Exercise e1.

Time derivative is 0 since $\varphi\rho$ is constant. Moreover, $k\rho/\mu$ is also constant and (1.1) is obtained.
The general solution of (1.1) is given by

$$p(x) = a_1 x + a_2$$, a_1 and a_2 are arbitrary constants.

From the boundary conditions:
Darcy's law is used to determine a_1 as function of Q

$$\frac{Q}{A} = -\frac{k}{\mu}a_1, \qquad a_1 = -\frac{Q\mu}{Ak} \quad .$$

Using $p(L) = p_0$ results in equation (1.2).

Exercise e2.

The equations (2.1) are first order ordinary differential equations with general solutions

$$\varphi = Ae^{c_r p}, \qquad \rho = Be^{cp},$$

where A, B are arbitrary constants. Unique solutions are obtained by introducing measured values φ_0, ρ_0 at reference pressure p_0 , i.e $\varphi(p_0) = \varphi_0$, $\rho(p_0) = \rho_0$.
The reference values are used to determine the constants A, B ,

$$A = \varphi_0 e^{-c_r p_0}, \qquad B = \rho_0 e^{-cp_0},$$

and (2.2) is established.

Taylor expansions of (2.2)

$$\begin{aligned}\varphi &= \varphi_0[1 + c_r(p - p_0) + 0.5c_r^2(p - p_0)^2 + \textit{higher order terms}] \approx \varphi_0[1 + c_r(p - p_0)] \\ \rho &= \rho_0[1 + c(p - p_0) + 0.5c^2(p - p_0)^2 + \textit{higher order terms}] \approx \rho_0[1 + c(p - p_0)]\end{aligned} \; .$$

Consider the left hand side of equation (3). Using formula (2.3) for ρ

$$\frac{\partial}{\partial x}\left(\frac{k\rho}{\mu}\frac{\partial p}{\partial x}\right) \approx \frac{k\rho_0}{\mu}\frac{\partial^2 p}{\partial x^2},$$

since c is small.

Since terms containing $c_r c$ can be neglected $\varphi\rho$ can be written

$$\varphi\rho = \varphi_0\rho_0[1 + c_r(p - p_0) + c(p - p_0) + c_r c(p - p_0)^2] \approx \varphi_0\rho_0[1 + (c_r + c)(p - p_0)]$$

and the right hand side of (3) becomes

$$\frac{\partial}{\partial t}(\varphi\rho) = \varphi_0\rho_0(c_r + c)\frac{\partial p}{\partial t} \quad .$$

Equation (2.4) is now deduced.

References to analytical solution of heat equation
http://www-solar.mcs.st-and.ac.uk/~alan/MT2003/PDE/node21.html

Solution Exercise e3.

Using assumptions on φ, ρ equation (6) can be written

$$\left(\frac{Ak\rho_0}{\mu\Delta x}\right)_{i+1/2}(p_{i+1}-p_i)-\left(\frac{Ak\rho_0}{\mu\Delta x}\right)_{i-1/2}(p_i-p_{i-1}) = A\Delta x\varphi_0\rho_0(c_r+c)(p^{n+1}-p^n)/\Delta t$$

All parameters except pressures are constants in the equation above and equation (3.1) is obtained.
Equation 3.4 is used to compute the solution for blocks 2, 3 and 4, first for time n = 1 and then for time n = 2. No computations are needed for blocks 1 and 5 where the value of p is equal to 1 for all times.

block no.

	1	2	3	4	5
n=0	1	2.5	4	3	2
n=1	1	2.5	2.75	3	2
n=2	1	1.88	2.75	2.38	2
n=3	1	1.88	2.13	2.38	2