University of
Stavanger

FACULTY OF SCIENCE AND TECHNOLOGY

SUBJECT: PET575 Automated Drilling and Modeling

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AID: Approved calculator

COURSE RESPONSIBLE: Dan Sui

TELEPHONE NUMBER: 466 96 188

Question 1 Drilling automation

Figure 1 shows a schematic of a top drive control system. In the figure, PLC stands for programmable logic controller.

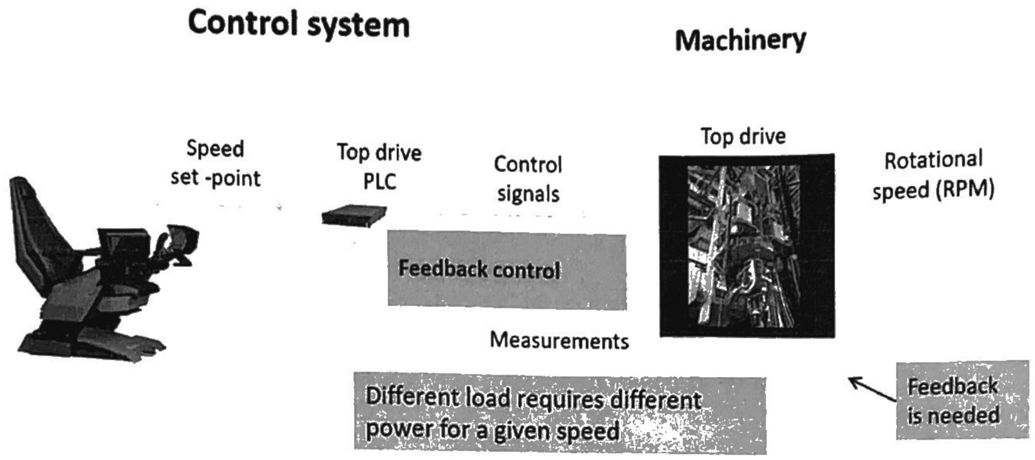

Figure 1 Top drive control system

Please answer the following questions.

Q1.1. Describe how the top drive drilling control system shown in Figure 1 works.

Q1.2. List the important automation tools used in such system and describe their functions respectively.

Question 2 Drilling fluids

The structure of the drilling fluid property evaluation system is shown in Figure 2. One differential pressure sensor is installed on the pipe in order to automatically evaluate the drilling fluid's density and viscosity. The pipe is placed vertically (The inclination is 0). DP1 in the figure is the differential pressure between pressures P1 and P2, or

$$DP_1 = P_2 - P_1.$$

The values of the parameters involved in the calculation are given in Table 1. It assumes that the fluid's density does not vary with flow rates. Both the density and the viscosity are uniform in the whole system. The fluid is a Newtonian fluid. The flow direction is given in the figure.

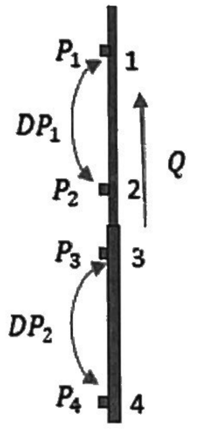

Figure 2 Structure of the system

Description	Notation	Values	Unit
Acceleration of gravity	g	9.8	m/s^2
Inner diameter of the pipe between points 1 and 2	D_1	0.02	m
Inner diameter of the pipe between points 3 and 4	D_2	0.04	m
Length between points 1,2	l	4	m
Length between points 3,4	l	4	m
Pipe roughness	ε	0	

Table 1 Parameters' values

The measured differential pressures (DP1) with respect to flow rates (Q) are listed in Table 2. The situation when the flow rate is 0 means that there is no circulation and the pipe is full of the fluid.

Flow rate (Q) (m^3/s)	DP1 (Pascal)
0	39200
30/60000 (0.0005)	45200

Table 2 Differential pressures vs flow rates

Q2.1 Calculate the density (ρ) and the viscosity (μ) of the fluid based on Table 1 and Table 2 and fill Table 3.

Flow rate (Q) (m^3/s)	density (ρ) of fluid (kg/m^3)	viscosity (μ) of fluid (Pa.s)
30/60000 (0.0005)		

Table 3 Fluid properties vs flow rate

Q2.2 Calculate the differential pressure between points 3 and 4 (DP2), where DP2 is the differential pressure between pressures P3 and P4, or

$$DP_2 = P_4 - P_3.$$

Then fill Table 4.

Flow rate (Q) (m^3/s)	DP2 (Pascal)
30/60000 (0.0005)	

Table 4 Differential pressure vs flow rate

Question 3 Modeling

Q3.1 What is a dynamic model?

Q3.2 ROP model

Effect description	**Sub-equations**	**Effect description**	**Sub-equations**
Depth	$p_2 = h_0 - h$	Formation compaction	$p_3 = h^{0.69}(g_p - \rho_n)$
Differential pressure	$p_4 = h(g_p - \rho_c)$	Bit type and weight	$p_5 = \ln \dfrac{\frac{w}{d_B} - \frac{w_0}{d_B}}{4 - \frac{w_0}{d_B}}$
Rotary speed	$p_6 = \ln \dfrac{r}{r_0}$	Bit tooth wear	$p_7 = -H$
Fluids properties	$p_8 = \ln \dfrac{\rho q}{350 \mu d_n}$		

Table 5.1 Model parameters

Para.	**Description**	**Para.**	**Description**
h	Depth of bit	h_0	Given depth of bit
g_p	Formation pressure gradient	ρ_n	Normal fluid pressure gradient
ρ_c	Equivalent circulating density gradient	w	WOB
w_0	Threshold bit weight	d_B	Bit diameter
r	RPM	r_0	Given RPM
H	Fractional tooth height worn away	ρ	Mud density
q	Flow rate	μ	Flow viscosity
d_n	Bit nozzle diameter		

Table 5.2 Model parameters

The ROP model is given as an exponential function

$$ROP = e^{a_1 + \sum_{i=2}^{8} a_i p_i},$$

where the function $p_i, i = 2, \ldots, 8$ and involved drilling parameters are given in Table 5. To develop the above ROP model, the model coefficients $a_i, i = 2, \ldots, 8$ need to be determined.

Suppose we have the dataset with n groups of data points shown as

$$Dataset = \{(P_1, ROP_1), (P_2, ROP_2), \ldots, (P_n, ROP_n)\}$$

which is available, where $P_j = (p_{2,j}, p_{3,j}, \ldots, p_{8,j})$ and ROP_j are associated with one specific depth $j, \forall j = 1,2, \ldots, n$.

Q3.2a Is it possible to determine the model coefficients $a_i, i = 2, \ldots, 8$ using least squares multiple linear regression method? State the reason.

Hint: Given the data $\{(x_1, y_1), \ldots, (x_n, y_n)\}$, the goal of the linear least squares regression is to find values of a and b of the linear function $y = ax + b$ that minimize the sum of squared errors $E(a, b) = \sum_{j=1}^{n} (y_j - (ax_j + b))^2$.

Q3.2b Which parameters in Table 5.2 could be selected as the inputs for the ROP optimization?

$$\omega_m = \sqrt{\frac{k}{m}}$$

$$\frac{c}{\omega_m \cdot m \cdot 2}$$

Question 4 Drillstring

Q4.1 What is the axial vibration of drill pipe?

Q4.2 For an undamped system

$$m\ddot{x}(t) + kx(t) = 0$$

Q4.2a Please calculate the natural frequency of such system when m=2, k=1?

Q4.2b When m=2 and k=10, does the frequency become lower or higher? Why?

Q4.3 Drillstring modeling

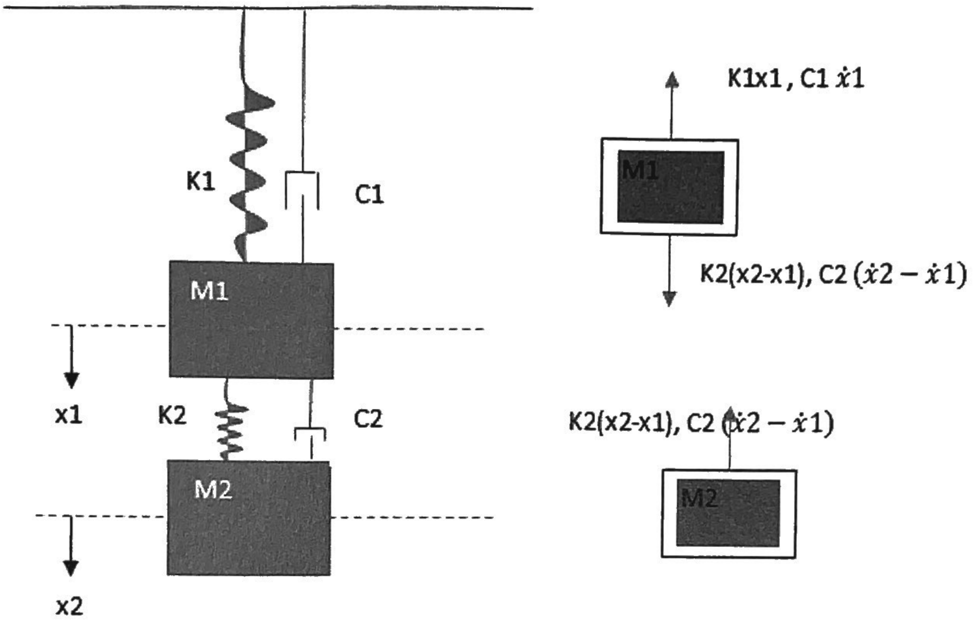

Figure 3 Spring-mass system

Consider a spring-mass system of Figure 3. Let k_1 and m_1 be the stiffness of the spring and the mass of the block 1 respectively. c_1 is the damping coefficient of block 1. Let k_2 and m_2 be the stiffness of the spring and the mass of the block 2 respectively. c_2 is the damping coefficient of block 2. There is no external force on block 1 and block 2. The forces on each block are further illustrated in Figure 3.

Assume that the mass of the first block is m and the mass of the second block is $2m$. Applying the Newton second law to the block system yields

$$M\ddot{X}(t) + C\dot{X}(t) + KX(t) = 0$$

where $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, x_1 is the displacement from the static position of block 1 and x_2 is the displacement from the static position of block 2. M is mass matrix, C is damping matrix and K is stiffness matrix.

Question (4.3) Which one listed below is the correct expression of the model coefficient matrix? State the reason.

(A) $M = \begin{bmatrix} m & 0 \\ 0 & 2m \end{bmatrix}, C = \begin{bmatrix} c_1 + c_2 & 0 \\ 0 & c_2 \end{bmatrix}, K = \begin{bmatrix} k_1 + k_2 & 0 \\ 0 & k_2 \end{bmatrix}$

(B) $M = \begin{bmatrix} m & m \\ 0 & 2m \end{bmatrix}, C = \begin{bmatrix} c_1 + c_2 & 0 \\ 0 & c_2 \end{bmatrix},, K = \begin{bmatrix} k_1 + k_2 & 0 \\ 0 & k_2 \end{bmatrix}$

(C) $M = \begin{bmatrix} m & 0 \\ 0 & 2m \end{bmatrix}, C = \begin{bmatrix} c_1 + c_2 & 0 \\ 0 & c_2 \end{bmatrix}, K = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}$

(D) $M = \begin{bmatrix} m & 0 \\ 0 & 2m \end{bmatrix}, C = \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}, K = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}$

Question 5 Drilling data

The raw dataset is given as

$$X = \{x_1, \quad x_2, \quad x_3, \quad \dots, x_n\}$$

Consider the following filter

$$y_k = \alpha y_{k-1} + (1 - \alpha) x_k$$

where $0 < \alpha < 1$, x is raw data, and y is processed data. After the filter, the processed dataset becomes

$$Y = \{y_1, \quad y_2, \quad y_3, \quad \dots, y_n\}$$

Q 5.1 What is this filter? Describe the main functions of such filter.

1 2 3 4 5 6 7 8 9

Q 5.2 The raw data set is $= \{1.13, \ 1.05, \ 0.83, \ 1.26, \ 1.04, \ 1.30, \ 0.92, \ 1.23, \ 0.95\}$. Ideally the data x is expected to be stable at 1.1. Try to set

(1) $\alpha = 0.1$

(2) $\alpha = 0.9$

to calculate the filtered dataset Y and evaluate the performances of such filter with respect to different α.

Hint: set $y_0 = x_1$ at the beginning to calculate y_1.

Question 6 Controller

Q 6.1 PID controller

The PID controller is shown below

$$u(t) = u_0 + K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de(t)}{dt}$$

Q6.1a What is u_0? Does u_0 vary with the time t?

Q6.1b The response of PID controller is shown in Figure 4.1 with respect to different values of K_p, where other parameters (u_0, K_i, K_d) are same.

Overshoot and Setting time

Reduce Rise time

eliminate offset

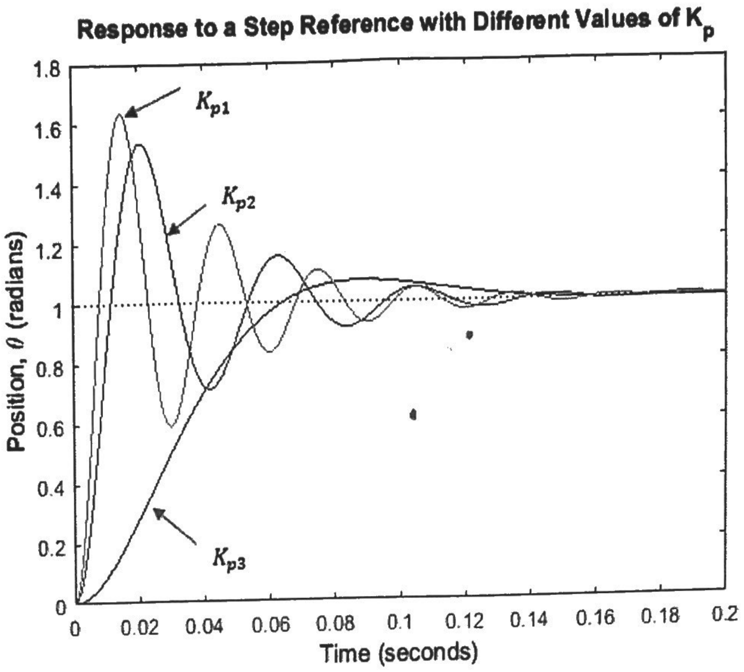

Figure 4 .1 PID system responses with different K_p (Setpoint is 1)

Please choose the corresponding K_p values for these three responses and state the reason.

(A) $K_{p1} = 11$, $K_{p2} = 1$, $K_{p3} = 21$.

(B) $K_{p1} = 11$, $K_{p2} = 21$, $K_{p3} = 11$.

(C) $K_{p1} = 1$, $K_{p2} = 11$, $K_{p3} = 21$.

(D) $K_{p1} = 21$, $K_{p2} = 11$, $K_{p3} = 1$.

Q6.1c The response of PID controller is shown in Figure 4.2 with respect to different values of K_i, where other parameters (u_0, K_p, K_d) are same.

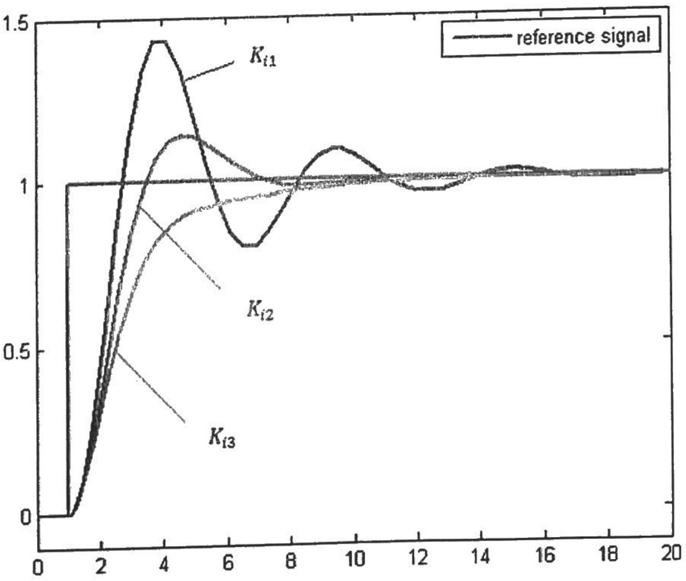

Figure 4.2 PID system responses with different K_i

Please choose the corresponding K_i values for these three responses and state the reason.

(A) $K_{i1}=2$, $K_{i2}=1$, $K_{i3}=0.5$.

(B) $K_{i1}=1$, $K_{i2}=2$, $K_{i3}=0.5$.

(C) $K_{i1}=0.5$, $K_{i2}=1$, $K_{i3}=2$.

(D) $K_{i1}=0.5$, $K_{i2}=2$, $K_{i3}=1$.

Q 6.2 What are the differences between the feedback control and the feed forward control? State the limitations of the feed forward control.

Question 7 Hydraulic model

A simplified downhole wellbore MPD model is given below. Figure 6 shows the diagram of the mass flows and the pressures in the well.

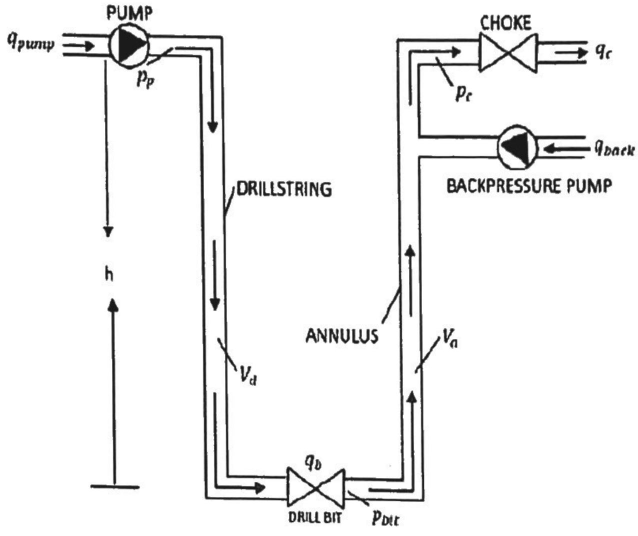

Figure 5 MPD system

The dynamic wellbore model is presented by

$$\begin{cases} \dot{P}_p = \dfrac{\beta_d}{V_d}(q_{pump} - q_b - \dot{V}_d) \\ \dot{P}_c = \dfrac{\beta_a}{V_a}(q_b - q_c + q_{back} + q_{res} - \dot{V}_a) \\ \dot{q}_b = \dfrac{1}{M}(P_p - P_c - \lambda_a q_b^2 - \lambda_d q_b^2 + (\rho_d - \rho_a)gh) \end{cases}$$

and bottom hole pressure can be estimated by

$$P_{bit} = P_c + M_a \dot{q}_b + \rho_a g h + \lambda_a q_b^2$$

where $M = M_a + M_d, M_a = \rho_a \int_0^{l_a} \frac{1}{A_a} dx, M_d = \rho_d \int_0^{l_d} \frac{1}{A_d} dx$. The involved drilling parameters are given in Table 6.

Para.	Description	Unit
V_a	Mud volume in annulus	m^3
V_d	Mud volume in drill string	m^3
β_a	Bulk modulus of mud in annulus	
β_d	Bulk modulus of mud in drill string	
P_c	Choke pressure	bar
P_p	Pump pressure	bar
q_b	Flow rate through the bit	m^3/s
q_c	Flow rate through the choke	m^3/s
q_{back}	Flow rate through the backpressure pump	m^3/s
q_{res}	Flow rate of influx from the reservoir	m^3/s
q_{pump}	Flow rate of the pump	m^3/s
λ_a	Friction parameter of mud in annulus	
λ_d	Friction parameter of mud in drill string	
ρ_a	Mud density in annulus	kg/m^3
ρ_d	Mud density in drill string	kg/m^3
g	Acceleration of gravity	m/s^2
h	Vertical depth of the bit	m
l_a	Length of annulus	m
l_d	Length of drill string	m
A_a	Cross sectional area of annulus	m^2
A_d	Cross sectional area of drill string	m^2
p_{bit}	Bottom hole pressure	bar

Table 6 Model parameters

$q_{out} - q_{in}$

$e = \frac{-10}{K_p}$

$K_p \neq 0$

Consider the choke valve model $q_c = z_c k_c \sqrt{P_c/\rho_a}$, where z_c is the valve opening, and $z_c \in [0,100]$. k_c is valve constant. In the MPD operation, the valve opening is chosen as an input variable. Assume the bottom hole pressure is the output and the reference of bottom hole pressure is given as P_r. Please

Q7.1 Draw its block diagram when PID controller is applied to the system.

Q7.2 What is $\dot{V}_a$? Does $\dot{V}_a$ have an impact on the bottom hole pressure? State the reason.

Q7.3a Suppose only P controller is implemented with the bias z_{c0}, or it can be expressed as

$$z_c(t) = z_{c0} + K_p e(t)$$

where $z_{c0} = 30$. In the above control, the unit of error $e(t)$ is bar. Assume that the bottom hole pressure is stable after some time. Please determine the offsets with respect to the different gains of P controller and fill the table.

K_p	z_c	Offset e(t) (bar)
3	33	
3	36	
10	33	
10	36	

Table 7 P controller

Hint: An offset is an error between the steady state and the desired setpoint.

Q7.3b For such P controller, is it possible to eliminate the offset? Why?

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Question 8 Dual Gradient Drilling

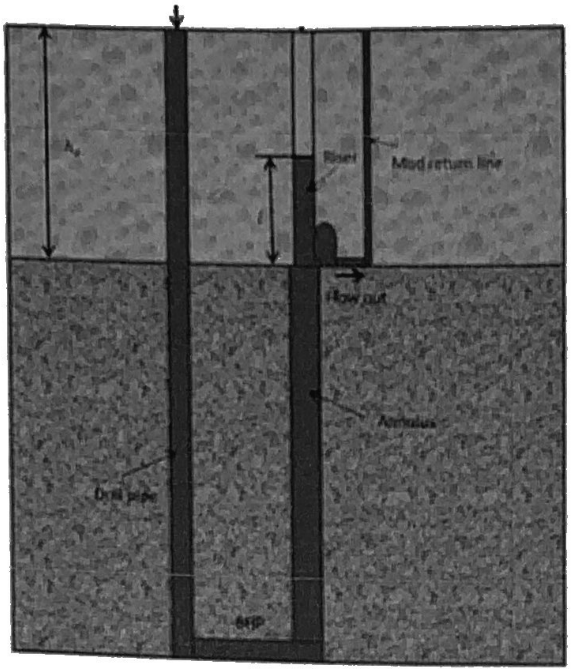

Figure 6 DGD system

A simplified downhole wellbore DGD (dual gradient drilling) system is given in Figure 6. Figure 6 shows the schematic of the system. The mud is injected through the drill string and returns from the annulus. The subsea pump placed on the seabed to suck the mud through the mud return line. In the riser the mud is left to manage the mud level. In the system, the air is above the mud in the riser. The DGD system is to manipulate the mud level to manage the bottom hole pressure. The additional parameters used in the system are given in Table 8.

Parameter	Description	Value	Unit
h_s	Vertical depth from seabed to sea level	500	m
l	Mud level in the riser	200	m
ρ	Mud density in the riser	1200	Kg/m^3
P_1	Hydrostatic pressure of mud in the annulus section	280	bar
g	Acceleration of gravity	9.8	m/s^2

Table 8 Parameter values

Q8.1 It is assumed that it is in the connection. Please calculate the bottom hole pressure based on the above table.

Q8.2 Now it is assumed that the main mud pump is on and pump rate is ramped up to 1000 l/min. Suppose the riser level l does not change. Is the bottom hole pressure increasing or decreasing due to the circulation? State the reason.

Q8.3 During the connection, assume that the bottom hole pressure is 300 bar. Suppose when the pump rate is ramped up to 1000 l/min, the total pressure loss in the annulus and the riser is 10 bar. How would you adjust the riser level l (increase or decrease) in order to manage the bottom hole pressure be stable (300 bar)? Please estimate a new value of l to keep the bottom hole pressure stable at 300 bar.

Question 9 Openlab Drilling Simulator

Q9.1 Back Pressure MPD:

Q9.1a In conventional drilling, the pressure in the well consists of two main components. What are these?

Q9.1b What are the two main properties of the drilling mud that affect the pressure in the well?

Q9.1c What are the basic principles behind Back-Pressure Managed Pressure Drilling? I.e . What extra equipment is needed, and how is the pressure in the well affected?

Q9.1d A flow sweep is when the mud flow rate is gradually reduced in steps and then increased. Explain how the MPD choke need to adjust (open and close) to ensure constant bottom hole pressure during a flow sweep.

Q9.2 Well Control:

Q9.2a What are the primary well barrier and the secondary well barrier to a kick in Managed Pressure Drilling?

Q9.2b In drilling, the pressure in the well is often referred to as "ECD-Equivalent Circulation Density". How do you calculate the ECD from the pressure.

Appendix (formulas)

Drilling fluids

For Newtonian fluids, Reynolds number $Re = \frac{vD\rho}{\mu}$ and the friction factor can be calculated by

$$f = \frac{64}{Re} \quad (Re < 2300)$$

$$\frac{1}{\sqrt{f}} = -1.8log_{10}\left(\left(\frac{\frac{\varepsilon}{D}}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right) \quad (Re \geq 2300).$$

$BHP = \Delta P + P_c + P_f$

In turn, for laminar flow the Reynolds number can be shown by

$$Re_{lam} = \frac{64}{f} \quad (Re_{lam} < 2300).$$

For turbulent flow the Reynolds number is

$$Re_{turb} = \frac{6.9}{10^{\left(\frac{1}{-1.8\sqrt{f}}\right)} - \left(\frac{\frac{\varepsilon}{D}}{3.7}\right)^{1.11}}$$

The friction pressure drop is $\Delta P = \frac{\rho f l v^2}{2D}$.