

# PET 575-Drilling Automation and Modeling

## Question 1 Drilling automation

Figure 1 shows a schematic of managed pressure drilling (MPD) system. In the figure, RT stands for real-time, and PLC is programmable logic controller.

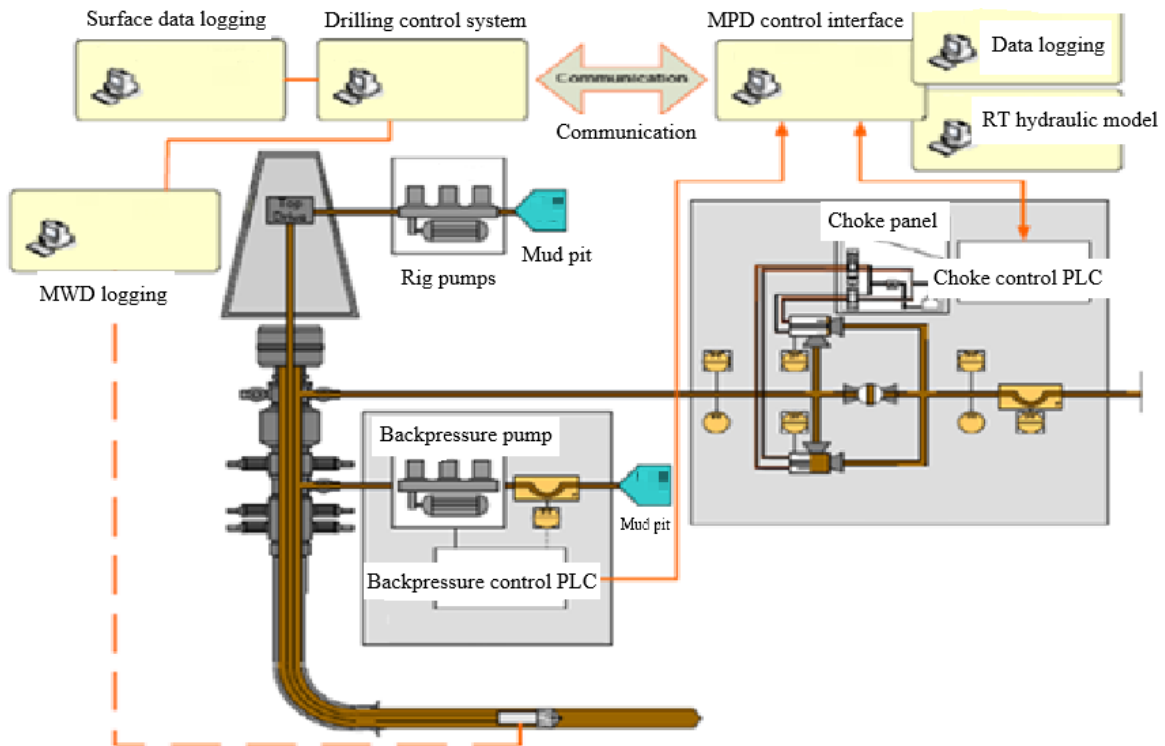


Figure 1 Automated MPD system

Please answer the following questions.

**Q1.1.** What is managed pressure drilling?

**Q1.2.** Describe how the automated MPD system shown in Figure 1 works.

**Q1.3.** List the important automation tools used in the MPD system and describe the functions of automation tools.

**Q1.4.** What is the role of hydraulic model in the system?

## Question 2 Drilling fluids

### Question A

Suppose that a power-law fluid is used with the model

$$\tau = K\gamma^n$$

where  $K$  is the consistency index and  $n$  is the flow behavior index. The fluid flows in a 4 m long pipe with inner diameter of 0.02 m. The values of the parameters involved in the calculation are given in Table 1.

Description	Notation	Values	Unit
Consistency index	$K$	0.46	Poise/s
Flow behavior index	$n$	0.80	
Fluid density	$\rho$	1000	kg/m <sup>3</sup>
Diameter of pipe	$D$	0.02	m
Pipe roughness	$\varepsilon$	0	
Pipe length	$l$	4	m
Flow rate	$q$	$10^{-3}$	m <sup>3</sup> /s

Table 1 Parameters values

Please answer the following questions.

**Q 2A.1.** Calculate the generalized Reynolds number.

**Q2A.2.** Calculate the frictional factor.

**Q2A.3.** Calculate the friction pressure drop.

**Equations:** For power law fluids, generalized Reynolds number is  $Re_g = \frac{v D_{eff} \rho}{\mu_a}$ . Effective diameter

of pipe is  $D_{eff} = \frac{4n}{3n+1} D$  and apparent viscosity is  $\mu_a = K \left( \frac{8v}{D_{eff}} \right)^{n-1}$  and  $v$  is the velocity.

For laminar flow, the friction factor is

$$f = \frac{64}{Re_g} \quad Re_g < 2300$$

For turbulent flow, the friction factor is

$$\frac{1}{\sqrt{f}} \approx -1.8 \log_{10} \left( \left( \frac{\varepsilon}{D_{eff}} \right)^{1.11} + \frac{6.9}{Re_g} \right) \quad Re_g \geq 2300$$

The friction pressure drop

$$\Delta P = \frac{\rho f l v^2}{2D}$$

### Question B

The structure of the drilling fluid property evaluation system is shown in Figure 2. The differential pressure sensors are installed on the pipes in order to automatically evaluate the drilling fluid density and viscosity. DP1 in the figure is the vertical differential pressure between pressure sensors 1 and 2, or

$$DP_1 = P_2 - P_1.$$

DP2 in the figure is the differential pressure between pressure sensors 3 and 4, or

$$DP_2 = P_4 - P_3,$$

where sensors 3 and 4 are installed on the inclined pipe with the inclination angle which is  $30^\circ$ . The length between sensors 1 and 2 is same as the length between sensors 3 and 4, which is 4m. The vertical distance between sensors 3 and 4 is 2m. The inner diameter of the pipe is 0.02m. It is assumed that the friction pressure drop between sensors 1 and 2 is the same as the friction pressure drop between sensors 3 and 4.

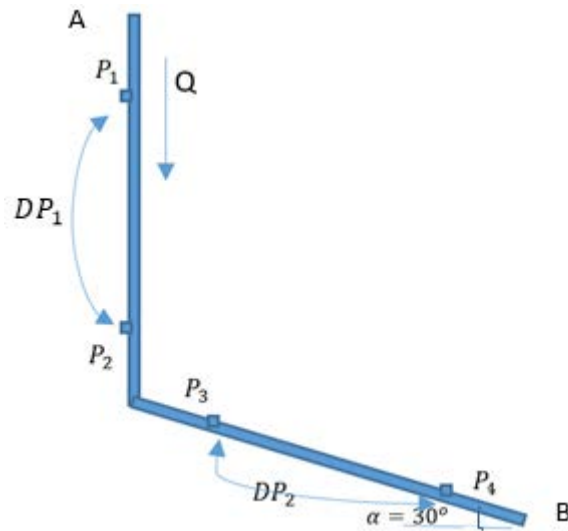


Figure 2 Structure of the system

The values of the parameters involved in the calculation are given in Table 2.

Description	Notation	Values	Unit
Acceleration of gravity	$g$	9.8	m/s <sup>2</sup>
Pipe inner diameter	$D$	0.02	m
Length between sensors 1,2	$l$	4	m
Length between sensors 3,4	$l$	4	m
Vertical depth between sensors 3,4	$h$	2	m
Pipe roughness	$\varepsilon$	0	
Vertical differential pressure between sensor 1, 2	DP1	22700	Pa
Inclined differential pressure between sensor 3, 4	DP2	3100	Pa
Flow rate	$Q$	0.0009	m <sup>3</sup> /s

Table 2 Parameters ' values

**Q 2B.** Suppose that the Newtonian fluid is circulated in the pipe. The circulating direction is from point A to point B. Based on the values given in the table, calculate the density ( $\rho$ ) and the viscosity ( $\mu$ ).

**Equations:** Reynolds number  $Re = \frac{vD\rho}{\mu}$ , the friction pressure drop

$$\Delta P = \frac{\rho f l v^2}{2D}$$

where  $f$  is friction factor and  $v$  is mean velocity. For laminar flow the Reynolds number is given by

$$Re_{lam} = \frac{64}{f} \quad (Re_{lam} < 2300).$$

For turbulent flow the Reynolds number is given by

$$Re_{turb} = \frac{6.9}{10^{\left(\frac{1}{-1.8\sqrt{f}}\right) - \left(\frac{\epsilon}{D}\right)^{1.11}}} (Re_{turb} \geq 2300)$$

### Question 3 Modeling and control

A tank is provided to store the feed liquid. Assume that the plant equipment is operated continuously. Given one tank with three inlets and one outlet. The situation is sketched in Figure 3, with the following notations.

Inlet A injecting the heavy mud with density  $\rho_a$ ; flow rate is defined as  $q_a$ ;

Inlet B injecting light mud with density  $\rho_b$ ; flow rate is defined as  $q_b$ ;

Inlet C injecting the mud that is circulated back from the outlet; flow rate is  $q_c$ . Take into consideration the uncertainty, the density is slightly different with the density of outlet liquid, or

$\rho_c = a * \rho$ , where  $a$  is the constant coefficient.

Outlet circulating the mud with uniform density  $\rho$ ; flow rate is defined as  $q_{out}$ .

The involved parameters are given in Table 3.

Description	Notation
Total flow rate of mud injected from inlets	$q_{in}$
Total flow rate of mud flowing out of outlet	$q_{out}$
Flow rate of mud flowing into inlet A	$q_a$
Flow rate of mud flowing into inlet B	$q_b$
Flow rate of mud flowing into inlet C	$q_c$
Density of mud flowing into inlet A	$\rho_a$
Density of mud flowing into inlet B	$\rho_b$
Density of mud flowing out of outlet	$\rho$
Density of mud flowing into inlet C	$a * \rho$
Cross sectional area of tank	$A$
Liquid level in the tank	$l$
Density of mud in the tank	$\rho$
Mass of mud in the tank	$m$

Table 3 Parameter's notations

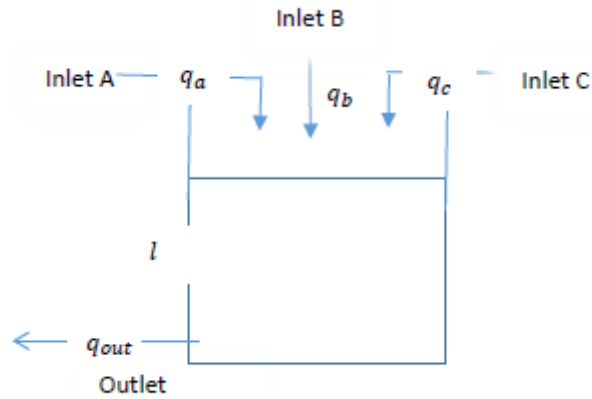


Figure 3 Sketch of tank system

Suppose that the liquid volume in the tank does not change with the time. Please answer the following questions.

**Q 3.1** Develop a dynamic model of the density of the mud in the tank.

**Hint:** Using mass balance.

**Q 3.2** Assume

$$q_{out} = 2000 \text{ l/min} \text{ and } q_c = 1000 \text{ l/min.}$$

What is the value of  $q_a + q_b$ ?

**Q 3.3** The objective is to manage the density of the mud in the tank. The setpoint of mud density in the tank is given,  $\rho_r$ . Assume the mud density in the tank  $\rho$  can be measured and is available all the time. The flow rate of mud flowing from inlet A is selected as an input variable. The PID controller is implemented to the system, which is shown below

$$u(t) = u_0 + K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de(t)}{dt}$$

- (1) Identify the state, input, and output.
- (2) What is  $e(t)$  and how to calculate  $e(t)$ ?
- (3) What is  $u_0$ ? Does  $u_0$  vary with the time  $t$ ?
- (4) What are  $K_p$ ,  $K_i$  and  $K_d$ ?

## Question 4 Drilling data

### Question 4.1 Trend forecasting

The time-based ROP data is given as below

Time(min:hour)	ROP value (m/hour)
01:20	30
02:20	32
03:20	31
04:20	33
05:20	34

Table 4 Realtime ROP data

To predict the ROP trend, the time is coded, which is given in Table 5. For example, coded time 1 represents the original time 01:20.

Coded time (t)	ROP value (m/hour)
1	30
2	32
3	31
4	33
5	34

Table 5 Coded time ROP data

**Q4.1** Please using the linear least squares method to calculate the coefficients  $a$  and  $b$  of the following ROP linear trend model

$$ROP = a + bt$$

where  $t$  is the coded time. Moreover, forecast the ROP trend based on the data given in the table.

**Hint:** (1) Given data  $\{(x_i, y_i), \dots, (x_n, y_n)\}$ , the goal of linear least squares method is to find values of  $a$  and  $b$  of the linear function  $y = ax + b$  that minimize the sum of squared errors  $E(a, b) = \sum_{i=1}^n (y_i - (ax_i + b))^2$ . It requires to find the values of  $(a, b)$  so that

$$\frac{\partial E}{\partial a} = 0, \frac{\partial E}{\partial b} = 0.$$

(2) For a  $2 \times 2$  matrix  $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ , the matrix inverse is  $A^{-1} = \frac{1}{|A|} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} = \frac{1}{a_{11}a_{22} - a_{12}a_{21}} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$ .

### Question 4.2 Outliers

Given the raw data from the measurement

$$x = [3, 3, 2, 60, 4, 3, 2, 2, 20, 3, 3, 1].$$

**Q4.2a** What are outliers?

**Q4.2b** Which filter (mean filter or median filter) provides the better performance of removing outliers? Please show the reason. (Suppose the window size is 5)

## Q5 Flow model

A simplified downhole wellbore MPD model is given below. Figure 4 shows the diagram of the mass flows and the pressures in the well.

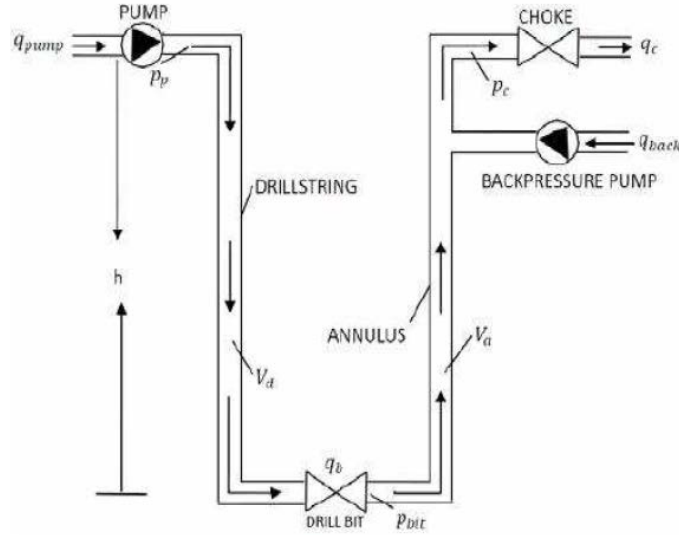


Figure 4 Sketch of tank system

The dynamic wellbore model is presented by

$$\begin{cases} \dot{P}_p = \frac{\beta_d}{V_d} (q_{pump} - q_b - \dot{V}_d) \\ \dot{P}_c = \frac{\beta_a}{V_a} (q_b - q_c + q_{back} + q_{res} - \dot{V}_a) \\ \dot{q}_b = \frac{1}{M} (P_p - P_c - \lambda_a q_b^2 - \lambda_d q_b^2 + (\rho_d - \rho_a)gh) \end{cases}$$

and bottom hole pressure can be estimated by

$$P_{bit} = P_c + M_a \dot{q}_b + \rho_a gh + \lambda_a q_b^2$$

where  $M = M_a + M_d$ ,  $M_a = \rho_a \int_0^{l_a} \frac{1}{A_a} dx$ ,  $M_d = \rho_d \int_0^{l_d} \frac{1}{A_d} dx$ . The involved drilling parameters are given in Table 6.

Para.	Description	Unit
$V_a$	Annulus volume	$m^3$
$V_d$	Drill string volume	$m^3$
$\beta_a$	Bulk modulus of fluid in annulus	bar
$\beta_d$	Bulk modulus of fluid in drill string	bar
$p_c$	Choke pressure	bar
$p_p$	Pump pressure	bar
$q_b$	Flow rate of the bit	$m^3/s$
$q_c$	Flow rate of the choke	$m^3/s$
$q_{back}$	Flow rate of the backpressure pump	$m^3/s$
$q_{res}$	Flow rate of influx from the reservoir	$m^3/s$
$q_{pump}$	Flow rate of the pump	$m^3/s$
$\lambda_a$	Friction parameter of annulus	$bar\ s^2/m^6$
$\lambda_d$	Friction parameter of drill string	$bar\ s^2/m^6$
$\rho_a$	Density mud in annulus	$kg/m^3$
$\rho_d$	Density mud in drill string	$kg/m^3$
$g$	Acceleration of gravity	$m/s^2$
$h$	Vertical depth of the bit	$m$
$\ell_a$	Length of annulus	$m$
$\ell_d$	Length of drill string	$m$
$A_a$	Cross sectional area of annulus	$m^2$
$A_d$	Cross sectional area of drill string	$m^2$
$p_{bit}$	Bottom hole pressure	bar

Table 6 Parameter's notations

Consider the choke valve model  $q_c = z_c k_c \sqrt{P_c / \rho_a}$ , where  $z_c$  is the valve opening, where  $z_c \in [0,100]$ .  $k_c$  is valve constant. In the MPD operation, the valve opening is chosen as an input variable. Assume the bottom hole pressure is the output and the reference of bottom hole pressure is given as  $P_r$ .

**Q5.1** Draw its block diagram when PID controller is applied to the system.

**Q5.2** What is  $\dot{V}_d$ ? Does  $\dot{V}_d$  have an impact on the bottom hole pressure? State the reason.



## Question6 Drill string

**Q6.1** what are the main types of drill string vibrations?

**Q6.2** For an undamped system

$$m\ddot{x}(t) + kx(t) = 0$$

where  $k$  and  $m$  are the stiffness of the spring and the mass respectively.

**Q6.2a** Calculate the natural frequency of such system when  $m = 1, k = 1$ ?

**Q6.2b** When  $m = 2$  and  $k = 1$ , the natural frequency becomes lower or higher? Why?

**Q6.3** For a damped system

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = 0$$

where  $c$  is the damping coefficient. Suppose  $m=1, k=1, c=3$ . Is it the overdamped system, critical damped system or underdamped system? Why?

**Q6.4** For a forced system

$$\ddot{x}(t) + 0.4\dot{x}(t) + 4x(t) = u(t)$$

where  $u(t)$  is the external force. The initial condition is  $x(0) = 0, \dot{x}(0) = 0$ .

Please show its transfer function  $H(s)$  such that  $H(s) = \frac{X(s)}{U(s)}$ .

**Equations** Laplace Transforms when  $x(0) = 0, \dot{x}(0) = 0$  are given below

$x(t)$	$X(s)$
$\dot{x}(t)$	$sX(s)$
$\ddot{x}(t)$	$s^2X(s)$
$u(t)$	$U(s)$

## Q7 Openlab drilling simulator

### Question 7.1 Managed Pressure Drilling (MPD)

- a) What are the main differences with back-pressure MPD compared to conventional drilling?

A well is drilled with back-pressure MPD. During drilling the temperature increases in the larger part of the well.

- b) How does this increase in temperature affect the bottom hole pressure?  
c) What adjustments can be done with the MPD choke opening to uphold a constant bottom hole pressure during the increase in temperature?  
d) In back-pressure MPD operations ECD in the upper part of the well is very large. Why is this?

**Question 7.2 Well Control**

- a) In one of the well control methods the heavier drilling fluid (kill mud) is pumped, while at the same time, the kick is circulated out of the annulus. What is the name of this well control method?
- b) What measurement at the rig is used to control bottom hole pressure during circulation when using this method?
- c) What is the name of an alternative well control method?

**Question 7.3 Cuttings transport**

- a) What are the main drilling parameters that affect the cuttings transport?
- b) Above what inclination can a cuttings bed be formed?
- c) What are two forms of cuttings transport which are desirable for drilling?