University of Stavanger

FACULTY OF SCIENCE AND TECHNOLOGY

SUBJECT: **PET 510 – Computational Reservoir and Well Modeling**

DATE: **22 November, 2019**

TIME: **4 hours**

AID: **Basic calculator is allowed**

THE EXAM CONSISTS OF 6 PROBLEMS ON 7 PAGES AND APPENDIX A - D

REMARKS:
You may answer in English or Norwegian. Exercises 1 and 2 (part A) and exercises 3-6 (part B) are given equal weight.

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Exercise 1.

A model for describing single-phase isentropic flow in a wellbore of length L takes the form

$$\rho_t + (\rho u)_x = K[P_r - P(\rho)] \qquad (*)$$

$$(\rho u)_t + (\rho u^2)_x + P(\rho)_x = -\kappa u - \rho g, \qquad x \in [0, L] \qquad (**)$$

where ρ, u, and $P(\rho)$ are density, velocity, and pressure respectively, while $K > 0$ is the PI-index characterizing the influx/efflux from/to reservoir. P_r represents reservoir pressure. Moreover, g is gravity constant and κ is friction coefficient.

(a) We ignore the acceleration terms in the momentum balance equation (**) and consider $P(\rho)_x = -\kappa u - \rho g$.
- Derive an equation for the density ρ.
- Identify the corresponding equation when gravity is set to zero, i.e., $g = 0$. What kind of equation is this? Give a brief, intuitive description of the effect from the different terms.

(b) Two simulation cases based on the model in (a) are shown in Fig. 1 where we have set the interaction with the reservoir to be zero, i.e., $K = 0$.
- Explain by words the physics reflected by the curves, both the initial state and the way they evolve over time.
- Try to refer to the equations in (*) and (**) to back up your explanation.
- Also explain the difference between the upper and lower row of figures.

(c) Now, we assume that the velocity u is a known constant and $P(\rho) = a_L^2(\rho - \rho_0) + P_0$ with a_L, ρ_0, and P_0 as given constants. Show that (*) then takes the form

$$\rho_t + u\rho_x = K a_L^2 (D - \rho), \qquad (***)$$

and find an expression for the constant D.
We set $K a_L^2 = 0$ in (***). Find an expression for the analytical solution of $\rho(x,t)$ when initial state is given by

$$\rho(x, t = 0) = \phi(x) \qquad \text{where } 0 \le \phi(x) < 1, \qquad x \in (-\infty, +\infty)$$

(d) We set $K a_L^2 = 1$ and $D = 1$ in (***). Find an expression for the analytical solution of $\rho(x,t)$ with the same initial data $\phi(x)$ as in (c).
Hint: Rewrite (***) by introducing $1 - \rho = v$ and solve for v.
- Verify that your solution satifies the equation.
- What happens with $\rho(x,t)$ as time $t \to \infty$?

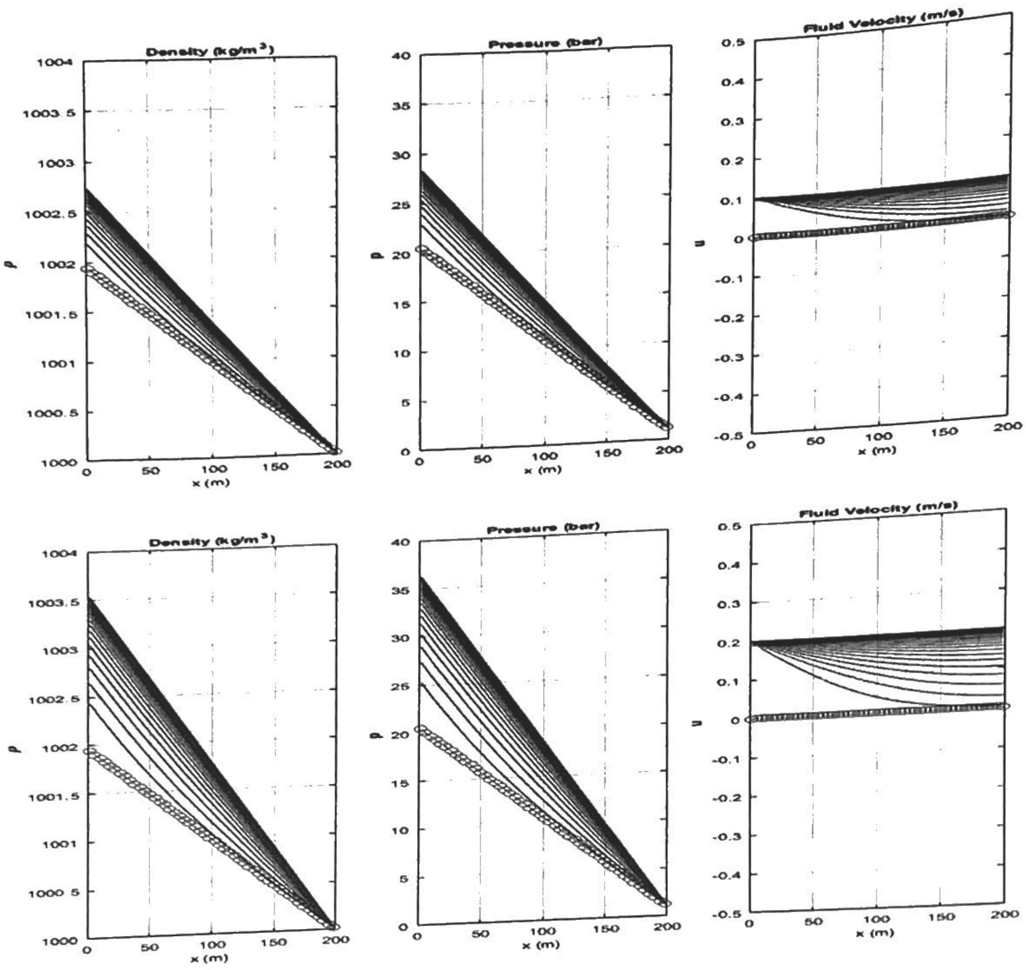


FIGURE 1. **Left:** Density ρ. **Middle:** Pressure P. **Right:** Velocity u. Thick line (red) shows initial state whereas the family of other curves (blue) shows how the quantity evolves over time.

(e) We consider (***) with $u = \frac{L-x}{1+t}$ and $K = 0$, that is,

$$\rho_t + \Big(\frac{L-x}{1+t}\Big)\rho_x = 0, \qquad x \in [0, L]$$

with initial data and boundary data given, respectively, as

$$\rho(x, t=0) = 1, \qquad \rho(x=0, t) = 2.$$

- Compute the expression for the characteristics. In particular, find the expression for the characteristics starting, respectively, at $x_0 = 0$, $x_0 = 100$, and $x_0 = 200$ where the length of the wellbore $L = 200$. Sketch the paths in the x-t space.
- Describe how the jump in density ρ which is located at inlet $x = 0$ at time $t = 0$ will move as time elapses. Give a sketch of $\rho(x, t = 1)$.

Exercise 2.

We can show that under appropriate assumptions, the model (*) and (**) from Exercise 1 is reduced to

$$(A) \qquad \rho_t = \kappa\rho_{xx} + \beta[1-\rho], \qquad x \in [0, L]$$

where $\rho = \rho(P)$ is fluid density and with some positive constant parameter $\kappa > 0$ and β which is constant in space and time.

(a) We now assume that (A) is subject to the initial and boundary condition

$$(AA) \qquad \rho(x, t=0) = \phi(x), \qquad \rho(x=0,t) = \rho_L, \qquad \rho(x=L,t) = \rho_R.$$

Divide the domain into M cells with points x_1, x_2,..., x_M located at the center of each cell. The cell interface $x_{1/2}$ corresponds to $x = 0$ and $x_{M+1/2}$ to $x = L$. Describe an explicit discretization of (A) and (AA).

(b) Describe an implicit discretization of (A) and (AA) and identify the corresponding linear system of the form $Ay = b$. What is a main difference between this explicit and implicit discrete scheme.

(c) We now consider (A) with a time dependent $\beta(t)$ given by $\beta(t) = 1/t$ and on the whole interval $(-\infty, +\infty)$. In order to derive an analytical (exact) solution of (A) the idea is to relate the time variable t and the space variable x in a special way. Introduce a variable y that describes this relation and a variable $v(y)$ and rewrite the PDE (partial differential equation) $\rho_t = \kappa\rho_{xx} + t^{-1}[1-\rho]$ as and ODE (ordinary differential equation) in terms of $v(y)$.

(d) We assume that initial data $\phi(x)$ and boundary data ρ_L and ρ_R given by (AA) satisfy

$$0 \le \phi(x) < 1, \qquad \rho_L = 1 = \rho_R.$$

We seek to obtain an estimate of the quantity $\int_0^L [\rho(x,t) - 1]^2 dx$. Show that the following estimate holds

$$\int_0^L [\rho(x,t) - 1]^2 dx \le \int_0^L [\phi(x) - 1]^2 dx$$

Exam Part B – Solving Nonlinear Equations & Modelling of Well Flow

There are 11 questions in total. Some formulas, equations and Matlab codes are found in Appendixes. This part constitutes 50 % of exam.

Exercise 3 Matlab

a] Write a Matlab script where you define a row vector x =[2 7 8 5 3 9] and then use a for loop to produce a row vector where the elements are reversed.

b] We have the following set of three equations with three unknowns.

$$2x - 5y + 4z = -3$$

$$x - 2y + z = 5$$

$$x - 4y + 6z = 10$$

Write a matlab script that solves the problem.

Exercise 4 – Bisection Method

a] We are given the function $f(x) = x^2 - 4x + 2$. This function has two roots. Start by making a sketch of the function.

We want to pick out the <u>**smallest**</u> root of the function above. The solution shall be found with an accuracy such that $|f(x_3)| < f_{tol} = 0.05$. Use a similar table as shown below until a satisfactory solution is found!

Iteration	x1	x2	x3	f(x1)	f(x2)	f(x3)	(x2-x1)/2
1							
2							
3							
Etc ..							

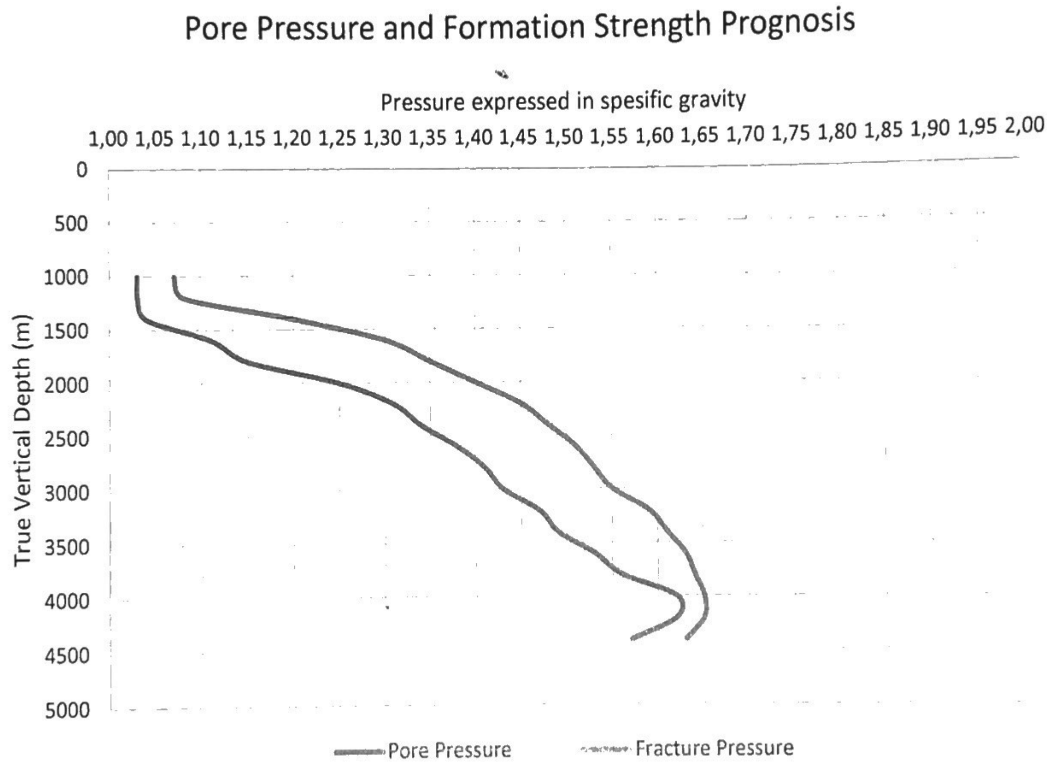
b] Now, we will instead try to find the **x value** that gives the ***minimum y value*** of the function above using the bisection method. Explain how we need to change the matlab code in Appendix B such that the code finds this x value.

Exercise 5 - Well Pressures & Cuttings Transport

We have a 4000 meter deep well measured from RKB (rotary kelly board). The BOP is situated at 1000 meter measured from RKB. The riser is 1000 meter long with an inner diameter of 19 inches. A 5 inch outer diameter drillpipe is used. A 3000 meter long 9 5/8 inch casing with inner diameter 8.5 inch is hanging in the wellhead just below the BOP. We will now start drilling the 8 ½ inch hole size.

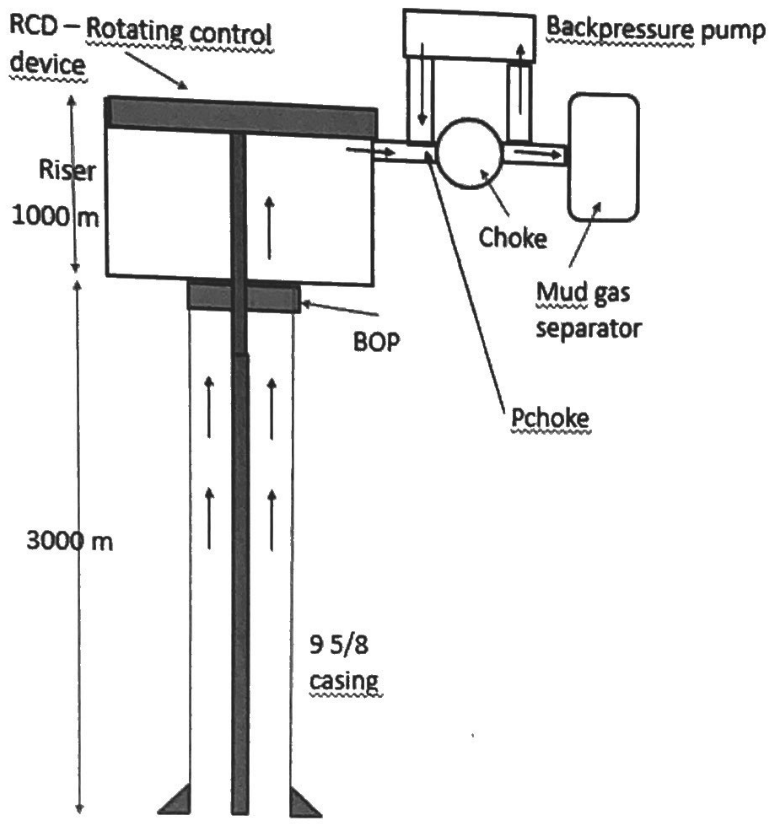
We are circulating with a rate of 1800 lpm and we have that the annular friction in the well below the BOP is 0.01 bar/m. The friction in the riser is negligible. The mudweight is 1.63 sg.

a] Calculate the ECD at bottom of the annulus in sg! Compare the results with the diagram showing pore and fracture pressure prognosis below. What can happen when we circulate?



In order to handle the small margin between the pore pressure and fracture pressure, we will adopt a backpressure managed pressure drilling system. Here, there is a rotary control device that seals the riser on top and the flow is lead through a choke and directly to a mud gas separator.

The pressure in the well is now given by the formula: $P_{well} = P_{hyd} + P_{fric} + P_{choke}$. The choke pressure can be adjusted so that we can basically have the same pressure both during circulation and static conditions. When circulation stops, the choke pressure can be increased by activating the back pressure pump.



b] The choke pressure is 10 bar and we circulate with 1800 lpm. We want the well pressure to correspond to an ECD of 1.63 sg (target pressure). What must the mud weight be then?

c] How much must the choke pressure be increased if we stop circulation and still want the well pressure to correspond to 1.63 sg ?

d] Small kicks are allowed to be circulated through the choke and mud gas separator without closing BOP using this system. Flow rate is 1800 lpm. If the kick is taken at bottom and we assume no slip conditions (K=1, S = 0). How long time will it take before the kick is at the surface ?

e] Choke pressure is set to 10 bar and the bottom pressure still corresponds to 1.63 sg. A kick with volume 0.3 m^3 is taken at bottom. The kick has a temperature of 70 Celsius when it is a bottom and 40 Celsius when reaching surface.

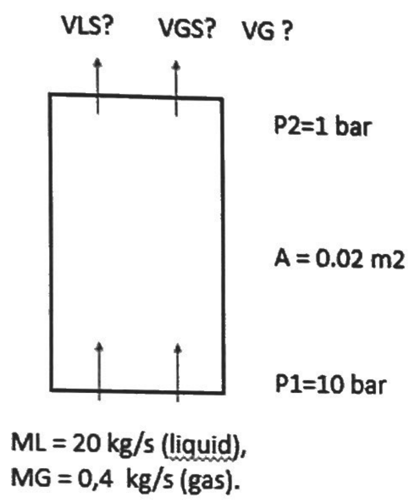
What is the volume of the kick just prior to entering the choke?

f] If one suspect cuttings transport problems in this well, what would you prefer to do?

Exercise 6 –Conservation laws

a] The following figure shows the uppermost numerical cell in a steady state two phase flow model. We know the inlet massrate of each phase (liquid = water, gas = ideal gas). At the outlet, we know that the pressure must be 1 bar (atmospheric conditions). The flowarea is 0.02 m^2.

Calculate the ***superficial liquid and gas velocities*** and the ***gas velocity*** at the outlet!



Appendix A – Some Units & Formulas

1 inch =2.54 cm = 0.0254 m

1 feet = 0.3048 m

1 bar = 100000 Pa

1 sg = 1 kg/l (sg - specific gravity)

$M = Q \cdot \rho$ M massrate (kg/s), Q Volumerate (m^3/s), ρ density (kg/m^3)

$Q = v \cdot A$ Q Volumerate (m^3/s), v velocity m/s. A area m^2

$p = \rho \cdot h \cdot 0.0981$ p (bar), ρ density (sg), h – vertical depth (m)

$\frac{P \cdot V}{T} = C$, from Ideal gas law, NB T is in Kelvin and the relation to Celsius is $K = {}^{\circ}C + 273{,}15$

$P \cdot V = C$, Boyles law (temperature is assumed constant)

Appendix B

Main.m

```
% Main program that calls up a routine that uses the bisection
% method to find a solution to the problem f(x) = 0.
% The search intervall [a,b] is specified in the main program.
% The main program calls upon the function bisection which again calls upon
% the function func.

% if error = 1, the search intervall has to be adjusted to ensure
% f(a) x f(b)<0

% Specify search intervall, a and b will be sent into the function
% bisection
 a = 4.0;
 b = 5.0;

% Call upon function bisection which returns the results in the variables
% solution and error.
  [solution,error] = bisection(a,b);
```

Bisection.m

```
function [solution,error] = bisection(a,b)

% The numerical solver implemented here for solving the equation f(x)= 0
% is called Method of Halving the Interval (Bisection Method)

% You will not find exact match for f(x)= 0. Maybe f(x) = 0.0001 in the
end.
% By using ftol we say that if abs(f(x))<ftol, we are satisfied. We can
% also end the iteration if the search interval [a,b] is satisfactory
small.
% These tolerance values will have to be changed depending on the problem
% to be solved.


 ftol = 0.01;

% Set number of iterations to zero. This number will tell how many
 % iterations are required to find a solution with the specified accuracy.

  noit = 0;

  x1 = a;
  x2 = b;
```

```
  f1 = func(x1);
  f2 = func(x2);


%   First include a check on whether f1xf2<0. If not you must adjust your
%   initial search intervall. If error is 1 and solution is set to zero,
%   then you must adjust the search intervall [a,b].


 if (f1*f2)>=0
     error = 1;
     solution = 0;
 else
 % start iterating, we are now on the track.
     x3 = (x1+x2)/2.0;
     f3 = func(x3);

     while (f3>ftol | f3 < -ftol)
        noit = noit +1 ;

        if (f3*f1) < 0
           x2 = x3;
        else
           x1 = x3;
        end

        x3 = (x1+x2)/2.0;
        f3 = func(x3);
        f1 = func(x1);

     end
     error = 0;
     solution = x3;
     noit % This statement without ; writes out the number of iterations to
the screen.
 end
```

func.m

```
function f = func(x)


  f = x^2-4*x+2;
```

Appendix C

```
% Program where the Larsen Cuttings Transport Model is implemented

%  First specify all input parameters:

do = 8.5; % Outerdiameter (in) ( 1 in = 0.0254 m)
di = 5; % Innerdiameter   (in)
rop = 33 % Rate of Penetration - ROP ft/hr (1 ft = 0.3048m)
pv = 15 % Plastic viscosity (cP)
yp = 16 % Yield point (lbf/100ft2)
dcutt = 0.1 % Cuttings diameter (in) (1 inch = 0.0254 m)
mw = 10.833 % Mudweight (ppg - pounds per gallon) 1 ppg = 119.83 kg/m3.
rpm = 80 % rounds per minute
cdens = 19 % cuttings density (ppg - pounds per gallon)
angstart = 50 % Angle with the vertical

% vcut - Cuttings Transport Velocity (CTF in Larsens paper)
% vcrit - Critical Transport fluid velocity (CTFV) in Larsens paper. This
% is the minimum fluid velocity required to maintain a continously upward
% movement of the cuttings.
% vslip - Equivalent slip velocity (ESV) defined as the velocity difference
% between the cuttings and the drilling fluid
% vcrit = vcut+vslip
% All velocities are in ft/s.
% ua - apparent viscosity

% It should be noted that the problem is nested. Vcrit depends on vslip
% which again depends on an updated/correct value for vcrit. An iterative
% approch on the form  x(n+1) = g(x(n)) will be used.


ang = angstart;
vcut = 1/((1-(di/do)^2)*(0.64+18.16/rop));

vslipguess = 3;
vcrit = vcut + vslipguess;

% Find the apparent viscosity (which depends on the "guess" for vcrit)
ua = pv+ (5*yp*(do-di))/vcrit

% Find vslip based on the "guessed apparent viscosity". This needs to be
% updated until a stable value is obtained. "Iterative approach".
if (ua <= 53)
 vslip = 0.0051*ua+3.006;
else
 vslip = 0.02554*(ua-53)+3.28;
end


%Now we have two estimates for vslip that can be compared and updated in a
% while loop. The loop will end when the vslip(n+1) and vslip (n) do not
% change much anymore. I.e the iterative solution is found.
n=1;
while (abs(vslip-vslipguess))>0.01
 vslipguess = vslip;
 vcrit = vcut + vslipguess;
% Find the apparent viscosity (which depends on the "guess" for vcrit)
  ua = pv+ (5*yp*(do-di))/vcrit;
```

```
% Find vslip based on the "guessed apparent viscosity". This needs to be
% updated until a stable value is obtained. "Iterative approach".
  if (ua <= 53)
   vslip = 0.0051*ua+3.006;
  else
   vslip = 0.02554*(ua-53)+3.28;
  end
  n=n+1;
  vslip  % Take away ; and you will se how vslip converges to a solution
end % End while loop

%
% Cuttings size correction factor: CZ = -1.05D50cut+1.286
 CZ = -1.05*dcutt+1.286
% Mud Weight Correction factor (Buoancy effect)
  if (mw>8.7)
  CMW = 1-0.0333*(mw-8.7)
  else
  CMW = 1.0
  end

% Angle correction factor

CANG = 0.0342*ang-0.000233*ang^2-0.213

vslip = vslip*CZ*CMW*CANG;   % Include correction factors.

% Find final minimum velocity required for cuttings transport (ft/s).

vcrit = vcut + vslip
```

Appendix D – Steady State Model for Two Phase Flow

Conservation of liquid mass

$$\frac{\partial}{\partial z}(A\rho_l\alpha_l v_l) = 0 \qquad [1]$$

Conservation of gas mass

$$\frac{\partial}{\partial z}(A\rho_g\alpha_g v_g) = 0 \qquad [2]$$

Conservation of momentum.

$$\frac{\partial}{\partial z}p = -(\rho_{mix}g + \frac{\Delta p_{fric}}{\Delta z}) \qquad [3]$$

Gas slippage model (simple):

$v_g = Kv_{mix} + S$ (K=1.2, S = 0.5 m/s) **[4]**

Liquid density model (simple)

$\rho_l(p) = \rho_{lo} + \frac{(p - p_O)}{a_L^2}$, assume water: $\rho_{lo} = 1000$ kg/m³, $p_O = 100000Pa$, $a_L = 1500$ m/s **[5]**

Gas density model (simple)

$\rho_g(p) = \frac{p}{a_g^2}$, ideal gas: $a_g = 316$ m/s. **[6]**

Friction model

The friction model presented here is for a Newtonian fluid like water. The general expression for the frictional pressure loss gradient term is given by:

$\frac{\Delta p_{fric}}{\Delta z} = \frac{2f\rho_{mix}v_{mix}abs(v_{mix})}{(d_{out} - d_{in})}$ (Pa/m) **[7]**

A - (m²)

ρ_i - phase densities (kg/m³), liquid –> i=l, gas ->i =g

v_i - phase velocities (m/s)

μ_i - phase viscosity (Pa s)

p - pressure (Pa)

g – gravity constant 9.81 m/s²

α_i - phase volume fractions taking values between 0 and 1. $\alpha_l + \alpha_g = 1$.

$\rho_{mix} = \alpha_l \rho_l + \alpha_g \rho_g$ - mixture density

$v_{mix} = \alpha_l v_l + \alpha_g v_g$ - mixture velocity

$\mu_{mix} = \alpha_l \mu_l + \alpha_g \mu_g$ - mixture viscosity