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a)

Laplace

$$\Delta p = \frac{2\sigma}{R}$$

Virtual work:

$$\Delta p \, A \, \Delta r = \Delta A \, \sigma$$

$$\Delta p \, 4\pi r^2 \Delta r = \left(4\pi (r+\Delta r)^2 - 4\pi r^2\right)\sigma$$

$$\Rightarrow \underline{\Delta p = 2\frac{\sigma}{R}}$$

b.

$$u_0 = 1.53 \left(\frac{\sigma g (\rho_L - \rho_G)}{\rho_L^2}\right)^{1/4} \quad \text{Rise velocity}$$

$u_0 = 0.25 \text{ m/s} \Rightarrow \underline{\sigma = 7.28 \cdot 10^{-2} \text{ N/m}}$.

Assume effect of water hydrostatic negligible.

$$\Rightarrow \Delta p = 2\frac{\sigma}{R} = \frac{7.28 \cdot 10^{-2}}{5 \cdot 10^{-3}} = 29 \text{ Pa}$$

$$\Rightarrow \underline{p_i = 100\,000 + 29 = 100029 \text{ Pa}}$$

c)

50 m water depth

$\Rightarrow$ pressure $p = 540500\ Pa$

Ideal gas law

$$\Rightarrow \rho_{CO_2}(50m) = \rho_{CO_2}(atm) \cdot \frac{p(50m)}{p(atm)}$$

$$\rho_{CO_2}(50m) = 10.51\ kg/m^3$$

$K \approx 8.75 \cdot 10^{-3}$

$$\Rightarrow \sigma(50m) = 7.09 \cdot 10^{-2}$$

$$\Rightarrow u_0(50m) = 0.248\ m/s$$