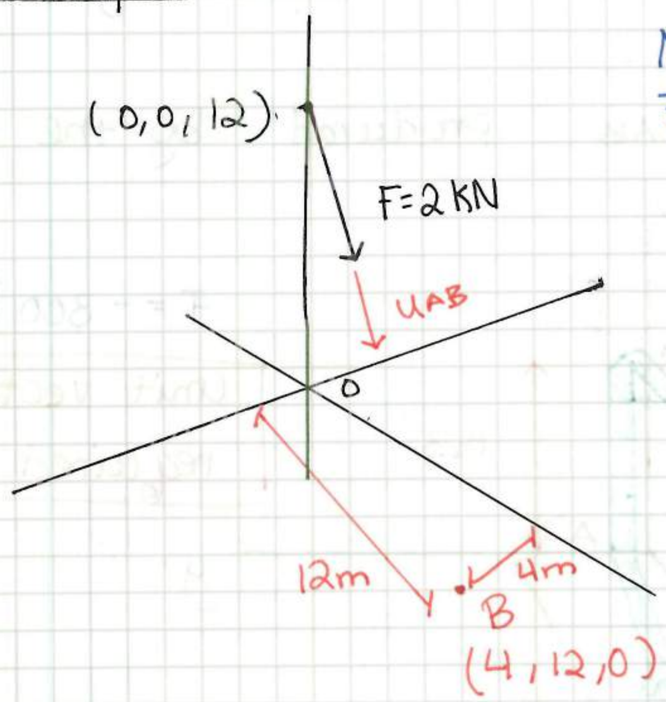
Example :

$$\vec{F} = (-6\hat{i} + 3\hat{j} + 10\hat{k})$$

$$r_{OB} = (0\hat{i} + 3\hat{j} + 1{,}5\hat{k})\,m$$

$$M_O = r \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & 1{,}5 \\ -6 & 3 & 10 \end{vmatrix} = (25{,}5\hat{i} - 9\hat{j} + 18\,K)\,Nm$$

Example :



$$M_O = \vec{r} \times \vec{F}$$

$$\vec{F} = F \cdot \vec{u_{AB}}$$

$$= \frac{2\,KN\,(4\hat{i} + 12\hat{j} - 12\hat{k})}{\sqrt{4^2 + 12^2 + 12^2}}$$

$$= 0{,}4588\hat{i} + 1{,}376\hat{j} - 1{,}376\hat{k}$$

$$\vec{r_{OA}} = 12\hat{k}$$

$$M_O = \vec{r_A} \times \vec{F}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 12 \\ 0{,}4588 & 1{,}376 & -1{,}376 \end{vmatrix} =$$

4,4 Principle of moment

Resolve the force vector, into 2 perpendicular components.

4,5 Moment of a force about a specified Axis:

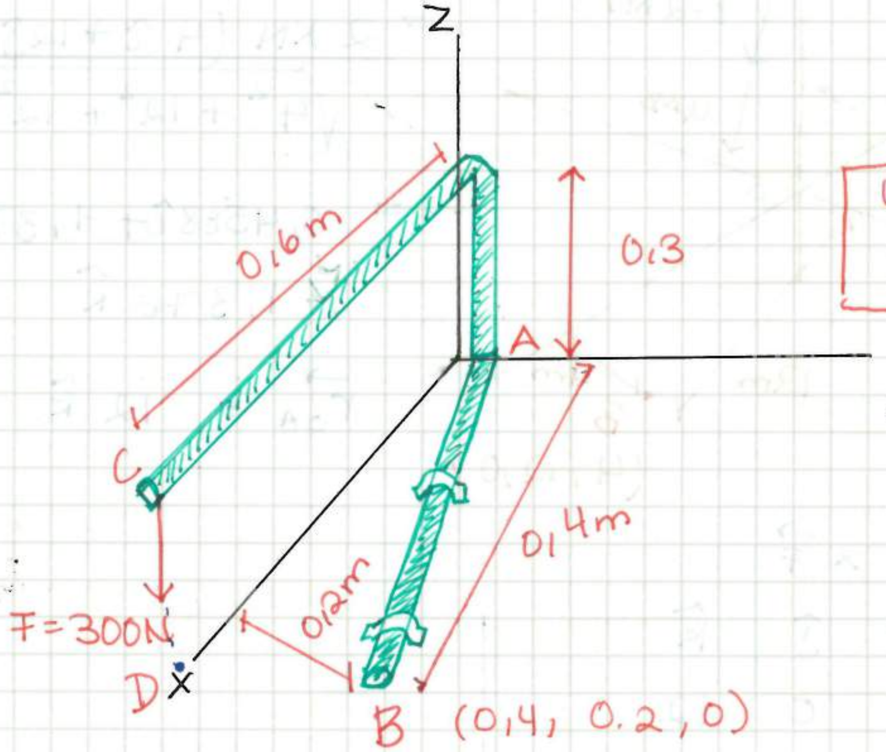
$M_a = F \cdot d_a$ (for any axis a)

d_a = perpendicular distance from the force line of action.

$$M_a = u_a (r \times F) = \begin{vmatrix} u_{ax} & u_{ay} & u_{az} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Example

Determine M_{AB} produced by the force F.



$F = -300\hat{k}$

unit vector of required axis

a) $M_{AB} = \vec{u}_{AB} (\vec{r}_D \times \vec{F})$

$$\vec{u}_{AB} = \frac{0{,}4\hat{i} + 0{,}2\hat{j}}{\sqrt{0{,}4^2 + 0{,}2^2}} = 0{,}89442\hat{j} + 0{,}4422\hat{j}$$

$$\vec{M}_{AB} = \vec{u}_{AB} (r_D \times \vec{F}) = \underline{(72c + 36\hat{j})\ Nm}$$

(2) moment about a specified (given) axis.

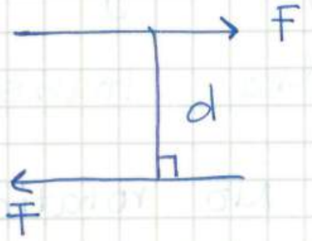
(3) $M_{axis} = M_a = M_o \cos\theta$

(4) $M_{axis} = \vec{u}_a \; (\vec{r} \times \vec{F})$

summary

4.6 Moment of a Couple

Couple - to paralell forces that have same magnitude but opposite directions, and seperated by a perpendicular distance d.



$M = r \times F$

expressed by vector cross product.

Example (7)

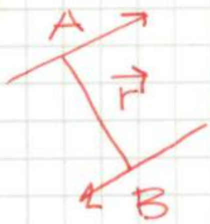
$M_1 = F \cdot d$

$= 150 \times 0{,}4 = 60 \text{ Nm } \hat{i}$

$M_2 = \vec{r}_{DC} \times \vec{F}_C = (0{,}3\hat{i}) \times \left[(125 \; 4/5)\hat{j} - (125 \; 3/5)\hat{k}\right]$

$= 22{,}5\hat{j} + 30\hat{k} \text{ Nm}$

$M_R = \vec{M}_1 + \vec{M}_2 = \underline{\underline{60\hat{i} + 22{,}5\hat{j} + 30\hat{k}}}$



$\vec{M}_C = (\vec{r} \times \vec{F})$

$\vec{r} = \vec{r}_B - \vec{r}_A$

Example 8

$$\Sigma M = -F_1 d_1 + F_2 \cdot d_2 - F_3 d_3$$

$$= -200 \cdot 0{,}4 + 450 \cdot 0.3 - 300 \cdot 0{,}5$$

$$= \underline{\underline{-95\ Nm}} \quad \text{(clockwise direction)}$$

20/1 Chapter 5 : Equilibrium of a Rigid body

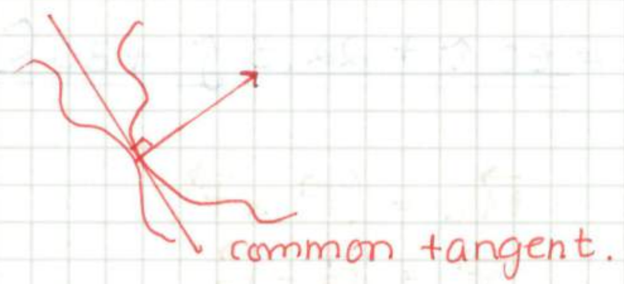
Rigid body - a large number of particles.

Equilibrium of a rigid body :

$\vec{F_R} = \Sigma F = 0$ No translation

$\curvearrowleft M_R = \Sigma M = 0$ No rotation

- External forces - weight (attractive force)
  - magnetic
- Reactions - between two contact surfaces
  - supports
- Strings / cabels / link



equilibrium in 2D :

roller - vertical forces are prevented.

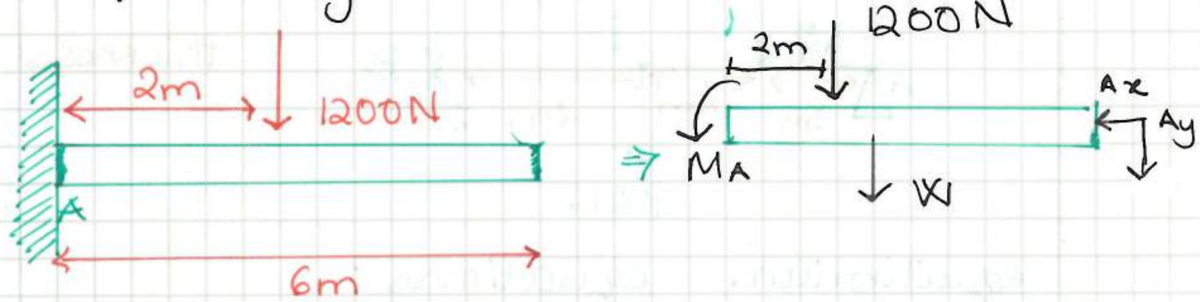
Pin - prevent translation in any direction.

Fixed - prevent both translation & Rotation.

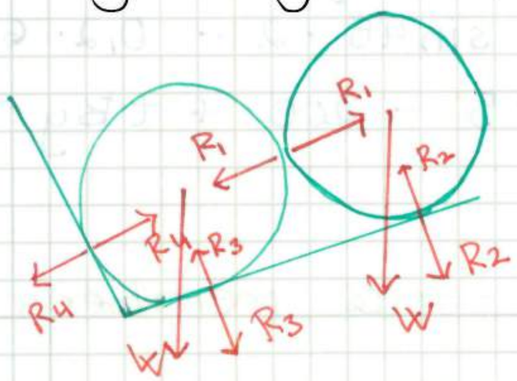
Require a complete specification of all the known and unknown external force that act on the body.

example :

(1) Draw free body diagram. The beam has a mass of 100 kg.



(2) Two pipe of mass 300 kg each are supported by the forked tine of the tractor. Draw free body diagram for each & both.



coplaner equilibrium problem

$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M_o = 0$ (2D)

Alternative sets of independent equilibrium

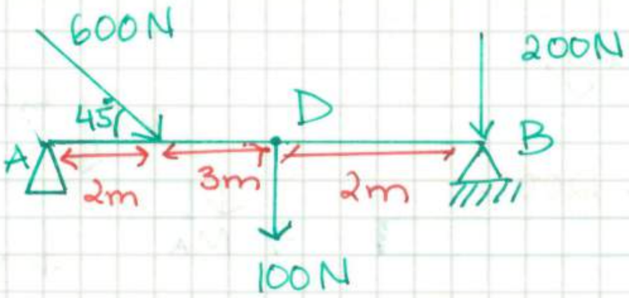
$\Sigma F_x = 0 \qquad \Sigma M_A = 0 \qquad \Sigma M_B = 0$

OR

$\Sigma M_A = 0 \qquad \Sigma M_B = 0 \qquad \Sigma M_C = 0$

example

Determine the horizontal & verticle component of reaction on the beam caused by the pin at B and the rocker at A. Neglect the weight of the beam.



thickness 0,2

equilibrium equations:

$\xrightarrow{+} \Sigma F_x = 600 \cos 45 - B_x = 0$

$+\uparrow \Sigma F_y = A_y + B_y - 600 \cdot \sin 45° - 200 - 100 =$

$\curvearrowleft M_A = -600 \sin 45° \cdot 2 - 0{,}2 \cdot 600 \cos 45°$

$- 100 \cdot 5 - 200 \cdot 7 + B_y \cdot 7 = 0$

$\underline{B_y = 405 \text{ N}}$

Two and three - force members

24/01

Summary Chapter 5

- Rigid body equilibrium – No translation ($\Sigma \vec{F} = 0$)
  – No rotation ($\Sigma \vec{M} = 0$)

Free body diagram
- External forces (eg. weight)
- Reactions (eg. contact surface / supports)

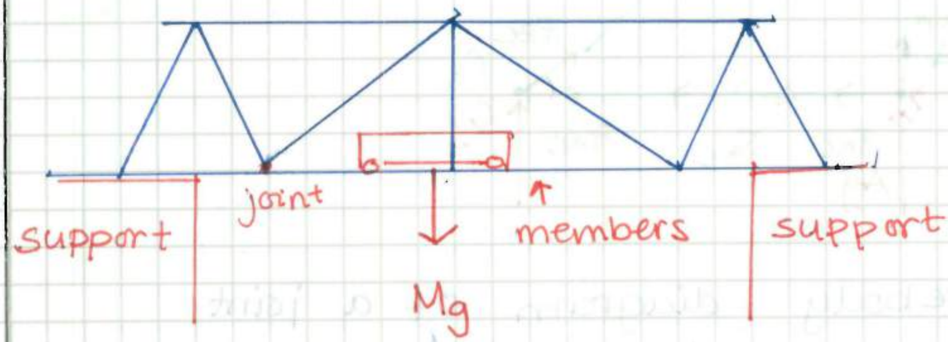
i)  roller support 

 rocker

ii)  pin

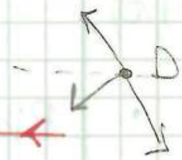
iii)  fixed

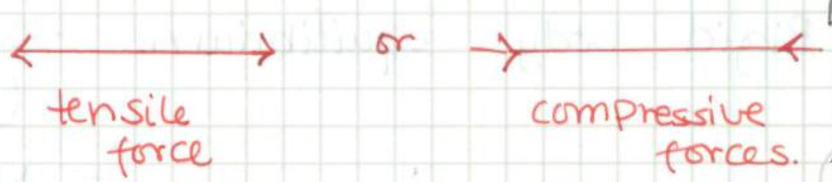
Truss (lattise structure)



Assume
- All external loading → acting at joint
- member joint – smooth pin

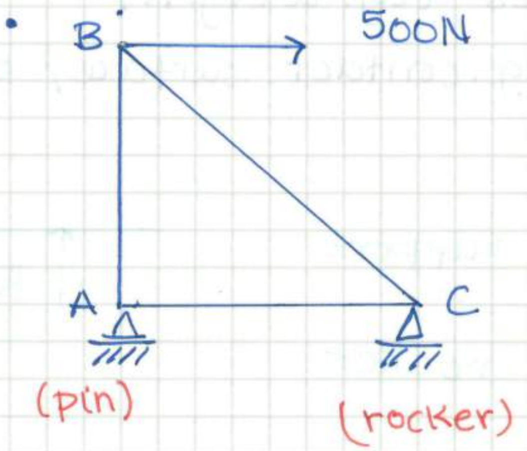
- member - two force members

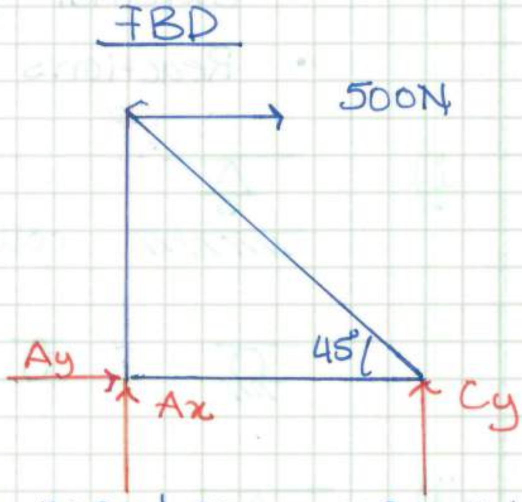




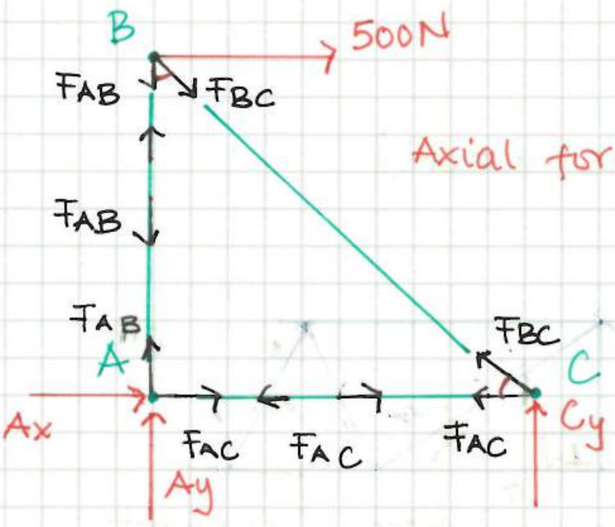
$\Sigma F_x =$
$\Sigma F_y =$

The method of joints



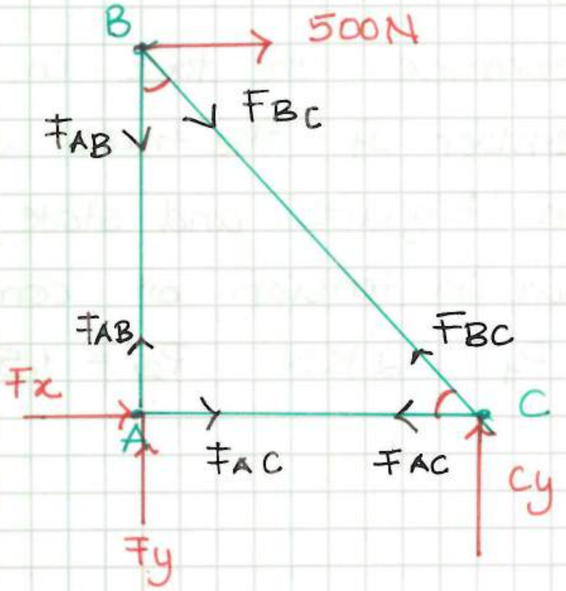


let's consider all the members are subjected to tensile forces.



Axial force of the member BC
$\doteq F_{BC}$

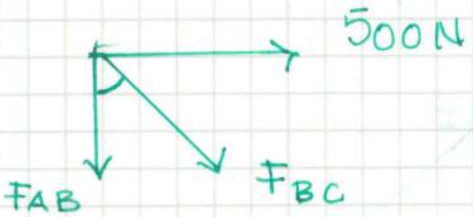
① Free body diagram of a joint
② sense of an unknown force
③ two force equilibrium equations



* Remember : All forces marked on members belong to joints.

considering equilibrium :

joint B :



$$\Sigma F_x = 500 + F_{BC} \cdot \sin 45^\circ = 0$$

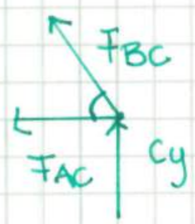
$$F_{BC} = -500/\sin 45 = \underline{\underline{-707.1\,N}}$$

(compression)

$$\Sigma F_y = -F_{AB} - F_{BC} \cdot \cos 45^\circ = 0$$

$$\underline{\underline{F_{AB} = 500\,N}}$$ (Tension, T)

joint C



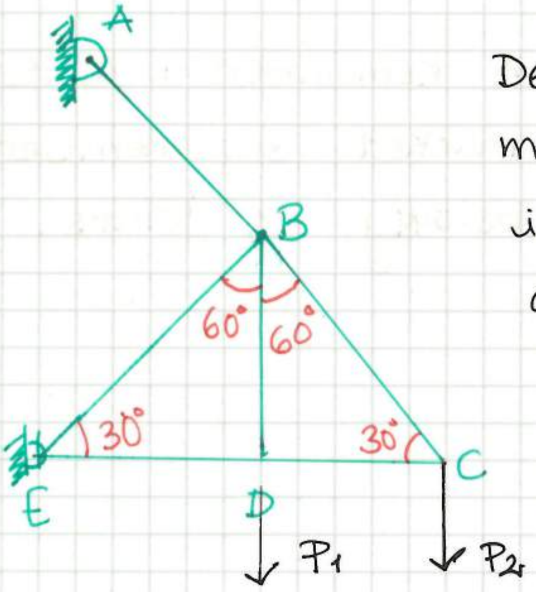
$$\Sigma F_x = -F_{BC} \cos 45^\circ - F_{AC} = 0$$

$$\underline{\underline{F_{AC} = 500\,N \quad (T)}}$$

Check :

2 unknown max

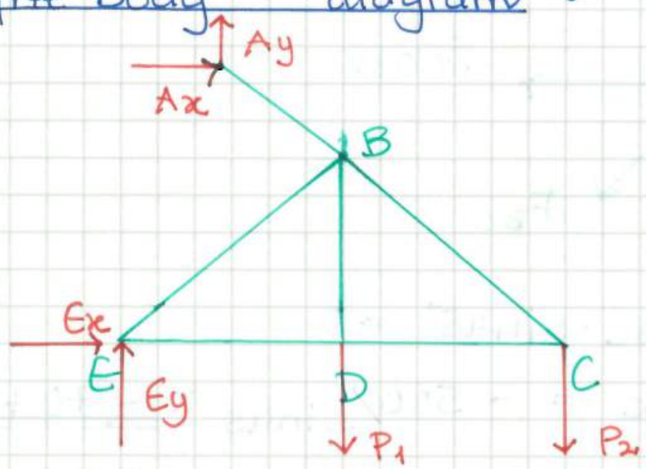
1 Known min



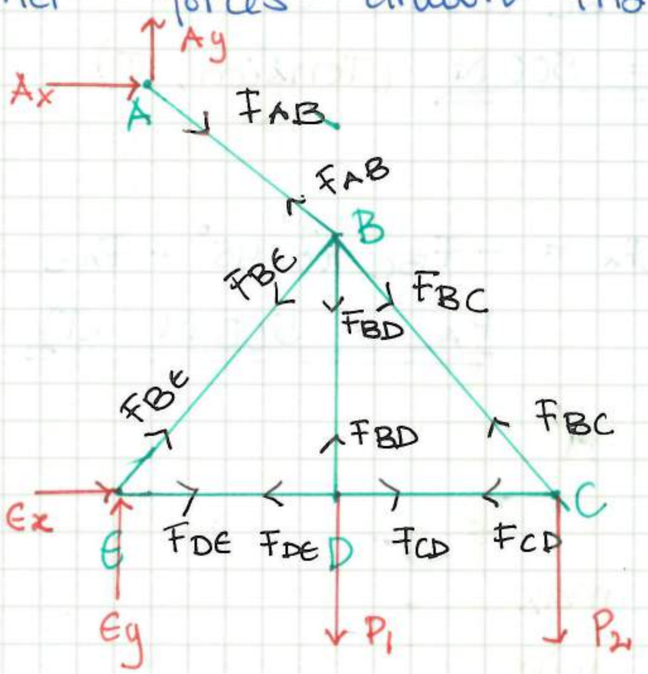
Determine the force in each member of the truss shown in figure and state if members are in tension or compression.

$P_1 = 2\,KN \qquad P_2 = 1{,}5\,KN$

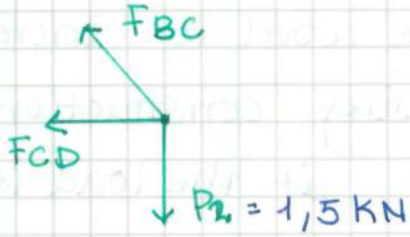
free body diagram :



other forces drawn that belongs to the joint :



joint C



$\Sigma F_x = -F_{CD} - F_{BC}\cos 30 = 0$

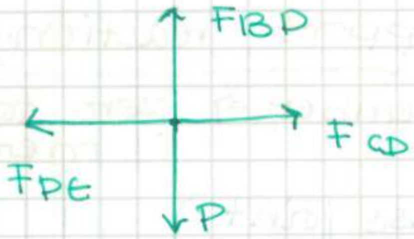
$F_{CD} = -F_{BC} \cdot \cos 30^\circ$

$= -2{,}598$ KN (C)

$\Sigma F_y = F_{BC} \cdot \sin 30^\circ - P_2 = 0$

$F_{BC} = 3$ KN (T)

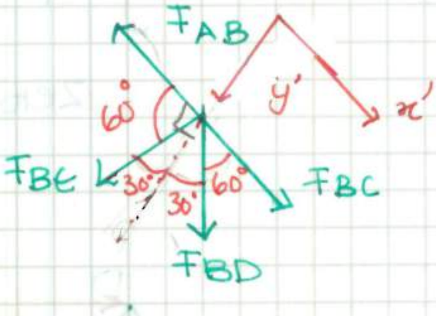
joint D



$\Sigma F_x = F_{DE} = -2{,}598$ KN (C)

$\Sigma F_y = F_{BD} = 2$ KN (T)

joint B



$\Sigma F_{x'} = 0$

$\Rightarrow F_{BC} - F_{AB} + F_{BD}\cos 60^\circ - F_{BE}\cos 60^\circ = 0$

$\Sigma F_{y'} = F_{BE} \cdot \sin 60 + F_{BD} \cdot \sin 60 = 0$

cos 30

$F_{BE} = -F_{BD} = -2$ KN (C)

$F_{AB} = 3 + (2 \times 2\cos 60^\circ)$

$F_{AB} = 5$ KN (T)

6.3 Zero-force members

Zero force members are used to increase the stability of truss during construction and to provide added support if the load is changed

(contine of the previous example...) ÷

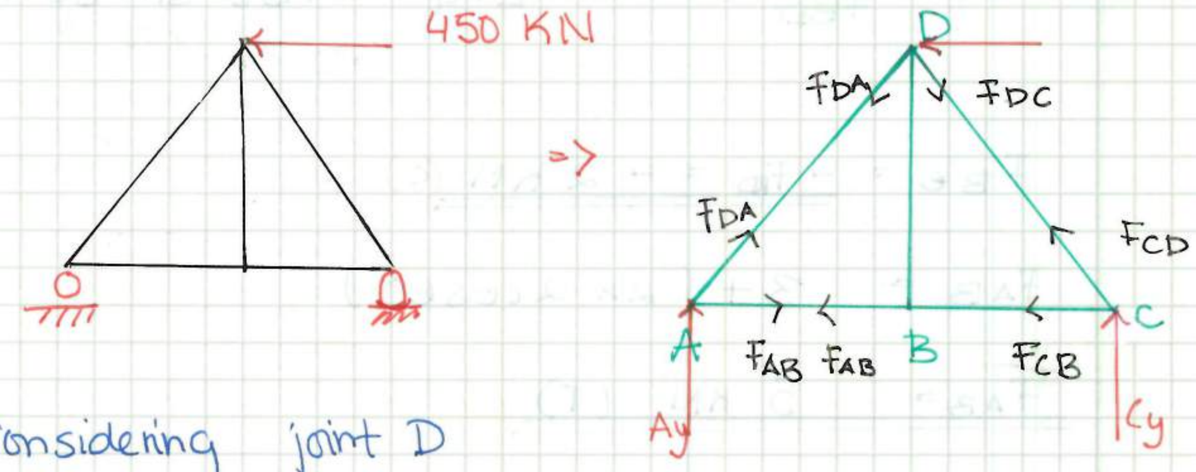
If only two members in truss &
No external forces &
No support reaction

$\underbrace{\qquad\qquad}$
Two members ⇒ zero force members
linear

only 3 members in truss joint
2 members - linear
No external forces
No support reaction

} 3rd member of the truss
⇓
Zero force member

example



considering joint D

$$\Sigma F_x = -450 - F_{AD}\sin 45^\circ + F_{CD}\cdot\sin 45 = 0$$

$$\Sigma F_y = -F_{AD}\cos 45^\circ - F_{CD}\cdot\cos 45^\circ = 0$$

$$F_{AD} = -F_{CD}$$

$2 F_{CD} \sin 45^\circ = 450$

$F_{CD} = \dfrac{450}{2 \sin 45^\circ} = \underline{\underline{318 \text{ KN (T)}}}$

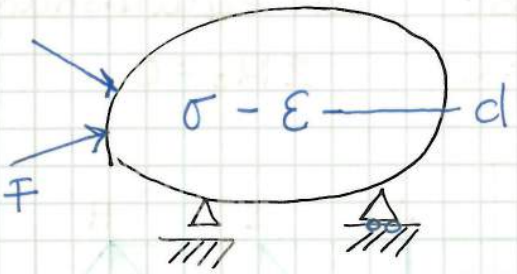
$\underline{\underline{F_{AD} = -318 \text{ KN (C)}}}$

Joint A :

$\Sigma F_x = F_{AB} + F_{DA} \cos 45^\circ = 0$

$\underline{\underline{F_{AB} = 225 \text{ KN(T)}}}$

27.01



Statics - chapter 6 - Structural analysis

objective :- Find internal forces of truss members.

Limitation - simple truss (2D)

Method 1 : - method of joint

(when we have to find alle member force)

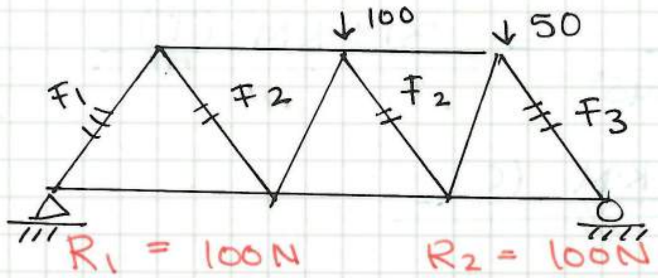
(i) joint equilibrium

(ii) tensile force → +ve

(iii) Sequence of considering equilibrium of joint } If 2 unkwon 4 known

we have to find support reaction

(iv) Symmetric Problem



- Geometry
- loading
- support reaction

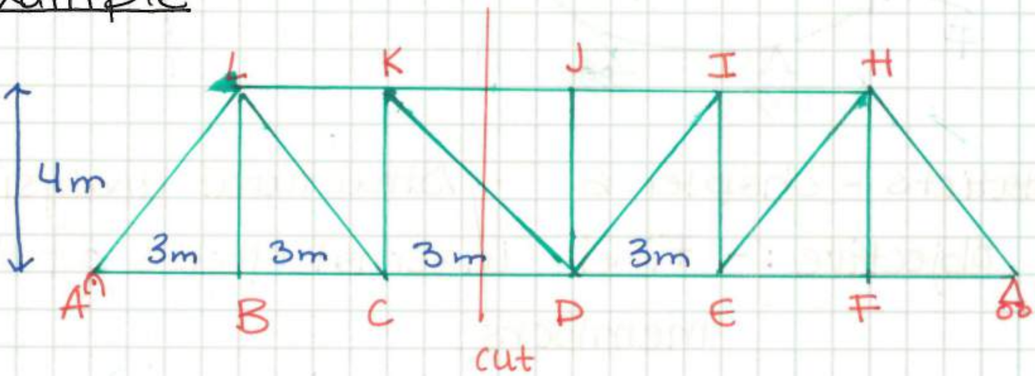
Method 2 - Method of Section

(when we need to find selected number of member force).

segments are also in equilibrium.

$\Sigma F_x = 0$ $\Sigma F_y = 0$ $\Sigma M_o = 0$

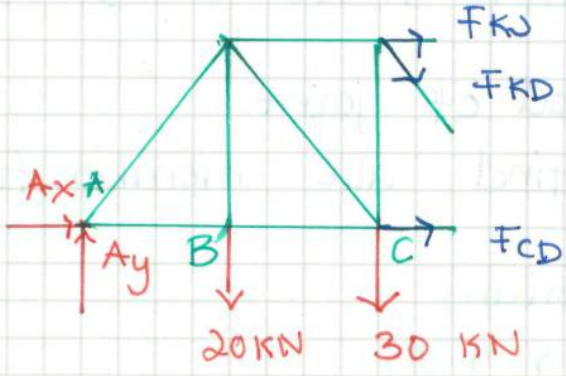
example



$F_{KJ} = ?$

$K_{KD} = ?$

$F_{CD} = ?$



considering entire truss

$\Sigma F_x = A_x = 0$

$\uparrow \Sigma F_y = A_y + G_y - 90 = 0$

$\circlearrowleft^{+} M_G = (40\times 9) + (30 \times 12) + (20 \times 15) - A_y \times 18$

$\underline{A_y = 56.7 \text{ KN}}$

$\curvearrowleft M_K = F_{CD} \cdot 4 + (20 \times 3) - (56{,}7 \times 6) = 0$

$F_{CD} = 70{,}1 \text{ KN (T)}$

$\uparrow \Sigma F_y = -F_{KD} \cos\theta - 30 - 20 + A_y = 0$

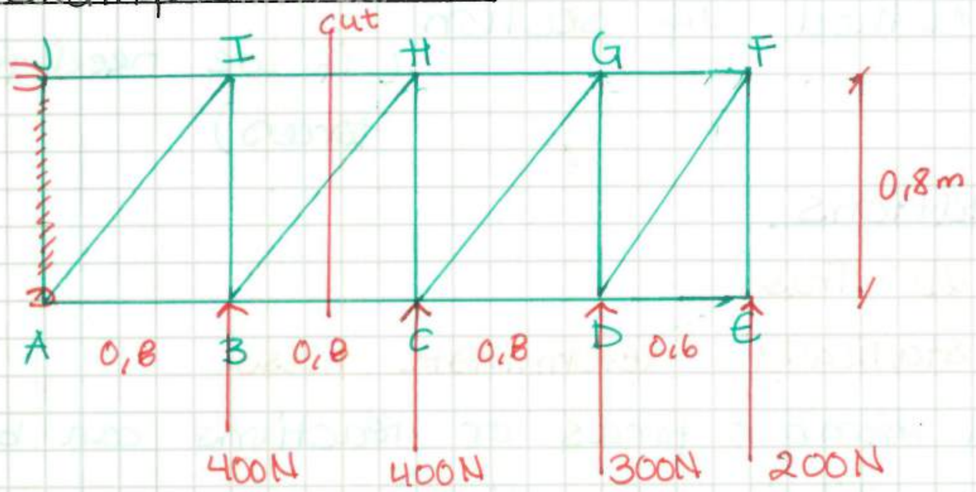
$F_{KD} = 8{,}38 \text{ (T)}$

$\rightarrow \Sigma F_x = F_{KJ} + F_{KD} \sin\theta + F_{CD} = 0$

$F_{KJ} = -75{,}125 \text{ KN}$

we cut between K J because we need to find F_{KJ}, always cut 3 member, because we only have 3 eq equation.

example 6 & 7

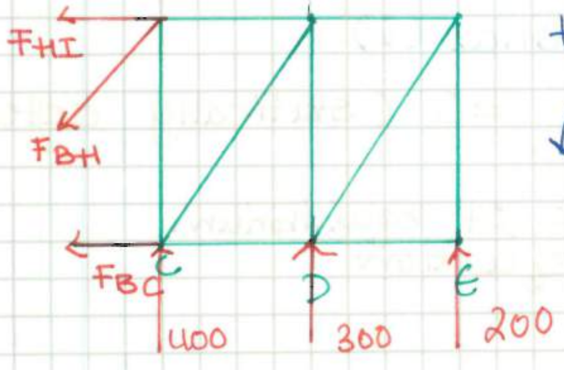


$F_{BC} = ?$

$F_{BH} = ?$

$F_{HC} = ?$

considering the right section. ("Less calc")



taking moment

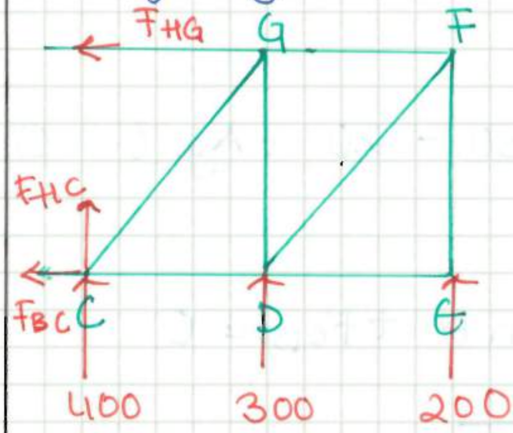
$\curvearrowleft M_H = -F_{BC} \cdot 0{,}8 + 300 \cdot 0{,}8 + 200 \times 1{,}4 = 0$

$F_{BC} = 650 \text{ N (T)}$

$\uparrow \Sigma F_y = 900 - F_{BH} \cos 45^\circ = 0$

$F_{BH} = 1273 \text{ N (T)}$

Imaginary cut 2 :



$\Sigma F_y = 400 + 300 + 200 + F_{HC}$

$\underline{F_{HC} = -900\,N}$ (C)

Truss - Internal forces
(axial forces) members

(1) Method of joints (if want to find all member forces)

(2) Method of section (if we need few member forces)

Limitations

- 2D Truss
- statically determinate truss

(if all member forces or reactions can be found by equilibrium equation)

Degree of indeterminance (i)

$i = \underbrace{(m+r)}_{\text{no. of unknown}} - \underbrace{2j}_{\text{no. of equilibrium equation}} = 0$ (statically determinate)

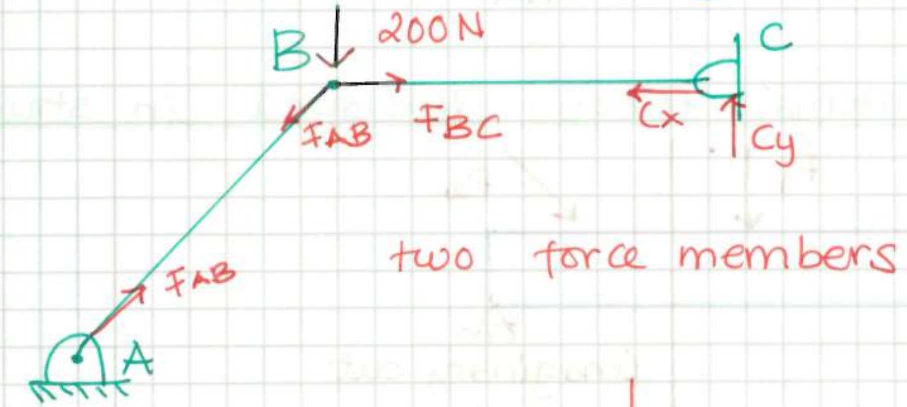
j : no. of joints

m : no. of member

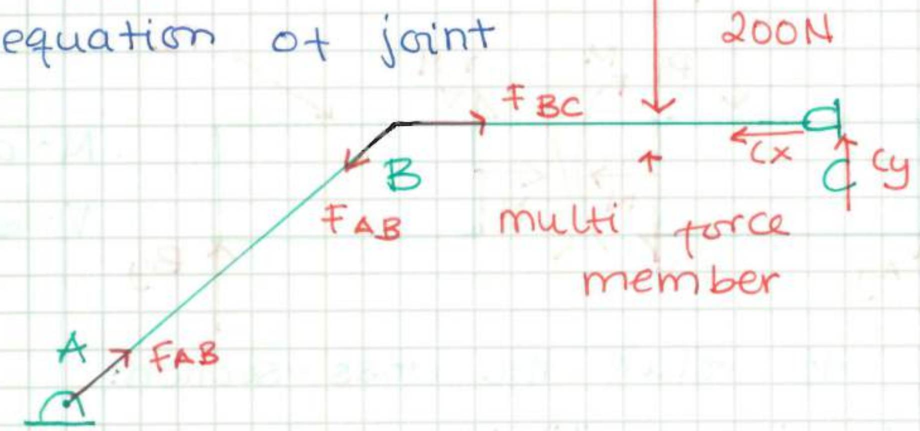
r : no. of reaction

6.6 Frames and Machines

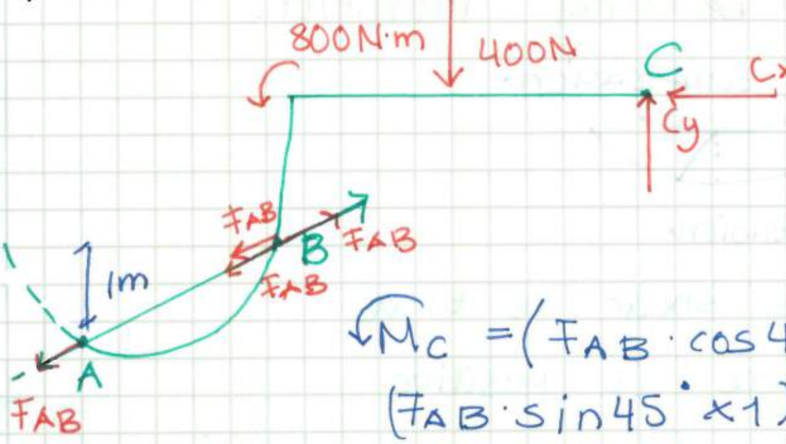
Frames are generally stationary.
Machines contains moving parts.



equation of joint



example (8)



$\curvearrowleft M_C = (F_{AB} \cdot \cos 45^\circ \times 3) - (F_{AB} \cdot \sin 45^\circ \times 1) + 800 + 400 \times 2$

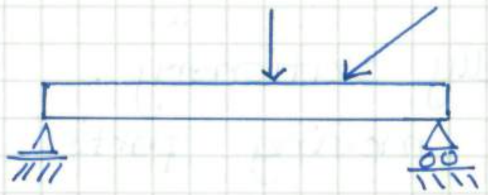
$\underline{\underline{F_{AB} = 1131\ N}}$

$\Sigma F_x = -C_x + 1131 \sin 45^\circ = 0$

$\underline{\underline{C_x = 800\ N}}$

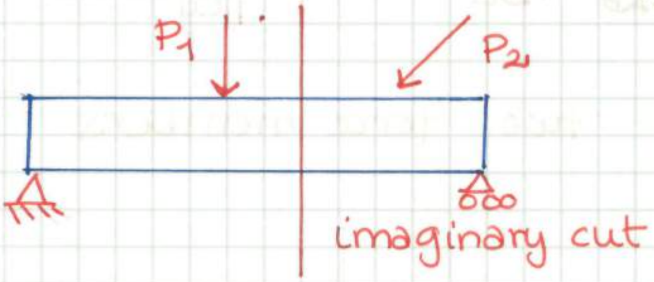
$\Sigma F_y = -C_y + 1131 \cos 45^\circ - 400 = 0 \Rightarrow \underline{\underline{C_y = 400\ N}}$

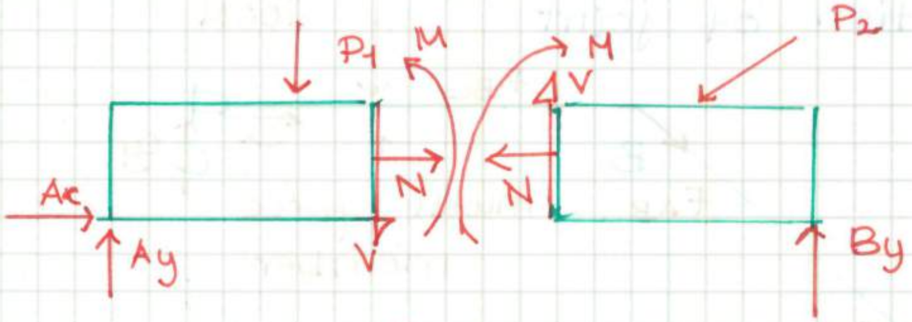
31/01 chapter 7 : Internal forces



multi force members
(internal forces)

7,1 - Internal forces developed in structural member



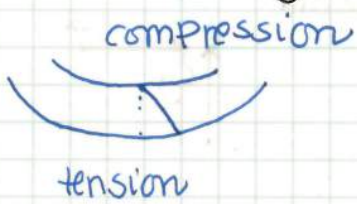


N - axial force
V - shear force

Tendency to rotate the cross-section.

Segment in equilibrium, 3 eq. for 2D problems

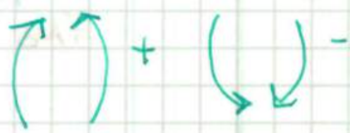
more ab. bending moment.



Clockwise shear is +ve

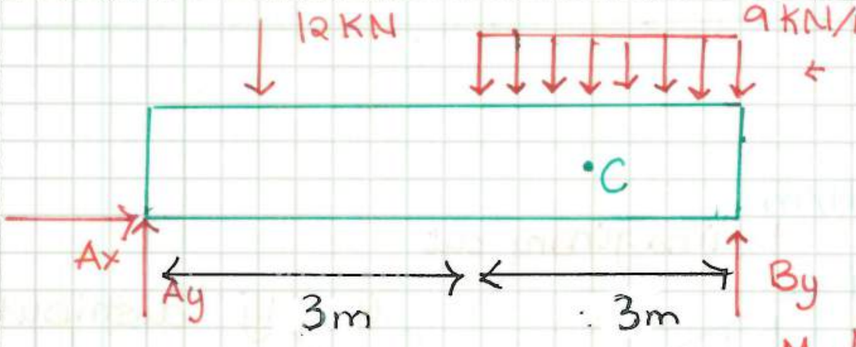
axial force is positive

upward bending moment positive



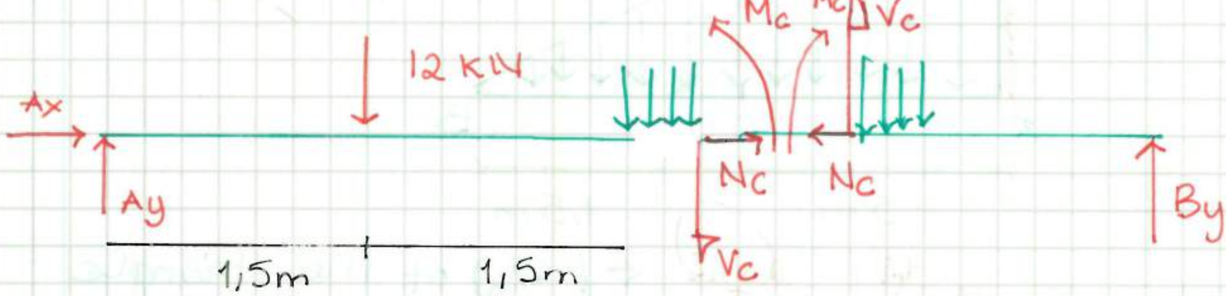
example

Determine the normal force, shear force and moment at point C.



uniform distributed load.

$9 \times 3 = 27$ KN



Applying equations of equilibrium - FBD

$\overset{+}{\rightarrow} \ \Sigma F_x = A_x = 0$

$+\uparrow \ \Sigma F_y = A_y + B_y - 12 - 27 = 0$

$M_A = -(1,5 \cdot 12) - (27 \cdot 4,5) + B_y \cdot 6 = 0$

$\underline{\underline{B_y = 23,25 \text{ KN}}}$

By using the right segment, we can write equilibrium equations to solve this problem.

Apply equilibrium equations

$\rightarrow \Sigma F_x = -N_c = 0 \Rightarrow \underline{N_c = 0}$

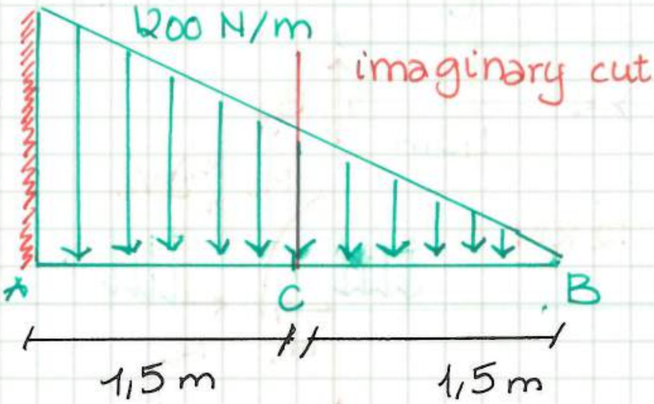
$\uparrow \Sigma F_y = B_y + V_c - (9 \times 1,5) = 0$

$\underline{\underline{V_c = -9,75 \text{ KN}}}$

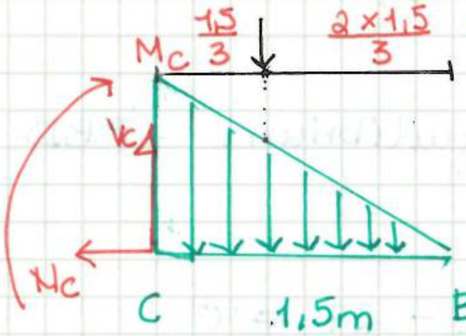
Bending moment around $C^{\circlearrowleft +} = -M_c - (9 \times 1,5) \times 1,5/2 + (23,25 \times 1,5) = 0$

$\underline{\underline{M_c = 24,75 \text{ KNm}}}$

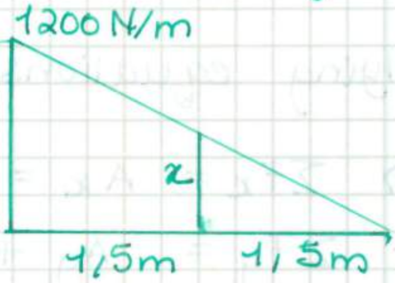
Example 2



Linearly distributed load

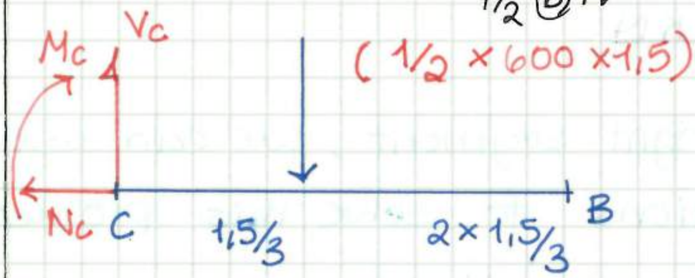


← (2/3) of the triangle



$\frac{x}{1200} = \frac{1,5}{3} \Rightarrow x = 600\ N/m$

force
$\frac{1}{2}\,b\,h$

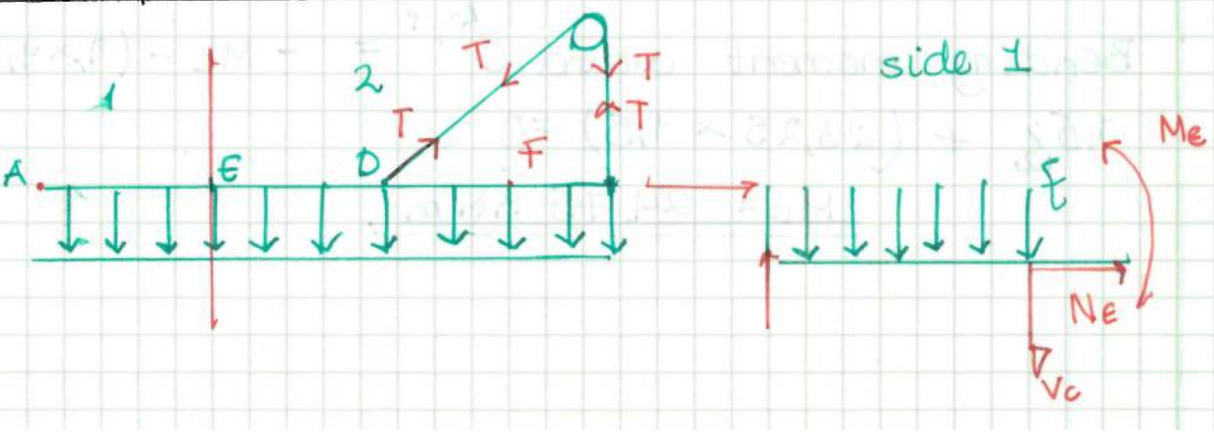


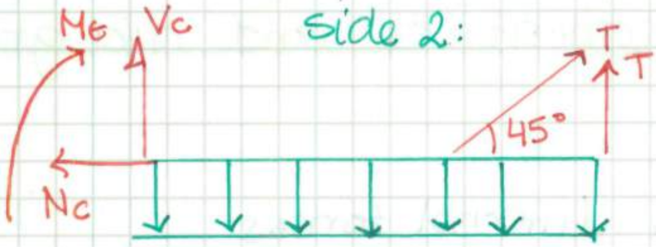
unknown M_C, N_C, V_C.

agian solving with three equations

$N_C = 0$ $\quad V_C = -450\ N$ $\quad M_C = -225\ N$

example 3





we look at side 1 & 2.

$$\curvearrowleft M_A = (T \times 6) - (1800 \cdot 3) + (T \sin 45 \cdot 3)$$

$$\overset{+}{\rightarrow} \Sigma F_x = A_x + T \cos 45^\circ = 0$$

$$+\uparrow \Sigma F_y = A_y + T \cdot \sin 45^\circ + T = 0$$

$A_x = -470.17\,N$ and $A_y = 664.92\,N$

Again using the same method of 3 eq.
we find that

$N_E = -470.17\,N$ $\quad$ $M_E = 660\,Nm$ $\quad$ $V_E = 215\,N$

7.2 Shear and moment Equations and Diagrams

Simply supported beam : pin at one end roller at other end.

Cantileaver beam : fixed at one end and free at the other.

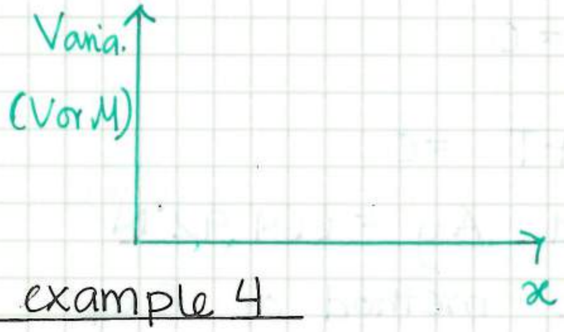
Beam with an overhang :

Type of loads :

- Concentrated load (single force)
- couple
- Distributed load
  - ↳ uniform
  - ↳ linear
- Reactions

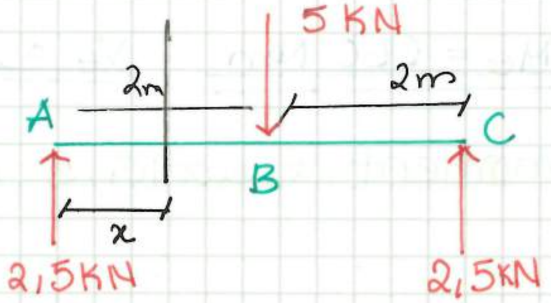
7.2. Shear and moment eq. and diagrams

Objective - To find
maximum internal forces
(maxiumum SF)
(maximum axial forces)

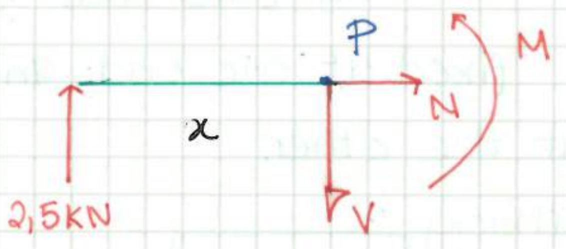


$V = f(x)$
$M = f(x)$

example 4



(1) Imaginary cut $0 \leq x < 2$

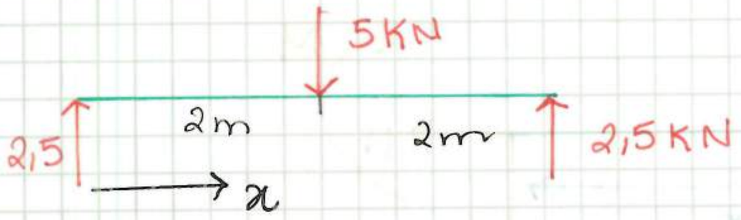


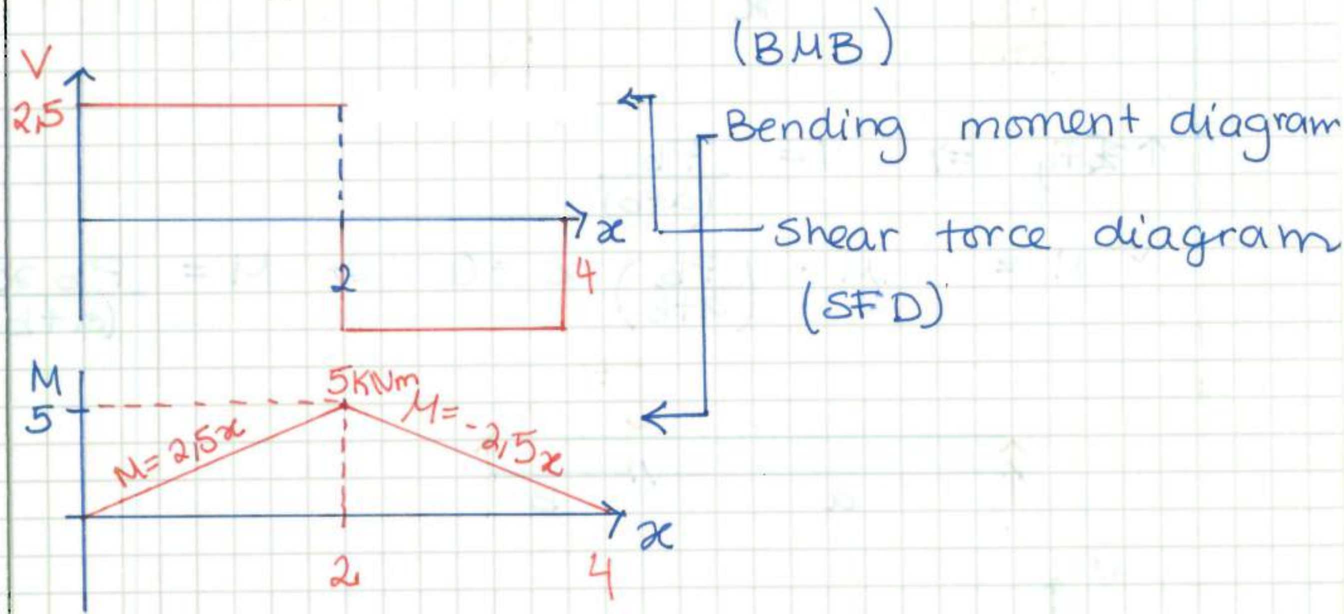
Apply equation of equilibrium

$\uparrow \Sigma F_y = 2,5 - V = 0 \Rightarrow V = 2,5 \text{ KN}$

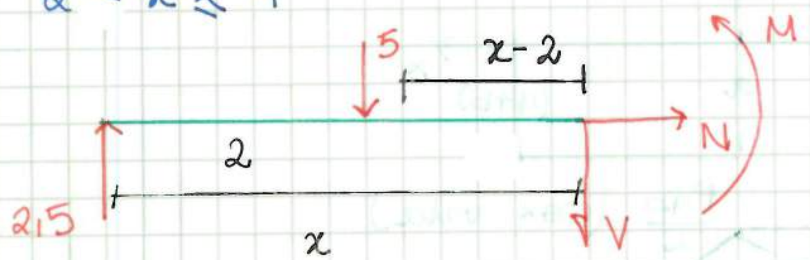
$\circlearrowleft M_p = M_p - (2,5 \times x) = 0$

$M_p = 2,5x$





$2 < x \leq 4$



Appyling equation of equilibrium

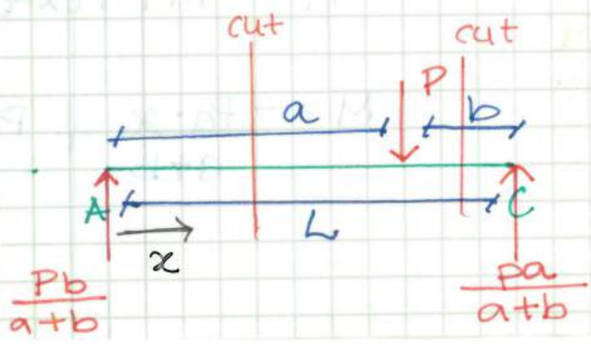
$$\uparrow \Sigma F_y = 2{,}5 - V - 5 = 0$$

$$\underline{\underline{V = -2{,}5 \text{ KN}}}$$

$$\circlearrowleft M) = M + [5 \times (x-2)] - (2{,}5 \times x) = 0$$

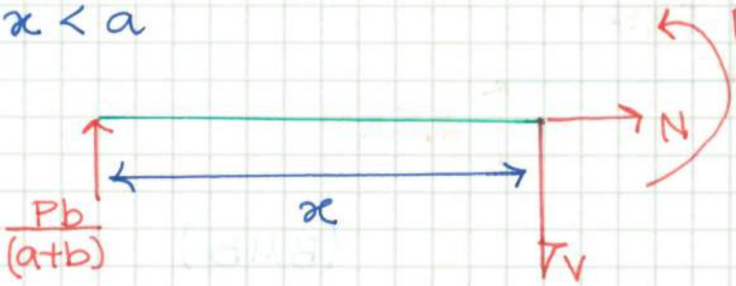
$$M = -2{,}5x + 10$$

example 5



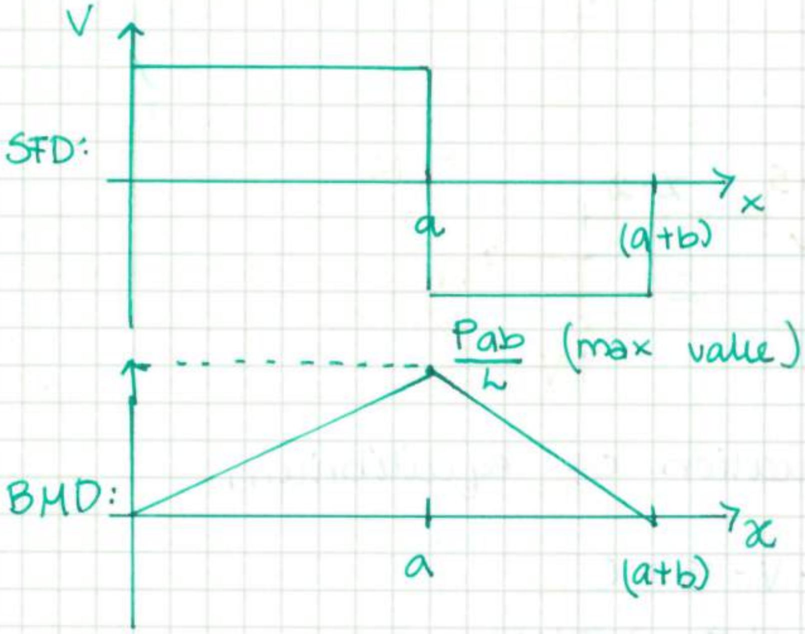
If there are many loads, you have to cut several times.

$0 \leq x < a$

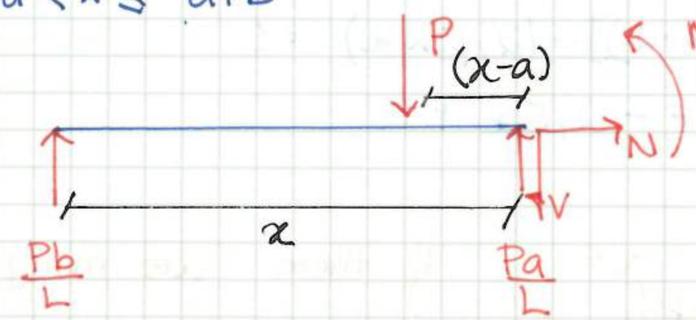


$\uparrow F_y \Rightarrow V = \dfrac{Pb}{(a+b)}$

$\curvearrowleft M = M - \left(\dfrac{Pb}{a+b}\right) x = 0 \Rightarrow M = \dfrac{Pbx}{(a+b)}$



$a < x \leq a+b$



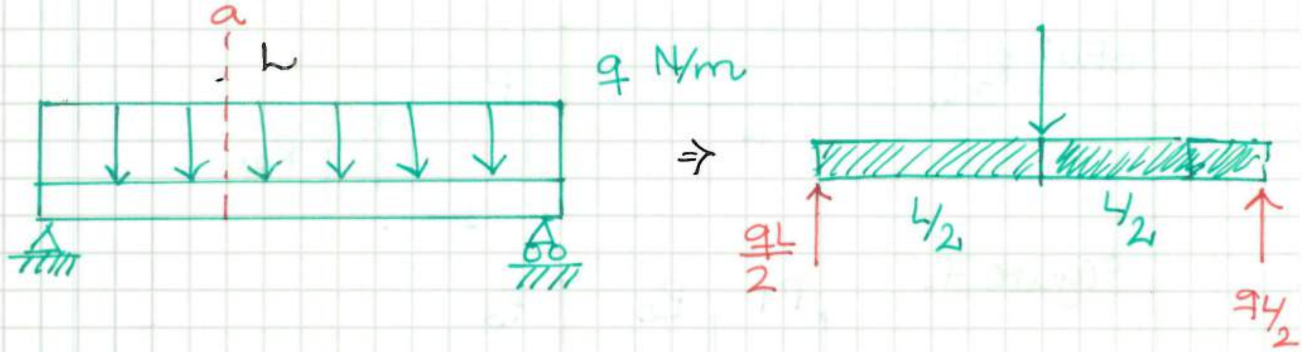
$V = \dfrac{-Pa}{a+b}$

$\curvearrowleft M = M + P(x-a) - \left[\dfrac{Pb}{a+b} \times x\right]$

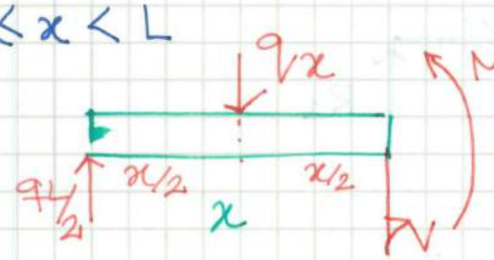
$M = -\dfrac{Pa \cdot x}{a+b} + Pa$

3/02

Example 6



$0 \leq x < L$



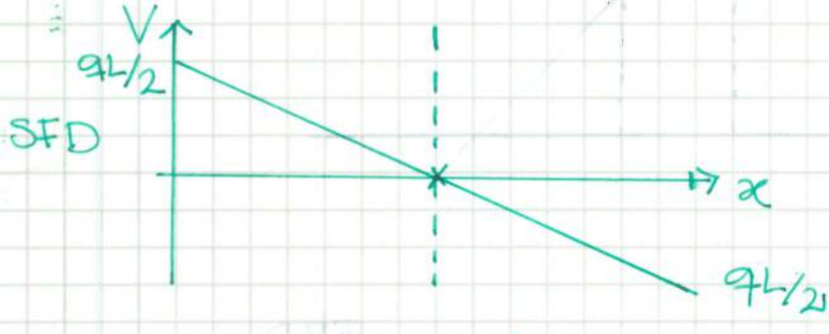
$$+\uparrow \Sigma F_y = \frac{qL}{2} - qx - V = 0$$

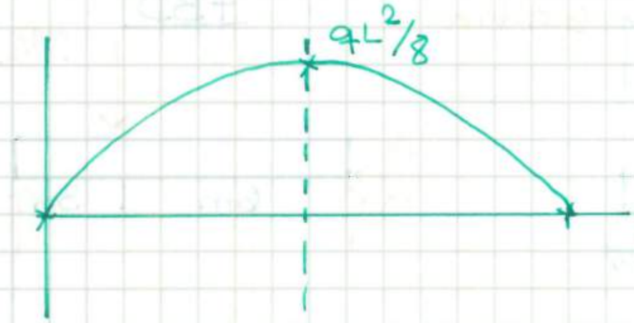
$$V = \frac{qL}{2} - qx$$

moment around the imaginary cut.

$$M + \left(qx + \frac{x^2}{2}\right) - \frac{qL}{2}x = 0$$

$$M = \frac{qL}{2}x - \frac{q x^2}{2}$$

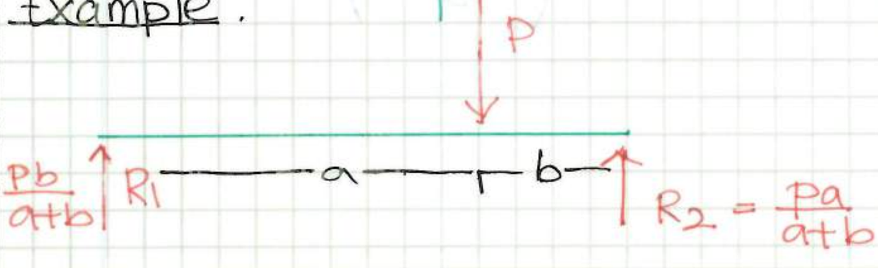




$$\frac{dM}{dx} = 0$$

$\frac{d^2M}{dx^2}$ $\lessgtr$ decide the curvature

Example.



SFD:

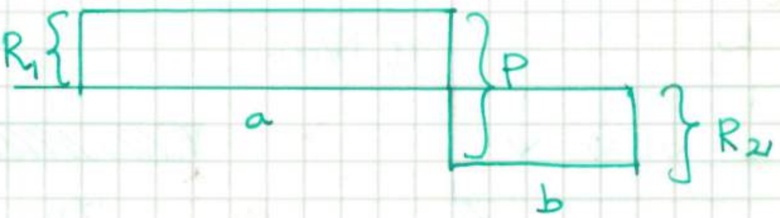
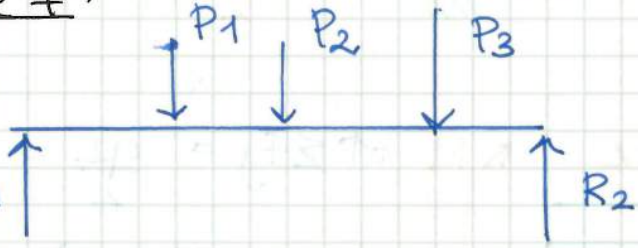
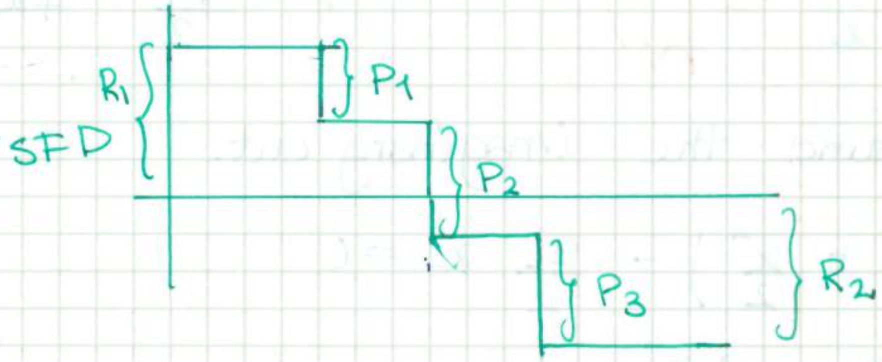
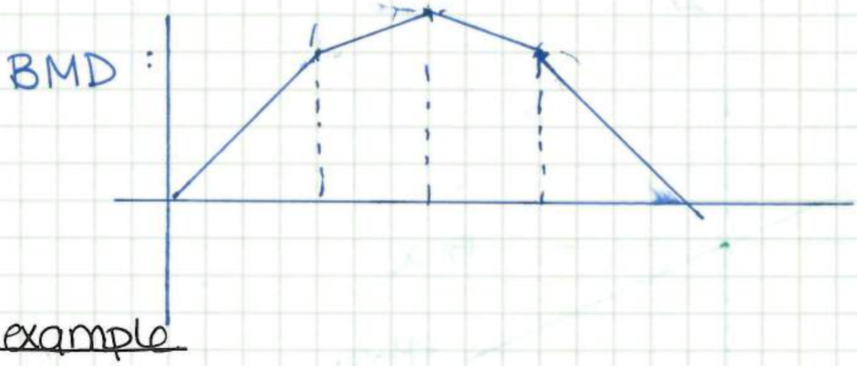


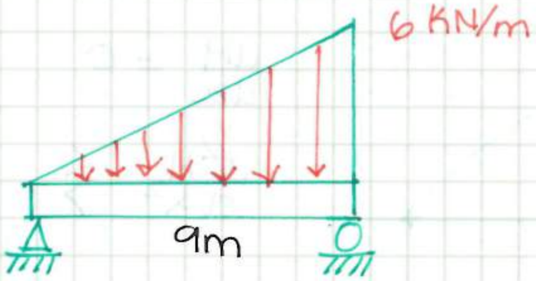
Figure 7:

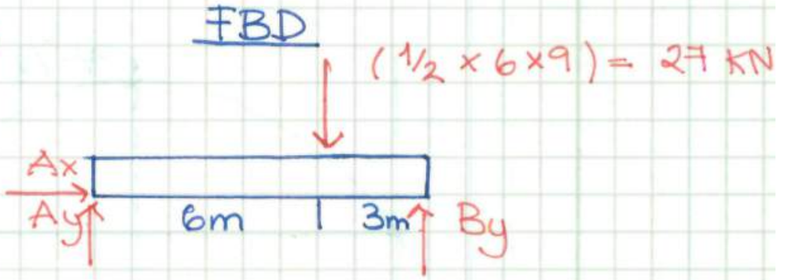






example

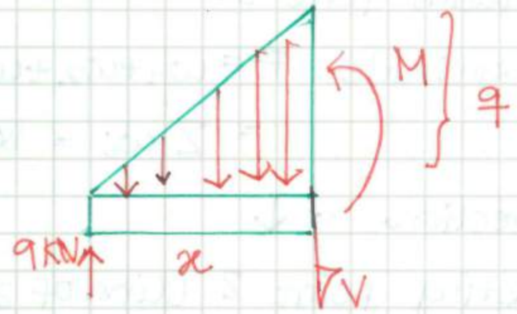




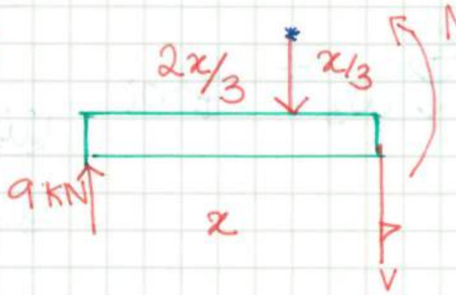
$\left(\frac{27 \times 3}{9}\right) = \underline{9\ KN}$ $\quad$ $\left(\frac{6 \times 27}{9}\right) = \underline{18\ KN}$

example

$$* \left(\tfrac{1}{2} q x\right) = \tfrac{1}{2} \times \tfrac{2}{3} \cdot x^2 = \frac{x^2}{3}$$



$\Rightarrow$



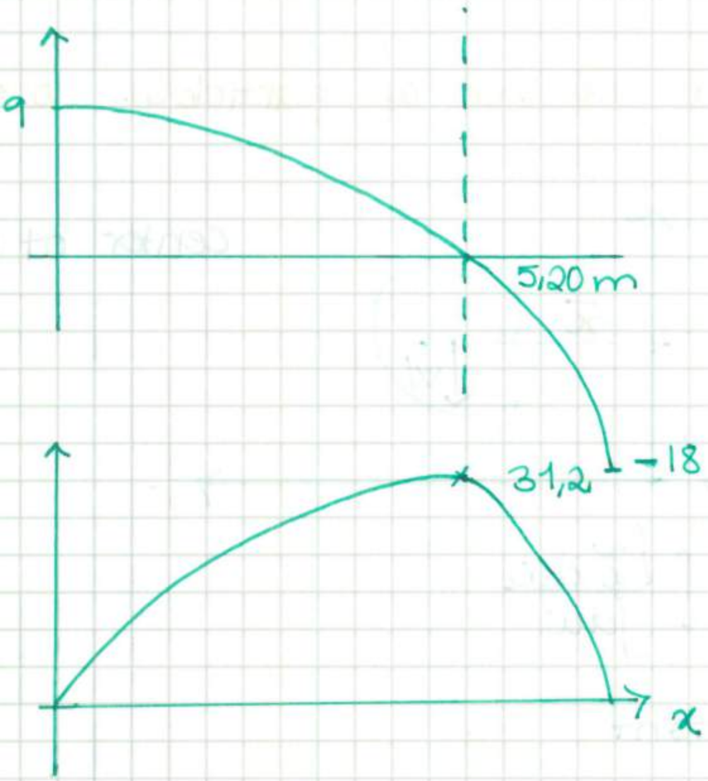
$$\frac{q}{x} = \frac{6}{9} \Rightarrow q = \tfrac{2}{3} x$$

$$+\uparrow \Sigma F_y = 9 - \frac{x^2}{3} - V = 0 \Rightarrow V = 9 - \frac{x^2}{3}$$

$$\curvearrowleft M = M + \left(\frac{x^2}{3} \times \frac{x}{3}\right) - (9x)$$

draw these two eq. in a graph.

$$M = \left(9x - \frac{x^3}{9}\right)$$



SFD curve

BMD curve

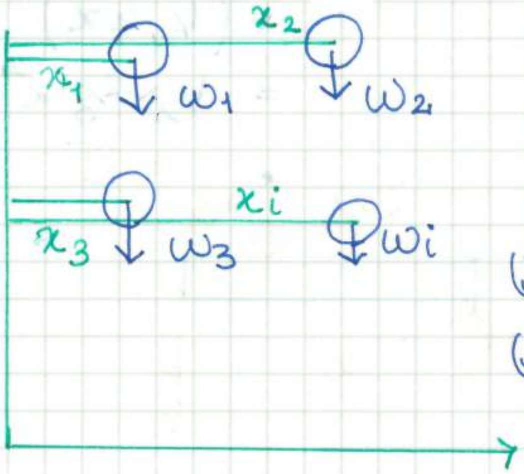
Relation between distributed load and shear :

$$+\uparrow \Sigma F_y = 0 \qquad \frac{dV}{dx} = w(x)$$

slope of shear diagram

distributed load intensity

Chapter 9

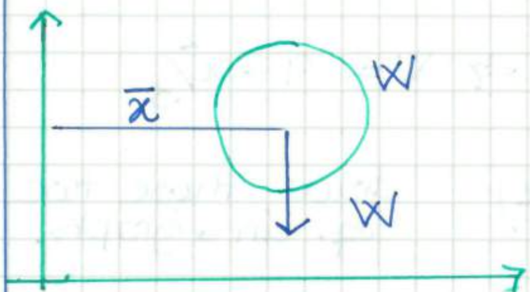


Resultant force i.

(i) magnitude $F = w_1 + w_2 + w_2 \dots + w_i$
$= \Sigma w_i = W$

(ii) direction $- \downarrow$

(iii) acting point ? (Line of action)



$\Rightarrow w_1 \cdot x_1 + w_2 x_2 + w_3 x_3 + \dots w_i$

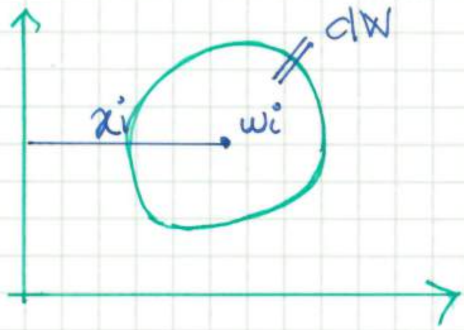
$\Rightarrow \Sigma w_i x_i$

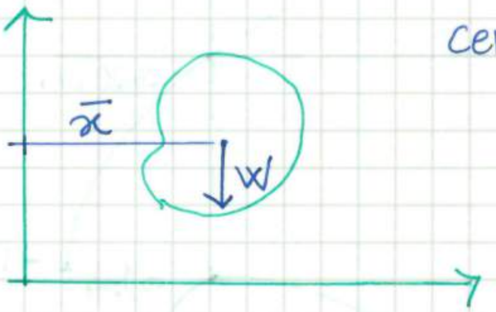
Taking moment

$= \Sigma w_i x_i = W \bar{x}$

$$\bar{x} = \frac{\Sigma w_i x_i}{W}$$

- body has infinite number of particles. $0 < i < \infty$





center of gravity

$$\bar{x} = \frac{\Sigma w_i x_i}{W} = \frac{\int \tilde{x}\, dw}{\int dw}$$

$W = mg$ - g - constant

$$\bar{x} = \frac{\int \tilde{x} \cancel{g}\, dm}{\int \cancel{g}\, dm}$$

Center of gravity $\downarrow$ another form

Center of mass

constant density ρ , $dm = \rho \cdot dv$

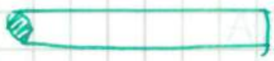
$$\bar{x} = \frac{\int \tilde{x}\, dm}{\int dm} = \frac{\int x \cdot dv}{\int dv}$$ (centroid of volume

Homogenous material

thickness - constant

$$\bar{x} = \frac{\int \tilde{x} \cdot \cancel{\rho} \cdot \cancel{t} \cdot dA}{\int \cancel{\rho t} \cdot dA} = \frac{\int \tilde{x} \cdot dA}{\int dA}$$

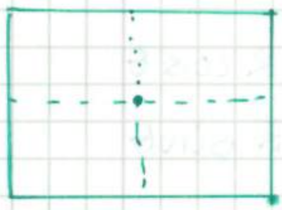
centriod of Area.



a - constant area

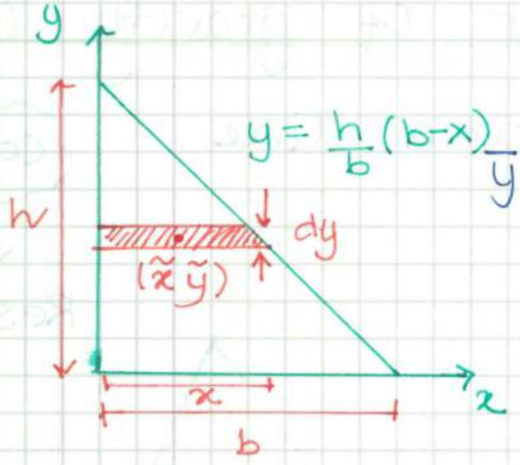
centriod of Line

$$\bar{x} = \int \tilde{x}\, dL \Big/ \int dL$$



Centeriod point in the center of the rectangle, when the figure is symmetric.

Example



$y = \frac{h}{b}(b-x)$

$x = \frac{b}{h}(h-y)$

$$\bar{y} = \frac{\int \tilde{y}\, dA}{\int dA}$$

$\tilde{y} = ?$ $dA = ?$

$\tilde{y} = y$ very sma

$dA = (dy \times x)$

$$\bar{y} = \frac{\int_0^h y \cdot x \cdot dy}{\int_0^h x\, dy}$$

$$= \frac{\int_0^h y \cdot \left[\frac{b}{h}(h-y)\right] dy}{\int_0^h b/h\,(h-y)\, dy}$$

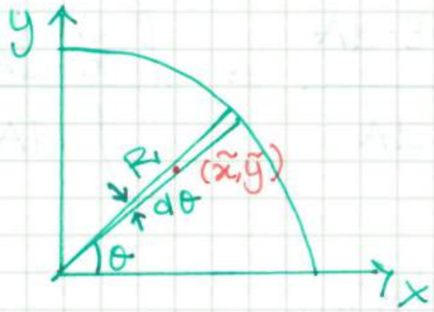
$$\bar{y} = \frac{by^2}{2} - \frac{b}{h}\frac{y^3}{3} \Big|_0^h$$

$$\underline{\underline{\bar{y} = h/3}}$$

centroid triangle

any triangular shape, the centriod can be calculated from height.

example

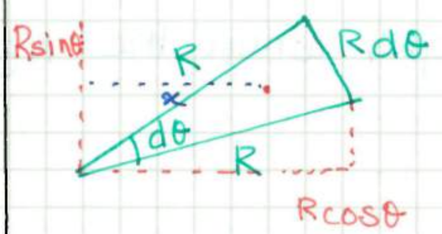


$$\bar{x} = \frac{\int \tilde{x} \cdot dA}{\int dA} \qquad \tilde{x} = ?$$

$$\bar{y} = \frac{\int \tilde{y} \cdot dA}{\int \cdot dA} \qquad \tilde{y} = ?$$

$dA = ?$

Note : dA - have to select a small element
- find the centriod of dA $(\tilde{x},\tilde{y})$



$$\tilde{x} = 2/3 \cdot R\cos\theta$$

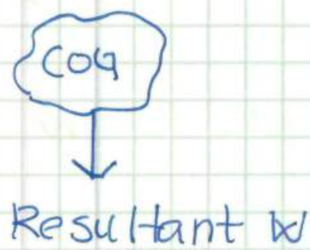
$$\tilde{y} = 2/3\ R\sin\theta$$

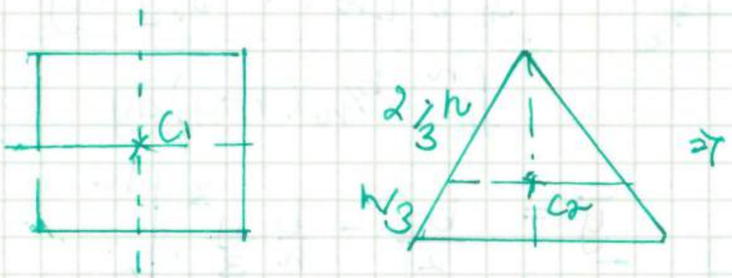
$$dA = \frac{1}{2} \cdot R^2 \cdot d\theta$$

7/2 chapter 9 : center of gravity (COG)

centriod of volume area line

$W = g \cdot m = g \cdot V \cdot \rho$



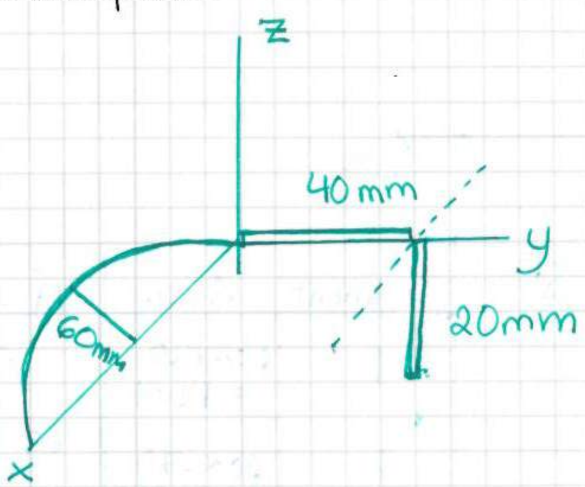


composite body

$\bar{y} = \frac{\Sigma \tilde{y} \cdot A}{\Sigma A}$ $\quad \bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A}$

$\bar{z} = \frac{\Sigma \tilde{z} \cdot A}{\Sigma A}$

example



segment	L/mm	$\tilde{x}$	$\tilde{y}$	$\tilde{z}$	$\tilde{x}L$	$\tilde{y}L$	$\tilde{z}L$
1	$\pi r = 188{,}5$	60	$-\frac{2r}{\pi} = 38{,}2$	0			
2	40	0	20	0			
3	20	0	40	-10			

$\bar{x} = \frac{\Sigma \tilde{x} \cdot L}{\Sigma L} = \frac{x_1 L_1}{L} + \frac{x_2 \cdot L_2}{L} + \frac{x_3 L_3}{L}$

$= \frac{60 \times 188{,}5}{248{,}5} + 0 + 0 = \underline{45{,}5 \text{ mm}}$

$\bar{y} = -22{,}5$ mm

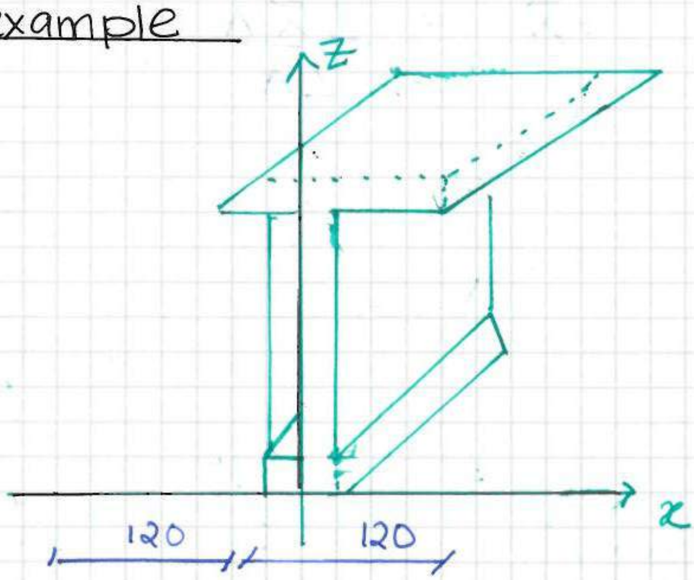
$\bar{x} = 45{,}5$

$\bar{z} = -0{,}805$ mm

Koordinate

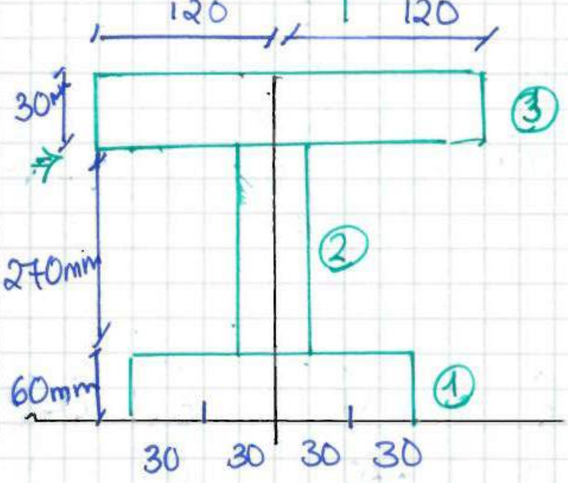
$(45{,}5, -22{,}5, -0{,}805)$

example



seperate into simple shapes.

Find $\bar{y} = ?$



Segment	Area	$\bar{y}$
1	$60 \times 120 = 7200$	30
2	$60 \times 270 = 16200$	$60 + \frac{270}{2} = 195$
3	$30 \times 240 = 7200$	$60 + 270 + \frac{30}{2} = 345$

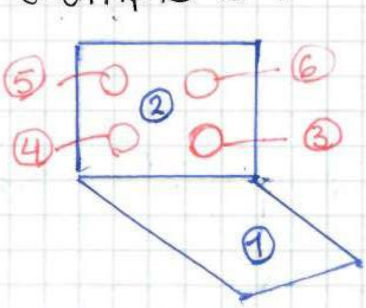
$A\bar{y}$	seg.
30×7200	1
195×16200	2
345×7200	3

$A = A_1 + A_2 + A_3$
$= 30600 \text{ mm}^2$

$\sum_{1}^{3} A\tilde{y} = 585900$

$$\bar{y} = \frac{\sum_{i=1}^{3} A\tilde{y}}{\sum_{i=1}^{3} A} = \frac{585900}{30600} = \underline{\underline{191 \text{ mm}}}$$

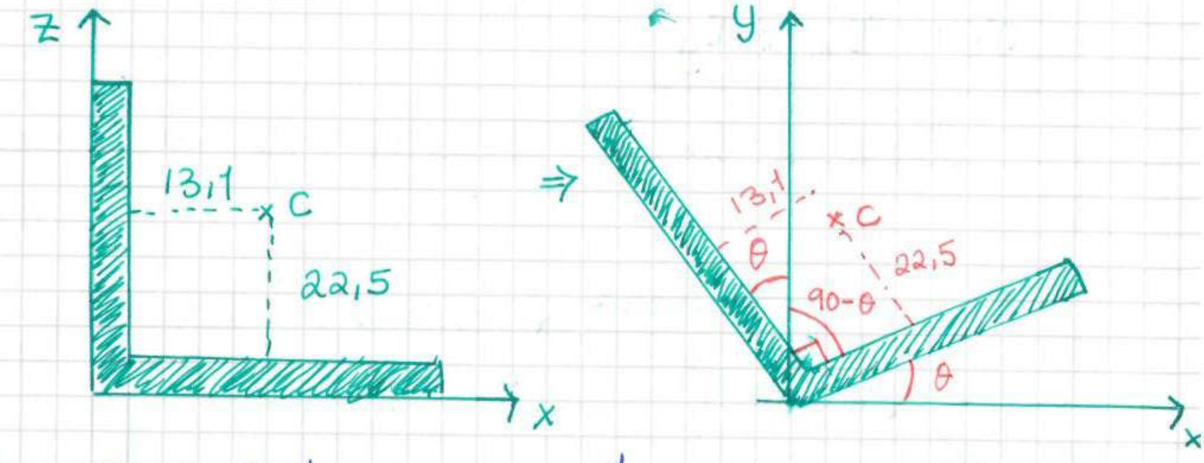
example 5 :



$W = g \cdot M = g \cdot \rho \cdot V = g \cdot \rho \cdot t \cdot A$

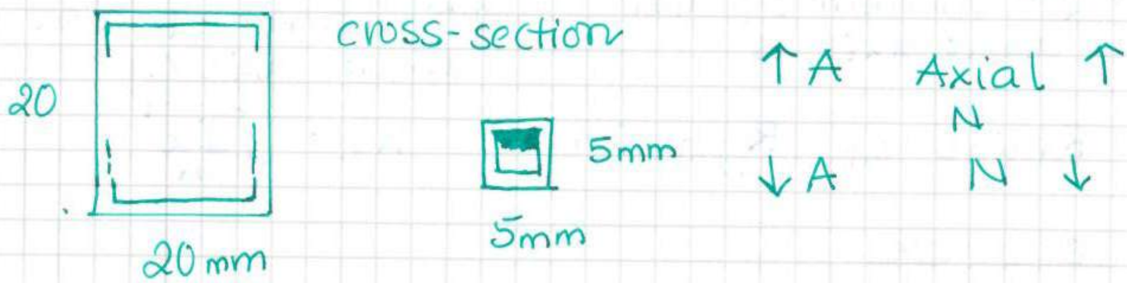
$$\bar{x} = \frac{\sum \bar{x} W}{\sum W} = \frac{\sum \bar{x} A}{\sum A}$$

example 5 part 2

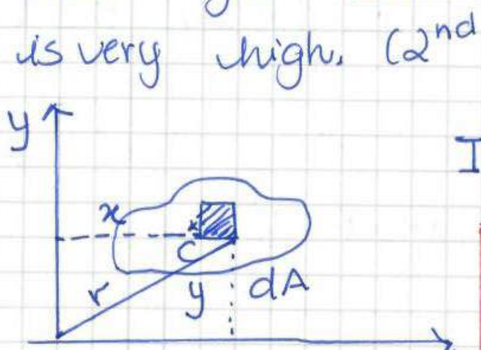


$\tan\theta = \dfrac{13,1}{22,5}$ $\theta = \tan^{-1}\left(\dfrac{13,1}{22,5}\right) = \underline{\underline{30,2^\circ}}$

chapter 10 : Moment of Inertia



big cross-section area can carry big axial force
moment of uintertia, the same cross-sectional area where you turn the object the bending moment is very high. (2nd moment of area)



I_x (2nd moment of area about x-axis)

$$\boxed{\begin{aligned} I_x &= \int_A y^2 \cdot dA \\ I_y &= \int_A x^2 \cdot dA \end{aligned}}$$

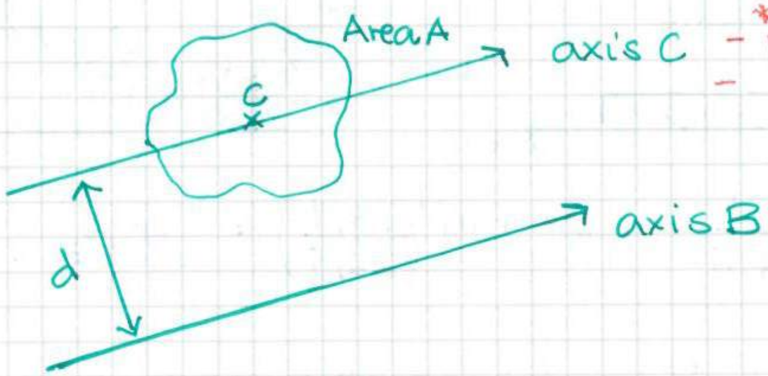
m^4 or mm^4

J_o (polar moment of area) about the axis passing through O, 90° to area

$$J_0 = \int_A r^2 \cdot dA = \int_A (x^2+y^2)\, dA$$

$$\boxed{J_0 = I_x + I_y}$$

Parallel-axis theorem for an area



* - passing through centriod
- parallell to axis B

perpencidular distance between the ax

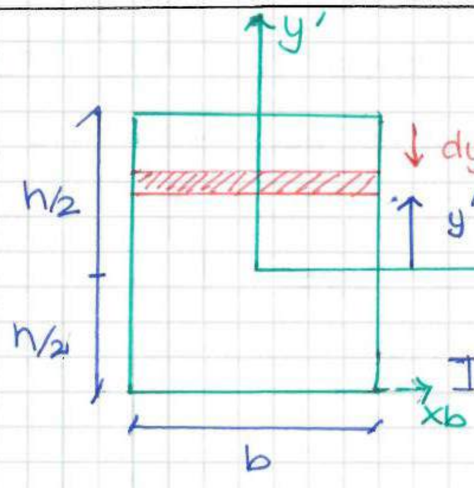
$$\boxed{\underbrace{I_B}_{\text{2nd moment of area about axis B}} = \underbrace{I_C}_{\text{2nd momen of area about axis C *}} + A d^2}$$

Radius of gyration: $K_x = \sqrt{\frac{I_x}{A}}$

example

Determine the moment of intertia for the rectangulor area about
(a) the centriod x-axis
(b) the axis x_b passing through the base of the rectangle.
(c) the pole (ie. z-axis $90°$ to $x'-y'$ plane and passing through centriod C.)

(a)



$$I_{x'} = \int_A y^3 \cdot dA$$

$$dA = b \cdot dy'$$

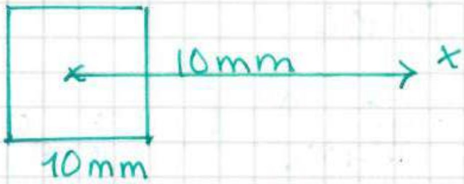
$$I_{x'} = \int_{-h/2}^{h/2} y^2 \, b \cdot dy'$$

$$I_x = b\int_{-h/2}^{h/2} y' \, dy = b\,\frac{y^3}{3}\Big|_{-h/2}^{h/2}$$

$$\underline{\underline{I_x = \frac{bh^3}{12}}}$$

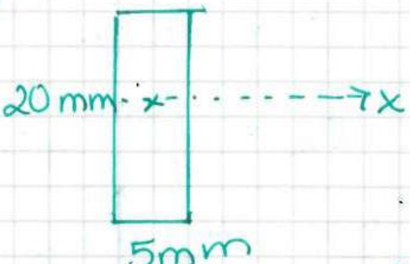
Note :

(a)



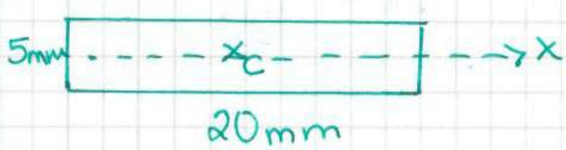
$$I_x = 1/12 \times 10 \times 10^3 \text{ mm}^4$$
$$= \underline{833{,}33 \text{mm}^4}$$

(b)



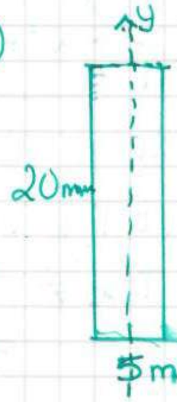
$$Ix = 1/12 \times 5 \times 20^3$$
$$= \underline{3333{,}33 \text{ mm}^4}$$

(c)



$$I_x = 1/12 \times 20 \times 5^3$$
$$= \underline{208{,}33 \text{ mm}^4}$$

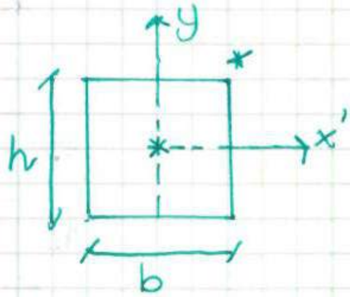
(d)



$$I_y = 5 \times 20^3/12$$
$$= \underline{208{,}33 \text{ mm}^4}$$

(b) $I_{x_b} = I_{x'} + Ad^2$ $\quad d(= h/2)$, $A(= bh)$

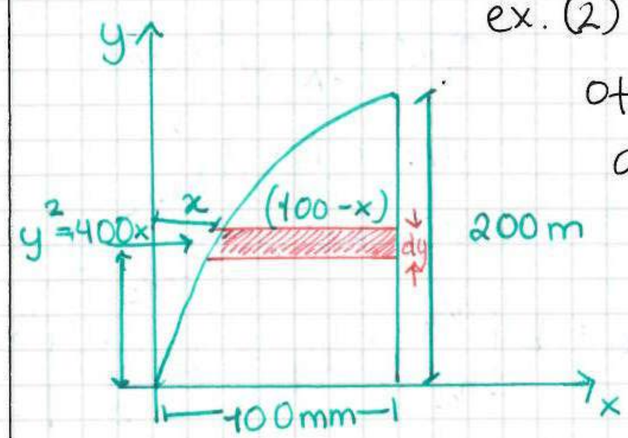
$$= \frac{bh^3}{12} + bh(h/2)^2 = \frac{1}{3}bh^3$$



(c) $J_0 = I_{x'} + I_{y'}{}^{*}$

$$= \frac{bh^3}{12} + \frac{hb^{3*}}{12} = \underline{\underline{\frac{bh}{12}(h^2+b^2)}}$$

ex. (2) Determine the moment of untertia for the shaded area shown in Fig. about x-axis.



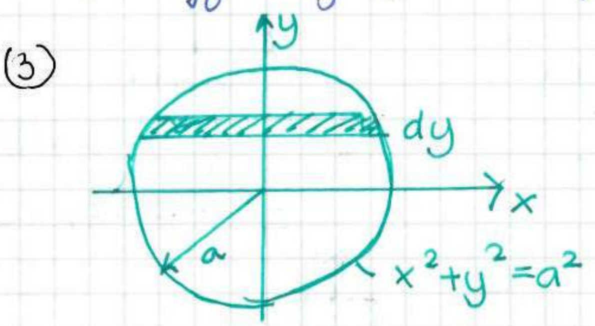
$$I_x = \int y^2 \cdot dA = \int_0^{200\,mm} y^2\,(100-x)\cdot dy \quad *$$

$$dA = dy\,(100-x)$$

$$y^2 = 400x \qquad x = y^2/400$$

$$I_x = \int_0^{200} y^2\left(100 - \frac{y^2}{400}\right) dy = \underline{\underline{107 \times 10^6\ mm^4}}$$

(3)



$$I_x = \int_A y^2 \cdot dA$$

$$J_o = \int_A r^2 \cdot dA = \int_0^a r^2 \cdot 2\pi r\, d$$

$$\underline{\underline{J_o = \frac{\pi a^4}{2}}}$$

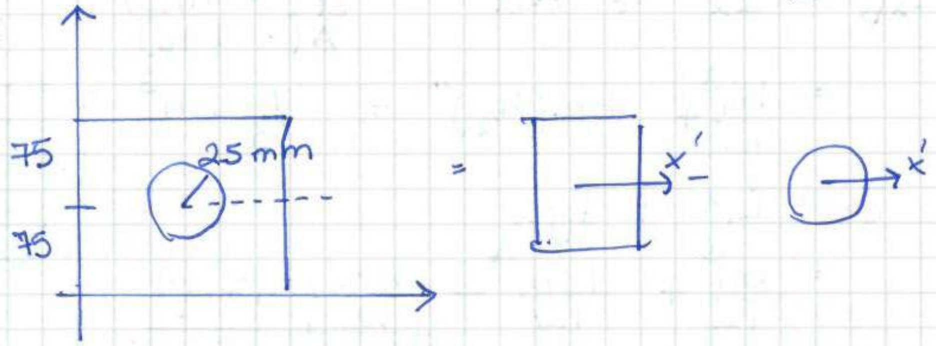
$$dA = 2\pi r \cdot dr$$

$$\Rightarrow J_o = I_x + I_y$$

$$J_o = 2I_x \Rightarrow \underline{\underline{I_x = \frac{\pi a^4}{4}}}$$

10.4 Moment of Intertia for composit area

ex. 5



$$I_{x'} = \int_A y^2 \, dA$$

$$= I_{x'} + (75^2 \cdot A)$$

$$= \tfrac{1}{12} bh^3 + 75^2 A = \tfrac{1}{12} \times 100 \times 150^3 + (75^2 \times 150 \times 100)$$

$$= \underline{\underline{112{,}5 \times 10^6 \text{ mm}^4}}$$

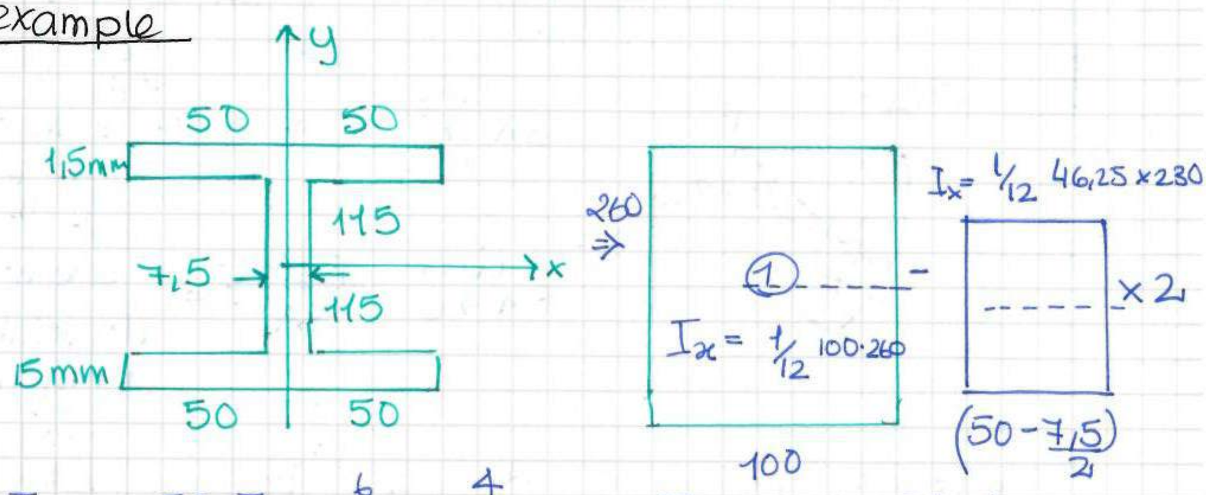
$$I_x = I_{x'} + Ad^2 \quad \text{(for circle)}$$

$$= \tfrac{1}{4} \pi \, 25^4 + \pi \times 25^2 \times 75^2)$$

$$= \underline{\underline{11{,}4 \times 10^6 \text{ mm}^4}}$$

$$\underset{\text{(composite)}}{I_x} = \underset{\text{(rec)}}{I_x} - \underset{\text{(circ)}}{I_x} = 112{,}5 \times 10^6 - 11{,}4 \times 10^6 = \underline{\underline{101 \times 10^6 \text{ mm}^4}}$$

example



$$\underline{\underline{I_x = 52{,}7 \times 10^6 \text{ mm}^4}} \qquad I_y = \Sigma \, (I_y)_3$$

$$I_y = \left(\tfrac{1}{12} \times 15 \times 100^3\right) + \tfrac{1}{12} \times 230 \times 7{,}5^3 + \left(\tfrac{1}{12} \times 15 \times 100^3\right) = \underline{\underline{2{,}51 \times 10^6 \text{ mm}^4}}$$